Design and Implementation of Auction Agents for Mobile AGent-based Internet Commerce System (MAGICS)

by Jie Zhang

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ABSTRACT

Abstract of thesis entitled “Design and implementation of intelligent auction agents for Mobile AGent-based Internet Commerce System (MAGICS)” submitted by Jie Zhang for the degree of Master of Philosophy at The Hong Kong Polytechnic University in January 2004.

Auction-based electronic commerce (e-commerce) has become very popular in the past few years. Numerous Web-based auction sites have been set up to support both consumer-oriented and business-oriented transactions. To complement the existing Internet auction services, software agents (or simply agents) are expected to become more commonly used to provide sophisticated and fully automated auction services. To achieve this goal, many agent-based auction systems are being developed worldwide. In this thesis, we investigate the design of auction agents for a mobile agent-based auction system called the Consumer-to-Consumer (C2C) Mobile AGent-based Internet Commerce System (MAGICS). Our focus is to design auction agents with different bidding strategies to facilitate e-commerce and mobile commerce (m-commerce). In C2C MAGICS, a bidder can create a mobile auction agent through a proxy server. Equipped with a certain bidding strategy, the mobile auction agent can then move to the required server(s) to carry out the bidding tasks.

Based on the commonly used proxy bidding service, we first investigate a deterministic bidding strategy. Essentially, an agent keeps bidding unless the specified maximum bidding price is exceeded. To analyze the bidding strategy, a general mathematical model is set up to evaluate the bidding results. Furthermore,
we have studied two types of distributions for the maximum bidding price namely: linear and normal distributions. Some simulations have been conducted to validate the correctness of the mathematical model and to study the behavior of the system.

As an extension of the deterministic bidding strategy, we propose a probabilistic bidding strategy. In essence, a generic willingness function is used to specify the bidding probability for each price. In fact, it can be used to cover the deterministic bidding strategy as well. Based on the willingness function, we have developed a mathematical model to compute the bidding results for both of the popular English and Dutch auctions. In particular, close form mathematical expressions have been obtained for some cases for analysis purposes. Furthermore, four willingness functions are defined to cover some possible bidding approaches. Simulation and experimental results are presented to validate the analytical results and to analyze the bidding results.

Finally, we propose a Backward InDuction Strategy (BIDS) for enabling an agent to bid effectively in multiple, concurrent auctions. Based on the concept of an auction chain, BIDS seeks to maximize the value of a utility function by solving a backward induction equation recursively. By doing so, an agent can determine whether to bid in the next available auction. Note that it may be better to stop bidding if the current utility (i.e., the utility associated with the current bid price) is less than the expected utility of all of the subsequent auctions. Simulation results demonstrate the advantages of BIDS over other strategies, particularly in the ability of BIDS to achieve a higher winning probability and a greater expected utility.
Some research results of this thesis have been published/presented as follows:


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CHAPTER 1
INTRODUCTION

1.1 Introduction to Auctions

Auctions have been used as a mechanism to match buyers and sellers for a very long time. For example, in Rome, auctions were used by sellers to market their goods. The first book about auction was written in Britain in the 1600’s [11]. The most general auction method, “first-price open price auction”, is also called the English auction. Simply stated, an auction is a method of allocating goods that are either scarce or difficult to evaluate based on competition [35]. In a market, a seller wants to sell an item at the highest possible price and the buyers wish to obtain the item at the lowest possible price. An auction helps the seller to identify the buyer who is willing to pay the highest price. Furthermore, the buying price of the item can be determined by the buyers instead of the sellers. This means that sellers can push the burden of pricing to the market. However, sellers may also control prices by choosing an appropriate auction type and by setting a lowest (or “reserve”) selling price.

There are various types of auctions. The bidding prices in an auction can be either ascending or descending and they can be either public or private [17]. An auction can also be classified based on the number of items sold (single-item auctions or multi-item auctions). Different auctions have their own characteristics and the most suitable type of auction for selling something
depends on many factors. Among them are time to sell the item, the cost of the item and the characteristics of the buyers.

With the advent of the Internet, auctions are nowadays widely used for electronic commerce [22]. With online auctions, users could buy/sell items in various regions of the world. Compared to traditional auctions, online auctions bring greater convenience while dramatically decreasing the transaction cost. However, they have some shortcomings that do not exist in traditional auction markets.

The organization of the rest of this chapter is as follows. Section 1.2 describes different types of auctions and 1.3 describes their characteristics. Section 1.4 introduces the online auctions and some famous online auction services. Section 1.5 concludes this chapter.

### 1.2 Types of Auction

There are many ways to classify auctions. According to the bidding information, auctions can be divided into open-auctions and closed auctions. Based on the variation of prices, we have ascending price auctions and descending price auctions. The difference between single-item auction and multi-item auction is the quantity of items to be sold. Auctions can also be classified into one-sided auctions and double auctions. Figure 1.1 shows the different types of auctions [9].

Auction-based electronic commerce has become very popular in the past few years [30]. Numerous auction sites have been set up to carry out different kinds of auctions. We describe several common types of auctions in detail as follows. For simplicity, we assume that the seller also acts as the auctioneer.
1.2.1 Several classic auctions

**English auction**

In an English auction, there are one seller and many buyers [35]. The seller sets a reserve price and deadline that are disclosed to buyers and a lowest acceptable price that is known only to the seller and auctioneer. The price is successively raised from the reserve price until only one bidder remains. That bidder wins the item at the final price provided that the final price is not less than the lowest acceptable price and deadline has not been reached. The auction can be run by having the seller announcing prices, the bidders calling out prices, or bids submitted electronically with the best current bid posted.

**Dutch auction**

A Dutch auction works in the opposite way. First the seller sets a reserved price, a decremental price and a private lowest acceptable price [30]. The auctioneer starts at the reserved price, and then lowers the price continuously.
The first bidder who accepts the current price wins the item provided that it is not less than the lowest acceptable price.

First-price sealed-bid auction (FPSB)

This auction has a deadline and each bidder independently submits a single bid before the deadline without seeing others’ bids. The item is sold to the bidder who places the highest bid [30]. Same as the above, the winning price must be equal to or larger than the lowest acceptable price.

Vickrey auction

Vickrey auction is a famous type of auction that is similar to FPSB [30]. The only difference between them is that the bidder who makes the highest bid gets the item at the second-highest bid, or the “second price”. The Vickrey auction has a so-called “truth-telling” characteristic. That is, bidders tend to submit bids based on their own value of the item.

Yankee auction

A Yankee auction can be viewed as a generalized type of the English auction because it works in a similar manner but caters for the bidding for multiple items [9]. Basically, the seller allocates items to the buyers according to the descending order of their bid prices until all items are sold out. Bidders with a higher bid will be served first. Each bidder pays what they bid plus the number of items to be bought.

Double auction

All the classic auctions we have introduced above are one-sided, in that a single seller (or buyer) accepts bids from multiple buyers (or sellers) to bid to exchange a designated commodity. The continuous double auction (CDA)
matches buyers and sellers immediately on detection of compatible bids whereas a periodic version of the double auction, which is also termed a call market or clearinghouse, collects bids over a specified interval of time and then clears the market at the expiration of the bidding interval [35]. The most common rules are $k_s$-th-price and ($k_s+1$)-th-price rules. Consider a set of $k_{total}$ single-item bids, of which $k_s$ are selling offers and the remaining $k_{b} = k_{total} - k_s$ are buying offers. The $k_s$-th-price auction clearing rule sets the price at the $k_s$-th highest among all the $k_{total}$ bids while ($k_s+1$)-th-price rule chooses the price of the ($k_s+1$)-th bid. The sellers whose bids are lower than the price will transact with buyers outbidding the price.

1.2.2 Single-item auction and multi-item auction

Auctions can also be classified into single-item auctions and multi-item auctions, according to the number of items sold. All auction rules described above only apply to selling one item. Rules for multi-item auctions are more complicated since the quantity requirement of each bidder may be different [35]. The English auction, Dutch auction and sealed-bid auction also support the selling of multi-items as described below.

In English and sealed-bid auctions, a bidder is asked to submit $k$ bids, where $b_i^1 \geq b_i^2 \geq ... \geq b_i^k$, to indicate how much he/she is willing to pay for each additional item. Thus $b_i^1$ is the amount that the bidder is $i$ willing to pay for one item, $b_i^1 + b_i^2$ is the amount he/she is willing to pay for two items and so on. Given that there are $j$ items to sell in an auction, they will be sold to the buyers with the highest $j$ bids. As an example, consider a situation in which there are six items to be sold to three bidders and the submitted bid vectors are:
Bidder 1: (50,47,40,32,15,5)

Bidder 2: (42,28,20,12,7,3)

Bidder 3: (45,35,24,14,9,6)

In this case, the six highest bids are

\[ (b_1^1, b_2^1, b_1^3, b_2^1, b_1^2) = (50,47,45,42,40,35) \]

Consequently bidder 1 is awarded three items, bidder 2 is awarded one item, and bidder 3 is awarded two items. However, the price that the bidders pay for each item depends on the pricing rules. The most common pricing rules are the 

*discriminatory* and *uniform-price* rules. In a discriminatory auction, each bidder pays an amount equal to the sum of his bids that are deemed to be winning—that is, the sum of his bids that are among the \(j\) highest of all bids submitted in all. Formally, if exactly \(k^i\) of the \(i\)-th bidder’s \(j\) bids \(b_k^i\) are among the \(j\) highest of all bids received, then \(i\) pays

\[ \sum_{k=1}^{k^i} b_k^i \]

In a uniform-price auction, all \(j\) items are sold at a “market-clearing” price such that the total amount demanded is equal to the total amount supplied, where all items are sold at the lowest winning price. That is, \(j\) items will be sold at the \(j\)-th highest price. This scheme encourages bidders to bid more.

In multi-item Dutch auction, the auctioneer announces the price decreasingly and an item is sold to a bidder who agrees to accept the current price. The auction is over when all items are sold.


1.3 Auctions Characteristics

1.3.1 Auction theorems

As introduced above, there exist various auctions in the market. However, the key differences among different auctions are related to the following [22]:

- **Anonymity**: Different information is disclosed during the auction process. For example, sealed-bid auctions are more anonymous than other types of auctions. In sealed-bid auction, only the identity of the final winner is disclosed and all the bids are kept secret. In Dutch auctions, we can know the bids of winners and their identities. In English auctions all bids are public.

- **Rules for ending an auction**: English auction may end at a predefined closing time. Alternatively, they may also end on the condition that no new bids are submitted within a certain time period. Dutch auctions always end by a new bid or when the price decreases to a predefined price. Sealed-bid auctions have a definite deadline.

- **Payment amount**: When an auction is over, the winner must pay for the item. However, the payment amount is not always equal to the winner’s bid. In Discriminative Auctions (i.e., Yankee auctions), a winner always pays what he/she bids. In Non Discriminative Auctions, however, each winner pays the lowest bid among all winners. In Vickrey auction, the winner pays the second highest bid rather than the highest bid.

- **Restrictions on bid amount**: In all auctions, the seller can specify the bidding parameters. In English auction, the seller typically sets the
minimum bidding and the minimum incremental price. In Dutch auctions, the maximum price and the decreasing price are set. A private reserve price can also be specified. Of course, it is confidential to the bidders.

Although these auction protocols are somewhat different, Vickrey discovers the following theorems that can basically be applied to most auctions [35]:

Revenue-Equivalence Theorem

(1) *English auctions, Dutch auctions and sealed-bid auction achieve the same expected revenue, if all bidders are risk-neutral, their valuation of the item follows the same distributions and they all evaluate the item independently. All bidders only know their own value and the distribution of other bidder’s valuation.*

(2) *In Vickrey auctions, it is a weakly dominant strategy for the bidder to bid based on its own valuation, i.e., all bidders tend to bid based on their real valuation of the item.*

Because of their particular characteristics, each auction may preponderate over all others in different situations. Vickrey auctions encourage bidders to bid based on the true value of the item. In Dutch auctions, the seller could change the decreasing price at any moment so that they could control the pace of the auction process. This is especially applicable for selling perishable items such as vegetables, meats. Dutch auctions were first used in the Dutch fish markets. Dutch auctions also discourage the formation of “rings” (see later explanation). In Dutch auction, a ring member still obtains benefits if he/she breaks away from the ring. However, if a ring member leaves the ring in an English auction, it will
not get any benefit and the operation of ring will not be influenced. Thus the English auction encourages the formation of rings. In comparison with other auctions, English auctions are more transparent and are usually favored by governments. Since Internet auctions do not suffer from constraints of time and location, English auctions are also preferred because of their transparency. For Dutch auctions, it is necessary that the bidders should gather at a common location at the same time. For English auctions, as a bidder should submit more than one bid, he/she should check the bidding status frequently. However, it is not convenient when the bidder is busy or the network is congested. Sealed-bid auctions, where a bidder submits one bid before the deadline, overcome these drawbacks. There is no need to keep track of the auction status.

In practice, open cry auctions are usually used for selling one item. If multiple items are to be sold, they are sold one at a time. This is acceptable in the physical world because each item sells fast, and it is impractical to take multiple bids for the items simultaneously. On the Internet, one can sell multiple items simultaneously. This is also necessary in some cases because an auction may take a longer time. Therefore, we expect to see an increase in the use of auctions for multiple items for facilitating e-commerce.

### 1.3.2 Tricks in auctions

Auctions are actually a zero-sum game between bidders and auctioneers. Some dishonest bidders may commit frauds to maximize their chance of winning an auction [22].
Bidding collusion

In an English auction, a set of bidders may collude to form a ring, where the members of the ring agree not to outbid each other. At the end of the auction, if the item is won by a ring member, it is resold among the ring members in a separate auction, or through some other allocation procedure. The surplus created in the second sale is a loss inflicted on the seller, which is divided among ring members. The Internet makes the formation of rings much easier.

Shills

The auctioneer may also commit frauds to increase its profit. It may estimate the reserve price of the highest bidder and pretend to be one of the bidders. That bidder will submit bids increasingly until the highest bidder reaches its reserve price. That spurious bidder is called a “shill”.

In general, bidder collusions and shills are illegal in commerce. However, there is no “cyber law” to punish these behaviors.

Winner’s curse

“Winner’s curse” means that the winning bid is always greater than the product’s market valuation. It is especially serious when the item is of private-valuation.

1.3.3 Auction bidders

Risk-averse and risk-neutral bidders

In an auction, each bidder may have a different objective. As we know, the winning probability should increase as the benefit to the bidder decreases. According to the tradeoff between winning probability and benefit, the bidders
can be divided into two categories: Risk-neutral and Risk-averse [35]. Risk-neutrality bidders always seek to maximize their expected profits while risk-averse bidders prefer to optimize the winning probability. Compared to risk-neutral bidders, the risk-averse bidders may submit a higher bid to increase their chance of the winning.

Private value and common value

Let us consider the following. Given an auction with one item to sell, and there are $N$ potential bidders. Bidder $i$ assigns a value of $X_i$ to the object—the maximum amount a bidder is willing to pay for the object. If each $X_i$ is independently valuated by the bidders, the valuation of the item is called private value. If the valuation of the item can be influenced by other factors, such as the valuation of other bidders, the number of items in the current market, the valuation is called common value. For example, the valuation of stocks is usually based on common value, which may fluctuate with the market situation and the amount of stocks held by other people.

If all the valuations of the item are of private value and they are identically distributed within the interval $[0, \omega]$ according to an increasing distribution function $F$, this auction is symmetric and all bidders are symmetric bidders. Otherwise, the auction is asymmetric.

1.4 Online auctions

1.4.1 Advantages of online auctions

With the advent of the Internet, more and more auction sites are appearing, such as www.ebay.com and www.onsale.com. The popularity of online auctions
continues to rise. EBay is one of several commercial sites that run user-created auctions. It claims to be transacting nearly $2 million a week. You can buy nearly everything you want on this C2C website. Onsale is the first and most prominent of the seller-oriented online auctions. It reported a gross revenue for the second quarter of 1997 of $18.6 million—a 50% increase over the previous quarter [30]. It is estimated that, up until now there are about four million people participating in online auctions among the 400 hundred auction web sites. Online auctions have been viewed as one of the most important Internet applications, like e-mail.

Generally speaking, online auctions have the following advantages over traditional auctions:

- Online auctions are more economical than traditional auctions. In a traditional market, an auctioneer has to pay considerable overhead costs to run the auctions. However, the cost of running online auctions is relatively negligible.
- Online auctions are not constrained by geographical distance. Users can easily go to each auction site. Furthermore, auction information can be updated instantly.
- Online auctions can operate 24 hours a day [22], while the traditional market cannot.
1.4.2 Challenges of online auctions

As mentioned above, the Internet greatly facilitates trading between people, especially those distributed in different areas. However, there are also some problems that do not exist in the physical market [22].

In order to protect online auctions from being disturbed, auction sites should provide an access control mechanism. For example, a seller could specify that the auction is only for registered buyers. Furthermore, some security measures should be taken to ensure that an auction site can not be attacked by hackers. These include preventing unauthorized postings as well as denial-of-service attacks. In physical auctions, bidders can notify the auctioneers of their bids directly such that no one can interfere with them. However, bidding information may be intercepted and modified by other people when biddings takes place on the Internet. Thus cryptographic mechanisms are needed to ensure that a bid submitted is not tampered with, or disclosed to other bidders such that the auction rules are violated.

In English auctions, some spurious bids may be created by the seller or auctioneer to prompt the bidders to further increase their bids. Technically, someone who does this is called a “shill”. Such false bid may occur frequently if the auctioneer knows the highest bidder’s reserve price. While false bids also exist in physical auctions, the problem is greatly aggravated because the bidders in online auctions are usually geographically dispersed and the cyber identities can be created easily. The possible solution to this problem is called caveat emptor, which requires mechanisms to let bidders determine the identity of other
bidders. For example an independent third party trust rating system can be used to assign a trustworthiness rating to the participants.

When an auction is over, the seller and buyer should complete the deal. To ensure the execution of the contract, a trusted third party service is needed for authentication and validation purposes.

1.5 Scope and Objectives

With so many advantages, it is expected that online auctions will become even more popular in the future. To facilitate users to place bids in auctions, agent technology has been employed in many auction sites. A number of agent-based systems have also been developed to help users to place bids automatically and effectively. As a prevalent bidding strategy, the Proxy Bidding Service is provided by many auction sites (i.e., eBay). It allows users to specify their maximum bidding prices so that the proxy can keep bidding unless the specified maximum bidding prices are exceeded. As the popularity of this deterministic proxy bidding strategy in online auction systems, there is a need to analyze its behavior. In Chapter 3, we analyze the proxy bidding strategy accordingly. In particular, we investigate two prediction schemes to estimate the distribution of the user reserve prices. This allows an agent to predict the winning probability in an auction, which is a very important issue related to the design of bidding strategies.

In the real world, sometimes users may not be certain whether to submit a bid. They may just have a certain degree of willingness to bid at a certain price. However, the proxy bidding strategy cannot be used to handle this situation. Therefore, we propose a probabilistic bidding strategy (i.e., bidding based on a
willingness function). Bidders can use different willingness functions to instruct their auction agents to bid based on their preferences. In fact, the proxy (deterministic) bidding strategy can be considered as a special case of the probabilistic bidding strategy. In particular, it can be used to effectively support mobile auctions.

The above two strategies deal with biddings in a single auction. However, with the development of online auctions, it is common to find many auctions selling the same item concurrently. Consequently, it is of interest to develop bidding strategies for multiple auctions. To achieve this goal, we propose a backward induction bidding strategy (BIDS) for users to bid in multiple concurrent heterogeneous auctions.

1.6 Organization of the Thesis

The organization of the rest of this thesis is as follows:

- Chapter 2 describes some agent-based online systems and the research on various bidding strategies

- Chapter 3 analyzes the general proxy bidding strategy and introduces some prediction schemes. A generic mathematical model is formulated to analyze the proxy bidding strategy. Based on the mathematical model, two prediction schemes have been put forward. With the previous auction records, they can estimate the winning probability in an auction based on certain parameters.

- Chapter 4 presents the probabilistic bidding strategy. The probabilistic bidding strategy is based on a willingness function. A mathematical
model has been formulated to analyze the bidding results for both the English and Dutch auction methods.

- Chapter 5 investigates BIDS, a bidding strategy, for supporting multiple concurrent auctions. A series of experiments have been conducted to evaluate the performance of BIDS in comparison with some other schemes.

- Chapter 6 concludes the thesis.

The organization of this thesis is shown in Fig. 1.2.

![Figure 1.2: Organizations of this thesis](image)
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

Chapter 1 described the background of the project. Although existing online auctions provide many advantages, they also have some shortcomings. Agent technologies can be used to solve some of these problems and they could greatly facilitate online auctions. With the development of agent-mediated electronic commerce, many agent-enabled auction sites have been set up and are expected to be popular in the future. One of the vital problems in agent-mediated online auctions is the design of the agent’s bidding strategy, which could greatly affect the performance of an agent-based system. At present, only a few auction sites support intelligent agents, which are implemented by means of simple bidding strategies. The design of bidding strategies is a complicated problem and they need to be adapted to different system environments and user requirements. In the past several years, many researchers are working on this issue.

The organization of this chapter is as follows. Section 2.2 briefly describes the basics of agents and their advantages for online auctions. Several famous agent-based online auction systems are also introduced in section 2.3. In section 2.4, we give an overview of some bidding strategies proposed by various researchers. Section 2.5 summarizes this chapter.
2.2 Agent-based online auction

Software agent technologies have been developing rapidly in the past several years. As said by Guilfoyle (1995), in ten years time, most new IT development will be affected, and many consumer products will contain embedded agent-based systems [24]. What is a software agent? An agent is a program that can delegate a task. The difference between “traditional software” and an agent is that an agent is personalized, continuously running and semi-autonomous [30]. These features make agents useful for a wide variety of information and process management tasks.

The first idea to develop machine independent executable messages could be traced back to the early days of AI work. Carl Hewitt has put forward a concurrent actor model in 1977, in which the concept of a self-contained, interactive and concurrently executing object called an actor was proposed.

Nowadays agents have many more characteristics. Autonomy, learning and cooperation are the basic attributes. Autonomy means that agents can operate on their owns without the need of human guidance. An agent is cooperative because it can work with other agents to complete a task. Some agents can even automatically learn information from the outside environment to enhance their performance. Agents with all of the above three characteristics are considered to be intelligent [36]. There are also several other attributes to classify agents. One of them is mobility. After initiation, some agents only work in the original server, whereas other agents can move across computers over a network. These agents are called mobile agents [13].
The agent technology is quite useful in online auctions [30]. Currently, most popular auction sites only provide a human interface. To bid for an item, users have to search on the Internet, find the items they want and send the bid message themselves. Unlike auctions in physical world which only lasts for a short time and bidders can know the auction result quickly, online auctions usually last for a longer time, from 2-3 days to even several weeks. It is quite inconvenient for users to monitor the bidding situation and respond quickly. Agent technology is a good way to facilitate online auctions. Agents can stay alive in an auction site, observing the fluctuation of prices and providing responses in a timely manner. Other than bidding in just one auction, mobile agents could even automatically wander on the Internet to search for appropriate auctions to participate. With the learning ability, intelligent agents can even perform better than humans in the aspect of negotiation. Intelligent agents can collect the records of all previous auctions and analyze them quickly. With good bidding strategies and rich history information, agents can perform better than human beings. Furthermore, with high computation power, intelligent agents can give much faster responses than human beings.

2.3 Several agent-based online auction systems

Development of the Internet has spurred a number of attempts to create virtual marketplaces. However, only human’s interfaces are provided by concurrent online marketplaces. With the development of agent technology, auction sites with agent interfaces have been available. Some of the famous auction sites are described as follows:
eBay and Onsale

Besides a user interface, eBay and Onsale also provide some simple agent-like interfaces [30]. For example, Onsale provides a proxy agent called BidWatcher, which can automatically bid on the user’s behalf. Before bidding in an English auction, the user tells the agent the maximum price he/she is willing to pay and the BidWatcher will automatically bid the minimum price possible to win the auction without exceeding the user’s assigned price ceiling. Given an English auction with current price $c$, minimum incremental price $d$ and the user’s maximum bidding price $l$, the agent will always bid the price $c+d$ if it is no higher than $l$. Otherwise, it will not submit any bid. However, proxy agents provided by the auction sites are not secure and trustful. The maximum bidding price may be disclosed to the auction site and a shill bidder can be created.

AuctionBot

Michigan AuctionBot (http://auction.eecs.umich.edu/) was developed by the Artificial Intelligence Laboratory at University of Michigan. It has been available for public use at the University of Michigan since September 1996 and to the entire Internet since January 1997 [27]. In particular, it has been used for selling used textbooks. AuctionBot can handle many auctions simultaneously and all its auctions are organized in a hierarchical structure. Sellers can choose the catalog in a systematic manner and create an auction at whatever layer he/she wants. One of the most significant features of AuctionBot is the parameterization of auctions. After analyzing the characteristics of all auctions, they decompose the auction design space into a set of orthogonal parameters such as “Bidding restrictions”, “Auction Event”, “Information Revelation” and “Allocation Policies”. With these parameters, many classic auction types can be supported by
setting the appropriate parameters. Furthermore, this kind of parameterization is so flexible that it can support auctions that do not exist at present. This parameterization method greatly enhances the scalability of the auction site. Figure 2.1 shows the structure of AuctionBot. Besides user interface, AuctionBot also provides agent interface. The database stored parameters of all auctions, which be accessed by users and agents through interfaces by certain rules. The agent interface provided by AuctionBot is a TCP/IP-level message protocol that allows agents to access all the features of the AuctionBot present in the web systems. Agents can place bids, create auctions, request auction information or review their accounts. The API provided by AuctionBot is self-contained such that developers can use it to build their own front end to AuctionBot. A backend decision program runs cyclically, which is up to conclude imminent auctions and make final decisions. When the auction is over, results will be notified to users through emails. However, AuctionBot provides only an information service. It collects bids, determines the results of the auction by using a well-defined set of auction rules, and notifies the participants. No actual money exchange is executed.

Figure 2.1: Structure of AuctionBot
Kasbah

Kasbah is a Web-based multi-agent system where users can create buying agents and selling agents to trade goods [2]. Generally these agents automate the Merchant Brokering and Negotiation stages of the consumer buying process. That is, agents can find all potential agents and negotiate with them automatically. Agents in Kasbah are not smart because no AI or machine learning techniques are used. However, the most interesting feature of Kasbah is its multi-agent aspect, that is, agents in Kasbah can interact and compete with each other. There are totally three kinds of agents in Kasbah: market agents, selling agents and buying agents. In Kasbah, there is a market agent, which is responsible for managing the selling agents and buying agents. All other agents are grouped according to their interests, that is, the items they would like to buy/sell. Once a buying/selling agent joins Kasbah, the market agent will add the agent into the corresponding group and other agents in the group will receive a notification message simultaneously. After receiving the message, the buying/selling agents will treat this new agent as a potential trader. Similarly, departure messages are sent by the market agent once an agent leaves the market. Each agent is allowed to perform its action in one “slice” of execution time in each market place “cycle”. During an execution time, an agent can negotiate with other agents. However, only one agent can be contacted within a time slice. If all potential agents have been inquired, the seller/buyer agent decreases/increases its offer with a certain strategy. Three strategies are available: linear, quadratic and cubic as shown in Figure 1.3. They represent three types of agent behavior. After changing the offer, a selling/buying agent first inquires the buying/selling agent with the highest/lowest price in the previous round.
Nomad

EAuctionHouse is another auction site. It is integrated with a mobile agent-based system called Nomad [32]. Besides the ordinary Web interface, EAuctionHouse supports two additional access mechanisms. A user can send a formatted text string directly through a TCP/IP connection or use Nomad to create agents for bidding on the user’s behalf. Nomad is composed of an agent generator, an agent dock, an agent database and a web interface. Two kinds of agents are provided in Nomad: tailored agents and template agents. The tailored agents are written by users and sent to the “Agent dock” through a TCP/IP connection. Nomad also provides several standard agents for users to create them via a Web interface. Several kinds of template agents are provided by Nomad: information agents, incrementor agents, N-agents and control agents. The Information agent can monitor an auction and notify the user by mail when a specified event occurs. The incrementor agent implements the dominant bidding
strategy in single English auctions. The N-agents is designed for FPSB auctions, which implements the Nash equilibrium strategy given the distribution of the agent valuation. Both user-written agents and standard agents are available in the agent dock. They can communicate with eAuctionHouse through TCP/IP connections using a predefined protocol.

**MAGMA**

The above auction sites only focus on the Brokering and Negotiation steps of the buying process. MAGMA is an auction site, which focuses on the whole buying process, covering also the purchase step [23].

In MAGMA, agents are not necessarily situated at the same location. They can be local, functionally independent and communicate with each other through socket connections. In other words, they can communicate using a common communication layer and transmit messages using a packet that is something like an IP packet consisting of a header and a body. A relay server is used to facilitate communications between the agents. It maintains all socket connections and routes messages between agents based on the agent names. There is also an advertising server that stores information of items in the market and can reply to requests generated from the trader agents. Another component of the market is the bank, which contains accounts of all trader agents. It can handle money transfer and check deposit. In MAGMA, a “trader agent” has two different states: buying and selling. Each agent is associated with a wallet, an inventory, an ad manager and a negotiator. The wallet stores the monetary information including the banking accounts, check, tokens and so on. In the purchase step, the wallet sends a payment request to the bank and receives the corresponding check. The inventory stores the information of items to be sold and the ad manager is
responsible for advertising issues. The negotiator is to negotiate with the counter-agent. It has two modes: manual mode and automatic mode. Using these two modes, it integrates with the user-interface and the agent interface. MAGMA is designed to be an open standard, allowing platform and language independent agents that conform to the MAGMA messaging API to connect to the system, register with the relay server, and conduct businesses over the MAGMA infrastructure.

**BiddingBot**

![Diagram of BiddingBot](image)

**Figure 2.3: BiddingBot**

Different from the above auction sites, BiddingBot is a multi-agent system that supports users in attending, monitoring, and bidding in multiple auctions [27]. Since there exists multiple auctions selling the same item simultaneously, it is more beneficial to bid in multiple auctions and select the one with the lowest price. However, it may result in “accidental purchase” (i.e., two or more auction agents win). In BiddingBot, agents can communicate with each other so as to avoid “accidental purchase”. BiddingBot is composed of a leader agent and
several bidder agents. Each bidder agent can bid in an auction site and the leader agent is for monitoring the behavior of the bidder agents. Before submitting a bid, the bidder agent sends a request message to the leader agent. The leader agent accepts the bid if there are no agents holding an active bid or the active bids can be withdrawn. In BiddingBot, users always buy the item with the lowest price through BiddingBot and thus the “Winner’s curse” issue can be avoided.

2.4 Agent Bidding Strategies

The design of bidding strategies is related to game theory, where each bidder only knows partial information and tries to win the game with minimum loss. A strategy could depend on various factors, such as other bidders’ strategies, the seller’s minimum accepting price, the number of auctions in the future. However, the information that an agent knows is incomplete while it is required to make a decision within a short time [38].

Many auction sites now provide agent-like services, which are implemented with predefined and non-adaptive negotiation strategies. When bidding in an English auction in eBay, a user can create a simple agent by inputting its maximum bidding price [30]. The agent can automatically submit the bid, which is equal to the sum of the current price and the minimum incremental price. In Kasbah, three bidding strategies can be chosen as shown in Figure 1.3 to bid in a continuous double auction: anxious, cool-headed and frugal. They respectively correspond to linear, quadratic and exponential functions for generating proposals/counter-proposals. The above strategies are quite simple and straightforward. However, they are non-adaptive. In other
words, they do not make use of any available information and do not adapt to change in the market situation.

K. M. Wong and E. Wong proposed a bidding strategy for continuous double auctions [18]. Unlike the above non-adaptive bidding strategies, this strategy is “market-driven”, that is, the strategy is changed when the market situation varies. Before bidding in the market, the user should notify the agent of its preference, the threshold of each attribute, the maximum bidding price $M$ and the user’s eagerness to trade $G$. After entering the market, the agent firstly searches for all counter-offers and evaluates them. The agent only negotiates with the count-offer with the maximum valuation. In the negotiation process, the market situation is evaluated before making an offer. For example, a seller agent considers the market situation at time $t$ as the following four factors:

i. the number of seller agents $S_t$;

ii. the number of buyer agents $B_t$;

iii. the number of agents interested in the agent’s current offer $I_t$;

iv. closing time of the marketplace $T$.

Using the above value, the agent could get the “remaining market time” $T(t)$, “competitions” $C(t)$ and “attractiveness” $A(t)$, which is calculated as:

$$T(t) = \frac{T - t}{T}$$  \hspace{1cm} (2.1)

$$C(t) = \frac{B_t}{B_t + S_t}$$  \hspace{1cm} (2.2)

$$A(t) = \frac{I_t}{B_t}$$  \hspace{1cm} (2.3)
The market situation at time $t$, which is denoted as $M(t)$ is:

$$M(t) = 1 - \frac{T(t) + C(t) + A(t)}{3} \quad (2.4)$$

Therefore, the price that the buyer agent should submit at time $t+1$, $b_{t+1}$ is

$$b_{t+1} = \frac{M(t) + E}{2} \times M \quad (2.5)$$

Using this strategy, bids submitted by sellers is dependent on the market situation. With long remaining time, more buyer agents may be interested in the offer, so the seller agent tends to give a higher offer and vice versa.

Another negotiation strategy in continuous double auctions has been put forward by W. Y. Wong et al. [38]. This strategy applies the Case-Based Reasoning (CBR) techniques to capture and reuse the previously successful negotiation experiences. Given a counter agent, the strategy divides the negotiation process into a number of decisions making episodes for evaluating an offer, determining strategies and generating a counter-offer as shown in Figure 2.4:

![Figure 2.4: Negotiation process [38]](image)

The offer in each episode is created by an episode strategy, which can be changed from one to another. An episode strategy is represented by a concession.
scheme. Given a series of offers: \((O_1, O_2, O_3, O_4, O_5)\), the concession in \(i+1\)-th episode \(C(i+1)\) is represented as follows:

\[
C(i + 1) = \frac{O_{i+1} - O_i}{O_i} \times 100\%
\]  

(2.6)

Thus a bidding process can be represented as a series of concession ratio. The agent would store the concession series of successful negotiation experience.

In the agent’s database, the past auction records are firstly classified into three categories according to the user’s objective: “Must-buy”, “Good-deal”, “Best-price”. The second level records some attributes of the items. When negotiating, the agent searches for all past negotiation records with the same objective and similar item attributes, and matches the concession series with those of the past records (see Figure 2.5). The one with the maximum similarity is selected and its corresponding concession ratio is used as the concession ratio of the current episode.

![Match figure](image)

Figure 2.5: Match figure [38]

With the development of electronic commerce, it is often possible to find auctions selling similar goods on the Web. However, the above strategies only consider a single auction, which may be inefficient in two aspects:

i. For a buyer, it may pay more for an item than its real value, which is called the “winner’s curse”.
ii. A seller may fail to make a deal in an auction site.

The above issues can be addressed by considering multiple simultaneous auctions. By bidding in simultaneous auctions, buying agents can compare the price of different auctions and then bid in the best auction. Selling agents could also find the buyer with a higher offer thus achieving higher profits and higher success transaction ratio. Furthermore, with more participants employing multiple bidding strategies, the market can be more efficient and the equilibrium price can be achieved.

Because of the above advantages, many researchers also investigate bidding strategies for multiple auctions. The design of multiple-site bidding strategies is more complicated than single-site bidding strategies because of the complicated environment. Besides the bidding price, a multiple-site bidding strategy should also select the appropriate auctions to participate. It is even more complicated if the agent needs to buy multiple items, because the number of bids should also be decided.

P. Anthony et al. proposed a multiple bidding strategy, which can be applied for English, Dutch and Vickrey auctions simultaneously [26]. Before bidding in the market, four parameters are assigned to the agent: the user’s maximum bidding price, the latest time to obtain an item, the desire indicator etc. In a bidding process, the agent decides the current maximum bidding price based on the above parameters and other market attributes. Four tactics can be used in this strategy, namely “the remaining time tactic”, “the remaining auctions tactic”, “the desire for bargain tactic” and “the desperateness tactic”. For instance, the remaining time tactic determines the recommended bid value based on the length
of residual time for the auction. The bid value at time $t$, $f_{rt}$ is calculated by the following expression:

$$f_{rt} = \alpha_{rt}(t) \times M$$

(2.7)

where $\alpha_{rt}(t)$ is a polynomial of the form:

$$\alpha_{rt}(t) = k_{rt} + (1 - k_{rt}) \left( \frac{t}{T} \right)^{\frac{1}{\beta}}$$

(2.8)

In (2.8), $k_{rt}$ and $\beta$ are constant values, $M$ is the maximum bidding price set by the user and $T$ is the deadline to obtain the item.

The remaining time tactic reflects the current maximum bidding price of the agent, taking into consideration of the remaining trading time. The other three tactics respectively calculate the maximum bidding price based on the remaining number of auctions in the market, the user’s desire and the user’s desperateness. The final maximum bidding price is a combination of the four prices with a certain weight assigned by the user. Given a maximum bidding price $b_t$ at time $t$, the agent only considers the opening auctions that satisfy the following conditions:

i. English auctions with imminent deadlines and minimum bidding prices higher than $b_t$

ii. Dutch auctions with current price no higher than $b_t$

iii. Vickrey auctions with imminent deadlines

Among all the potential auctions, the agent calculates the expected utility of each auction and bids in the auction with the maximum utility value. For an English auction, the agent will bid up to $b_t$; For a Dutch auction, the agent will
submit a bid once the price decreases to $b_t$. For a Vickrey auction, the agent will submit a bid at $b_t$.

The performance of this strategy depends on ten parameters: four user parameters, four weights, $k_r$ and $\beta$. A genetic algorithm has been applied to find the optimum set of parameters under different market situations and bidding objectives [25]. The algorithm is summarized as follows (see [25] for details):

```
Randomly create initial bidder populations;
While not (Stopping Criterion) do
    Calculate fitness of each individual by running the market place 2000 times;
    Create new population
    Select the fittest individuals (HP);
    Create a “mating pool” for the remaining population;
    Perform crossover and mutation in the mating pool to create new generation (SF);
    New generation is HP+SF;
End While
```

After running for 2,000 times, a set of stable parameters were obtained, which is the optimum set of parameters in the given bidding situation. The paper inspected four market situations: short bidding time and a small number of active auctions in the marketplace (STLA); short bidding time and large number of active auctions (STMA); long bidding time and small number of active auctions in the marketplace (LTLA); long bidding time and large number of active auctions in the marketplace (LTMA). Three different utility functions are used, which represent the preference of users. Having classified the market situation into multiple categories, the optimum parameters of each situation could be determined [25]. The intelligent agent could always assign the appropriate parameters according to the suitable market situations.
Another bidding strategy based on dynamic programming has been proposed by A. Byde [1]. The strategy applies for English auctions for one item. The price is increased by a fixed amount and one of the bidding agents is randomly selected to be the highest bidder. If there are no agents submitting bids in a given time step, the item should be awarded to the agent with the highest bid and the auction closes. Three bidding algorithms namely “GREEDY”, “HISTORIAN” and “OPTIMAL” are put forward. Agents using the “GREEDY” algorithm always bid with the lowest current price without taking other issues into consideration. For the “HISTORIAN” algorithm, an agent bids with the highest expected utility. For the “OPTIMAL” algorithm, an agent is assigned a market state according to the current auctions in the market. Since each auction may change to another state with a certain transitional probability, the market situation transitional probability takes into account the corresponding transitional probabilities of each auction. A reward is assigned to the transition from state $A$ to state $B$, which is calculated to be the total value minus the payment for the items bought for the transition and the expected payments on active bids in state $B$. Since the state space is finite and a-cyclic, the value of each state can be determined by an inductive computation process and the agent should bid in the auction that maximizes the value. Experiments showed that the “OPTIMAL” algorithm outperform the others while the simple “GREEDY” algorithm gives the worst performance.

Some researchers also investigate a bidding strategy for agents to bid multiple items in multiple simultaneous English auctions [4], [5]. The strategy is composed of two sub-algorithms, which are respectively used to decide how to allocate multiple bids in the current active auctions and whether to leave the
current bidding auction for another auction with a lower bidding price. All auctions in the market are discriminatory multi-item English auctions, that is, winner should buy the item with what he/she bid. Given an English auction selling \( j \) items, the optimum strategy for the agent is to outbid the \( j \)-th highest price in order to achieve the highest utility. If an agent bids \( k \) items in an \( j \)-item English auction, the corresponding optimum strategy is to outbid the \((j-k+1)\)-th price for \( k \) times. If an agent holds one or more active bids in \( N_a \) auctions and it wants to get a total of \( N_T \) items, the first rule gives out the best allocation of the other \((N_T-N_a)\) bids. For each auction, a beatable-\( j \) list is calculated, where \( 1 \leq j \leq N_T \). The beatable-\( j \) list consists of the lowest \( j \) active bids excluding the agent’s own \( k \) bids. To bid \( j \) items in the auctions, the optimum strategy is to submit the bid which is equal to the sum of the \( j \)-th price in the beatable-\( j \) list and the incremental price for \( j+k \) times. Using a depth-first searching method, the agent can easily find out the best allocation of the \( N_T-N_a \) bids.

The above mechanism is optimum when all English auctions end simultaneously. However, auctions may also terminate at different times. Given two auctions, one with a higher price and imminent deadline and the other one with a lower price and longer deadline, the agent needs to choose which auction to participate. The mechanism proposed in the paper combines a simple learning algorithm with the utility theory. That is, it firstly works out a winning probability function \( B(x,q) \) based on several learning techniques. \( B(x,q) \) indicates the probability that \( x \) bidders value the good with a valuation greater than \( q \) in a given auction. With a bid price \( q \), an agent can win with probability \( 1-B(j,q) \) in the auction with \( j \) items to sell. Based on the winning probability, the utility in both auctions can be easily worked out. For the auction with imminent deadline,
the utility is $U_1 = M - q$, where $M$ is maximum bidding price and $q$ is the agent’s bidding price. For the auction with a longer deadline, its utility is represented as follows:

$$U_2 = \sum_{q=0}^{V} (B(j, q - 1) - B(j, q)) \times (M - q)$$  \hspace{1cm} (2.9)$$

Given $U_1 > U_2$, the agent should bid the first auction up to price $M - U_2$. Otherwise, it should move jump to the second auction. Using this strategy, the agent can achieve optimal or near-optimal purchase decisions.

Marlon Dumas et al. proposed a bidding strategy to buy a single item in heterogeneous auctions including English auction, FPSB auction and Vickrey auction [19], [20]. All English auctions in the market have predefined deadlines. Before an agent entering the market, it is assigned the maximum bidding price $M$, the deadline to leave the market $T$ and the minimum expected probability of obtaining the item $G$. It is assumed that it takes $L$ time units to execute a transaction so two auctions can be participated by the agent sequentially only if their deadlines are separated by at least $L$ time units. In the bidding phase, the bidding agent selects a set of auctions $A_s$ and a bidding price $b_t$, where $b_t \leq M$. The following conditions should also be satisfied:

- The deadlines of auctions in $A_s$ are non-conflicting. That is, the agent could sequentially participate in all auctions in $A_s$.

- Denote the probability that at least one of the selected bids succeeds as $P(A_s, b_t)$, $P(A_s, b_t)$ should be no lower than the expected winning probability $G$. That is:

$$P(A_s, b_t) = 1 - \prod_{a \in A_s} (1 - P_a(b_t)) \geq G$$  \hspace{1cm} (2.10)$$
where \( P_a(b_t) \) is the probability of winning auction \( a \) with a maximum bidding price \( b_t \).

- The bidding price \( b_t \) is the lowest one among all possible prices that satisfy the above two conditions.

Given the optimum auction set and the bidding price \( b_t \), the agent successively places bids in each of the selected auctions until one of them is successful. In case of sealed-bid auctions, the agent places a bid at price \( b_t \). For English auctions, the agent bids at \( b_t \) just before the auction closes since last-minute bidding works best for English auctions.

**Several prediction methods**

In the previous bidding strategies, the winning probability function has been widely used and several schemes to predict the winning probability have been put forward. The histogram method and the normal method are the most popular methods [20].

The histogram method assumes that the winning probability with price \( z \) in a given FPSB auction or English auction with current price zero is the same as the ratio of number of the times that an agent can win with price \( z \) in the past auctions to the number of all past auctions. Denote \( P(x) \) as the probability that the past auction price is \( x \). Thus \( P(x) \) is equal to the number of previous auctions with final price \( x \), divided by the number of all past auctions. For example, if the sequence of observed final prices is (22, 20, 25), the winning probability with price \( z \) in auction \( a \) with zero current price is:
If it is known that the winning price of the auction is no lower than price $q$, the corresponding winning probability should be revised to:

$$P_a(z) = \frac{\sum_{q \leq x \leq z} P(f_{p_a} = x)}{\sum_{x \leq q} P(f_{p_a} = x)}$$  \hspace{1cm} (2.12)$$

where $z \geq q$. Otherwise, $P_a(z) = 0$.

The histogram method is straightforward. However, it has two disadvantages. First, the computation of winning probability is dependent on the size of the set of past auctions. Second, if the minimum winning price is higher than all final prices of the past auctions, the winning probability cannot be found. Compared to the histogram method, the normal method addresses the above two drawbacks.

The normal method assumes that the distribution of winning price follows a normal distribution with mean $\mu$ and variance $\sigma$. Thus the winning probability with price $z$ in an auction $a$ with zero minimum price is:

$$P_a(z) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{z-\mu} e^{\frac{-x^2}{2\sigma^2}} dx$$ \hspace{1cm} (2.13)$$

The winning probability of auction $a$ with minimum price $q$ is:

$$P_a(z) = \frac{\int_{q-\mu}^{z-\mu} e^{\frac{-x^2}{2\sigma^2}} dx}{\int_{-\infty}^{\infty} e^{\frac{-x^2}{2\sigma^2}} dx}$$ \hspace{1cm} (2.14)$$
However, the winning price in English auctions and Vickrey auctions reflects the maximum bidding price of the second highest bidder instead of the maximum bidding price of the winner. A simulation algorithm can be used to produce the maximum bidding price of the highest bidder. It first examines all the maximum bidding price of the failed bidders and then figures out the distribution of the maximum bidding price. For each previous auction, a series of numbers are produced according to this distribution and the first one that equals to or higher than the final price of that auction is assigned as the maximum bidding price of winner. After processing all auction records, the right distribution can be worked out easily.

Another prediction method based on machine learning has been put forward by R. E. Schapire [29]. The learning problem can be viewed as a conditional-density-estimation problem: given current conditions, estimate the conditional distribution of prices. A boosting-based algorithm is proposed. Abstractly, given pairs \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), where each \(x_i\) belongs to a space \(X\) and each \(y_i\) is in \(R\). The \(x_i\)'s are the auction-specific feature vectors for some \(n, X \subseteq (R \cup \{\bot\})^n\). Each target quantity \(y_i\) is the difference between the closing price and the current price. Given a new \(x\), the goal of the algorithm is to estimate the conditional distribution of \(y\) given \(x\). The algorithm first divides the value of \(y\) into equal-length bins:

\[
[b_0,b_1),(b_1,b_2),(b_2,b_3),\ldots,[b_k,b_{k+1})
\]

where \(b_0 < b_1 < b_2 < \ldots < b_k < b_{k+1}\)

The purpose of the algorithm is to estimate the probability that a new \(y\) (given \(x\)) will be at least \(b_j\). Given the estimate \(p_i\) for each \(b_j\), the probability that
$y$ is in the bin $[b_j, b_{j+1})$ is $p_{j+1} - p_j$. A logistic regression algorithm based on boosting techniques is applied to calculate $p_i$. First a real-valued function $f : X \times \{1, 2, \ldots, k\} \to R$ is set up with the interpretation that

$$\frac{1}{1 + \exp(-f(x, j))}$$

(2.15)

This is the estimate of the probability that $y \geq b_j$, given $x$. The boosting-like algorithm is used to find out the function $f$ that minimizes

$$\sum_{i=1}^{m} \sum_{j=1}^{k} \ln \left(1 + e^{-s_j(y_i)f(x, j)}\right)$$

(2.16)

where $s_j(y) = \begin{cases} +1 & \text{if } y \geq b_j \\ -1 & \text{if } y < b_j \end{cases}$

### 2.5 Summary

In this chapter, we have given an overview of agent-based auction systems. The use of agent technology in online auctions not only save user’s time but also can enhance the bidding efficiency. The key issue in designing an agent-based auction system is the development of bidding strategies. Several bidding strategies for agent-based online auctions have been introduced in the chapter. Generally, the bidding strategies can be divided into two categories: single-site bidding strategies and multiple-site bidding strategies. Unlike single-site bidding strategies, the multiple-site bidding strategies can be used in handling multiple heterogeneous auctions. As many bidding strategies rely on the prediction of winning probabilities, several simple prediction schemes have also been described.
CHAPTER 3

DETERMINISTIC BIDDING STRATEGY AND WINNING PRICE PREDICTION

3.1 Introduction

As mentioned previously that one of the fundamental issues in designing agent-based auction services is to develop an effective bidding strategy; in other words, how to instruct an agent to place bids on behalf of the user. Kasbah allows users to specify a negotiation strategy in terms of a price-time function [2]. AuctionBot provides an application programming interface for users to develop customized agents [27]. In addition, there are also bidding strategies for handling auctions at multiple sites; e.g., [26], [1].

Many auction sites such as eBay provide a proxy bidding service, which can be viewed as a simple bidding strategy [30]. This proxy bidding strategy allows a user to specify a maximum bid price called the maximum bidding price, such that the user can keep bidding automatically unless the specified maximum bidding price is exceeded. This popular proxy bidding strategy can also be extended to mobile agent-based auction systems. In this case, an agent is instructed to continue bidding up to a certain price. In fact, the mobile agent-based approach has an advantage over the traditional proxy bidding service because the maximum bidding price can be kept secret (i.e., the auctioneer cannot know the maximum bidding price). Due to the popularity of the proxy bidding strategy, there is a need to analyze its behavior.
In this chapter, we first analyze the general proxy bidding strategy for an auction system called C2C MAGICS. The analysis provides valuable insights into the design of auction services in general and the agent-based auction systems in particular. Based on the distribution of the maximum bidding price or the reserve price, the model can be used to determine the probability of stopping at a particular price and the expected winning price. Simulation and analytical results are presented to evaluate the system's behavior. To perform the analysis, it is important to obtain the winning probability in an auction for a certain maximum bidding price. For the proxy bidding strategy, two prediction schemes are used to estimate the distribution of maximum bidding prices from the distribution of winning prices. These two schemes are the method of moments and the maximum likelihood estimation. Some experiments have been done to evaluate the performance of the two schemes and compare them with other prediction schemes. Experimental results indicate that the method of moment and maximum likelihood estimator work more effectively than other schemes.

The organization of the remaining sections of this chapter is as follows. Section 3.2 describes the system architecture of the mobile agent-based auction system. Section 3.3 formulates the mathematical model to evaluate the bidding results and discusses the simulation and analytical results. Section 3.4 presents the prediction methods and evaluates its performance. Section 3.5 concludes this chapter.
3.2 System Architecture

Figure 3.1: System architectures

Figure 3.1 shows the architecture of the mobile agent-based auction system as presented in [9]. There are five sub-systems: The access module is for processing incoming mobile agents. The auction protocol module contains the auction rules to support different types of auctions. The backend system stores the necessary information (e.g., user information). The administrative sub-system is for management purposes. Finally, there are other supporting systems, such as payment gateways to handle payment functions. The above auction system is called C2C MAGICS, which is part of the Mobile Agent-based Internet Commerce System (MAGICS). The aim is to facilitate C2C e-commerce by using mobile agents to complement the current Web-based e-commerce system. Unlike other auction sites, the task of bidding can be delegated completely to a mobile agent. Users can instruct their agents to bid using a particular bidding strategy.
As many user terminals may not be able to support mobile agents, a proxy server is provided for users to create the required agents. In fact, the proxy server can be used to create other mobile agents for different purposes (e.g., to create some agents to search for a desirable product or to compare the prices of a product). Users can access the proxy server through a Web or WAP interface. To create an agent, a user needs to input the bidding information including the reserve price (i.e., the maximum bidding price). The agent searches for the respective auction server and moves to the respective “auction space” to bid for the item with other agents. The aforementioned bidding strategy is used in which each agent keeps bidding up to the specified reserve price. In the following analysis, we consider using either the English or Dutch auction method to buy and sell one item.
3.3 Market Analysis

3.3.1 Mathematical model

In this section, an analytical model is formulated to evaluate the bidding results for both the English and Dutch auctions. These include computing the distribution of the winning price and, hence, the average winning price. The model is summarized as follows.

There are a total of \( n \) agents participating in an English/Dutch auction. The possible prices are \( b_0, b_1, b_2, \ldots b_d \). The incremental/decremental price for the English/Dutch auction is \( h \), so we have \( b_i=b_{i-1}+h \) \((i=1,2,3,\ldots,d)\). Note that for the English and Dutch auctions, the bidding prices are: \((b_0, b_1, b_2, \ldots)\) and \((b_d, b_{d-1}, b_{d-2}, \ldots)\), respectively. Note that the price \( b_0 \) and \( b_1 \) are so low that each agent is able to bid and the initial price \( b_d \) in Dutch auction is so high that no agents could accept it. Each agent is provided with a maximum bidding price, which follows a distribution \( R \). In other words, the probability that the winning price does not exceed \( b_i \) is \( R(b_i) \). In general, each agent always gives a bid, provided that the bidding price does not exceed its maximum bidding price. Denote \( F(b_i) \) as the probability that the winning price does not exceed \( b_i \). Note that in the later analysis, an appropriate subscript will be added to identify the auction type: English (E) or Dutch (D). Based on \( R(b_i) \) and \( n \), the objective of the following analysis is to determine \( F(b_i) \) as well as the average winning price. A number of distributions for the maximum bidding prices will be considered.

English auction

For the English auction, the bidding prices keep rising until no one submits a higher bid. Each submitted bid should be the current highest price, plus the
incremental price \( h \). Finally, the item is sold to the bidder who submits the highest bid. Given the distribution of the reserve price and the number of agents, we can compute the distribution of the final price and the expected winning price \( A_E \) as follows.

As we know, if more than one agent has a reserve price higher than \( b_i \), no matter which agent bid price \( b_i \), at least one agent would outbid it. And the final winning price must be larger than \( b_i \). Two possible cases with winning price \( b_i \) are considered here:

Case 1: The reserve price of all \( n \) agents does not exceed \( b_i \), so no one will bid higher than \( b_i \). This case occurs with probability \( R^n(b_i) \).

Case 2: If there is an agent \( t \) with reserve price larger than \( b_i \), it will certainly be the winner. In the case that there are \( i \) \((i=0,1,\ldots, n-1)\) agent(s) with reserve price \( b_i \), the final price does not exceed \( b_i \) only when \( t \) is the winner. This case occurs with probability

\[
\sum_{i=0}^{n-1} \frac{1}{i+1} \left( \binom{n-1}{i} R^{n-1-i}(b_{i-1}) (R(b_i) - R(b_{i-1})) \right)^i.
\]

Integrating the above cases, we get the distribution of the winning price \( F(b_i) \) as follows:

\[
F_E(b_i) = R^n(b_i) + n(1-R(b_i)) \sum_{i=0}^{n-1} \frac{1}{i+1} \left( \binom{n-1}{i} R^{n-1-i}(b_{i-1}) (R(b_i) - R(b_{i-1})) \right)^i
\]

\[=
R^n(b_i) + \frac{1-R(b_i)}{R(b_i)-R(b_{i-1})} (R^n(b_i) - R^n(b_{i-1}) \right) \]

It is not difficult to see that with probability \( F_E(b_i) - F_E(b_{i-1}) \), the winning price will stop at \( b_i \). Thus we could get the auction’s average winning price \( A_E \) as:
If the reserve prices of the agents have upper limit $b_d$, the average winning price can be transferred as follows:

$$A_E = \sum_{i=1}^{\infty} b_i \times (F_E(b_i) - F_E(b_{i-1})) = \sum_{i=1}^{\infty} (b_0 + ih)(F_E(b_i) - F_E(b_{i-1})) = b_0 + h\sum_{i=0}^{d-1} F_E(b_i) = b_d - h\sum_{i=0}^{d-1} F_E(b_i)$$  \hspace{1cm} (3.2)$$

$$A_E = \sum_{i=1}^{d} (b_0 + ih)(F_E(b_i) - F_E(b_{i-1})) = b_0 + hd - h\sum_{i=0}^{d-1} F_E(b_i) = b_d - h\sum_{i=0}^{d-1} F_E(b_i)$$  \hspace{1cm} (3.3)$$

**Dutch auction**

A Dutch auction works in the opposite way from English auction. Here we consider that only one item is sold. First the auctioneer announces a very high price $b_d$ and then decreases the price gradually by $h$ each time. The first bidder who accepts the price becomes the winner and the item is sold at the accepted price. Given the distribution of the reserve price and the number of agents, we can compute the distribution of the final price and the expected winning price $A_D$ as follows

The analysis for a Dutch auction is simpler than that for an English auction. According to the strategy, the winning price in a Dutch auction is equal to the highest reserve price among all agents. If the winning price is equal to or lower than $b_i$, the reserve price of all $n$ buyers must not exceed $b_i$. Thus, we can easily get the distribution of the winning price as

$$F_D(b_i) = R^\alpha(b_i)$$  \hspace{1cm} (3.4)$$

Similar to above, the average winning price can be computed as follows:

$$A_D = \sum_{i=0}^{d} b_i \times (F_D(b_i) - F_D(b_{i-1})) = \sum_{i=0}^{d} b_i \times (R^\alpha(b_i) - R^\alpha(b_{i-1})) = b_d - h\sum_{i=0}^{d-1} R^\alpha(b_i)$$  \hspace{1cm} (3.5)$$
In the following sub-sections, we will use the above model to evaluate the bidding results for several linear distributions for the reserve price. More specifically, the distribution of the winning price and the expected winning price will be computed.

Here we assume that the reserve price set by a user follows a linear distribution between $b_l$ and $b_r$. The density functions of several linear distributions $H(b)$ are shown in Figure 3.3, where $b_r$ should be larger than $b_l$. Assume the $H(b_l)$ and $H(b_r)$ is $c_1$ and $c_2$, respectively. According to the property of density function, we could easily get that $c_1 + c_2 = \frac{2}{b_r - b_l}$. Therefore, the linear distribution can be modified as uniform distribution and linear increasing distribution and linear decreasing distribution by adjusting the value of $c_1$.

Thus, the density function of the reserve price is:
\[ H(b) = \begin{cases} 0 & \text{if } b < b_i \\ \frac{2c_i}{(b_i - b_j)^2} - \frac{2c_i}{b_i - b_j} (b - b_j) + c_i & \text{if } b_i \leq b \leq b_j \\ 1 & \text{if } b > b_j \end{cases} \] (3.6)

We can easily get the distribution of the reserve price as follows:

\[ R(b) = \begin{cases} 0 & \text{if } b < b_i \\ \frac{(b - b_j)^2}{(b_r - b_i)^2} + \frac{c_i (b - b_i)(b_r - b)}{b_r - b_i} & \text{if } b_i \leq b \leq b_r \\ 1 & \text{if } b > b_r \end{cases} \] (3.7)

Based on (3.1), the distribution of the winning price in an English auction is:

\[
F_E(b_i) = \left( \frac{(b_i - b_j)^2}{(b_r - b_i)^2} + \frac{c_i (b_i - b_j)(b_r - b)}{b_r - b_i} \right)^n + \left( \frac{(b_i - b_j)^2}{h(b_i - b_r)} - \frac{(b_i - b_j)^2 - c_i (b_i - b_j)(b_r - b_i)h}{2h(b_i - b_r) - h^2 + c_i h(b_i - b_j)(h + b_r + b_i - 2b_j)} \right)^n
\]

\[
= \left( \left( \frac{(b_i - b_j)^2}{(b_r - b_i)^2} + \frac{c_i (b_i - b_j)(b_r - b)}{b_r - b_i} \right)^n - \frac{(b_i - b_i - b_j)^2}{(b_r - b_i)^2} + \frac{c_i (b_i - b_i - b_j)(b_r - b_i + h)}{b_r - b_i} \right)^n
\] (3.8)

where \( b_i \leq b_i \leq b_r \)

Note that when \( b_i < b_j \) or \( b_j > b_r \), we have \( F(b_j) = 0 \) and \( F(b_r) = 1 \), respectively.

This applies to the following cases as well.

The average winning price in English auction is

\[ A_E = b_0 + hE - h \sum_{i=0}^{c-1} F_E^n(b_i) = b_r - h \sum_{i=0}^{c-1} F_E^n(b_i) \] (3.9)

For a Dutch auction, the distribution of the winning price is

\[ F_D(b_i) = \left( \frac{(b - b_j)^2}{(b_r - b_i)^2} + \frac{c_i (b - b_i)(b_r - b)}{b_r - b_i} \right)^n \] (3.10)

where \( b_i \leq b_i \leq b_r \)

Hence, the average winning price is
\[ A_D = b_x - h \sum_{i=1}^{n-1} \left( \frac{(b_i - b_j)^2}{(b_x - b_i)^2} + \frac{c_i(b_i - b_x)(b_x - b_i)}{b_x - b_i} \right) \]  

(3.11)

### 3.3.2 Experimental Results

![Figure 3.6: Probability of stopping at a particular price for an English auction with ten bidders](image1)

![Figure 3.7: Probability of stopping at a particular price for a Dutch auction with ten bidders](image2)

We have applied the above model to carry out some analysis. Furthermore,
we have also validated the analytical results by means of simulations. For the simulations, a Java program has been written to simulate the auction process. Each simulation was repeated 100,000 times to obtain the steady state results.

Figure 3.6 and Figure 3.7 show the probability of stopping at a particular price for English and Dutch auctions, respectively with \( n=10 \). As shown in the previous section, the probability can be found as \( F(b_i)-F(b_{i-1}) \). For the linear distribution, we set \( c_1 = c_2 \) so that it is actually a uniform distribution with minimum and maximum reserve prices set at $150 and $250, respectively. The mean and variance of the normal distribution are $150 and $50, respectively. Figure 3.6 shows the distribution of the winning price when ten agents participated in an English auction. The analytical and simulation results are almost the same, thus indicating the correctness of both models. Figure 3.7 shows the distribution of the winning price for the Dutch auction with the same set of simulation parameters as the English auction. It can be seen that the Dutch auction method tends to give a higher winning price.

Figure 3.8: Probability of stopping at a particular price for an English auction with a uniform distribution for the reserve price
In the following, we would compare the winning price of auctions given the agents’ reserve prices follow several linear distributions and normal distribution. To be fair, all distributions have the same means and variations.

Figure 3.8 shows the probability of stopping at a particular price for the English auction when the reserve price follows a uniform distribution and the number of agents varies. The minimum and maximum bidding price is $100 and $200, respectively. The distribution becomes flattened at the high prices when the number of agents is small. However, with more agents participating in the auction, the auction is likely to end close to the maximum price (i.e., peaking at around $200). This is because, as more agents participate in an auction, the probability of having a reserve price close to $200 increases. Figures 3.9 and 3.10 show the probability of stopping at a particular price when linear increasing distribution and linear decreasing distribution are used for the reserve price. For Figure 3.9, we set $c_1 = c_2/4$ and the agents’ minimum reserve price and maximum reserve price are $86$ and $192$, respectively. For the linear decreasing distribution, the minimum reserve price and maximum reserve price of are set as $107$ and $214$ and $c_1 = 4c_2$. While they show similar effects as Figure 3.8, the variations are slightly different. The curves with the linear increasing distribution change dramatically, while the curves with the linear decreasing distribution vary slowly. This is due to the nature of the distributions. The linear increasing distribution is likely to get a higher reserve price. In contrast, the reserve price for the linear decreasing distribution is likely to be lower. Note that the distribution of the winning price is affected by the reserve price.
Figure 3.9: Probability of stopping at a particular price for an English auction with a linear increasing distribution for the reserve price

Figure 3.10: Probability of stopping at a particular price for an English auction with a linear decreasing distribution for the reserve price
Figures 3.11 show the probability of stopping at a particular price when the agents’ reserve price follows a normal distribution, with mean and variance of $150 and $28.87, respectively. It can be seen that the winning probability follows
a normal distribution. With more agents bidding in the auction, the average winning probability increases while the variation decreases. Also, the peaks of the curve become larger when there are more competing agents in the auction.

Figure 3.13: Probability of stopping at a particular price for a Dutch auction with a linear increasing distribution for the reserve price

Figure 3.14: Probability of stopping at a particular price for a Dutch auction with a linear decreasing distribution for the reserve price
Figure 3.15: Probability of stopping at a particular price for a Dutch auction with normal distribution for the reserve price

Figures 3.12, 3.13, 3.14 and 3.15 show the probability of stopping at a particular price for a Dutch auction when the reserve price follows the uniform, linear increasing, linear decreasing and normal distributions, respectively. The parameters are the same with those for English auctions. In general, they are similar to the graphs for an English auction.

Next, we analyze the relationship between the average winning price and the distribution of the reserve price. Figure 3.16 shows the average winning price for English and Dutch auctions when the number of agents varies. From the graphs, it can be seen that the average winning price increases towards the maximum price as more agents participate in the auctions. It can be seen that the average winning price for the four distributions are close to each other. However, with more agents participate in the auction, the average winning price for the normal distribution is the highest and the average winning price for the linear increasing distribution tends to be the lowest.
Furthermore, we compare the average winning price in English and Dutch auctions. As shown in Figure 3.16, the average winning price for an English auction is generally lower than that for a Dutch auction. This is because for English auctions, the winning price is the second highest reserve price or the second highest reserve price plus the incremental price. For Dutch auctions, the winning price is always the highest reserve price. However, the difference is greatly reduced as more agents participate in the auction.

![Figure 3.16: Comparison of the average winning price for English and Dutch auctions](image)

3.4 Prediction of winning probability

3.4.1 Several prediction schemes

In the above section, we have analyzed the general proxy bidding strategy and computed the distribution of winning prices. A related issue is how to get the winning probability for a certain maximum bidding price. To calculate the probability, we must have the distribution of maximum bidding prices.
Unfortunately, it is unknown in the real world. So, for the proxy bidding strategy, two prediction schemes are used to predict the distribution of maximum bidding prices from the distribution of winning prices. They are the method of moments and the maximum likelihood estimation.

To perform the analysis, we define the notations as follows:

- \( A \) is the set of previous auction records.
- \(|A|\) is the number of auction records.
- \( a_i \) is the \( i \)-th auction in the auction set \( A \).
- \( x(a) \) indicates the winning price of auction \( a \).
- \( N(a) \) gives the number of agents participating in auction \( a \).
- \( A_i \) is the set of auction records with \( i \) agents participating in the auction.
- \( F_c(b_z) \) gives the winning probability of the current auction \( c \) with reserve price \( b_z \).

The aim of a prediction scheme is to compute \( F_c(b_z) \) for each reserve price \( b_z \).

**Histogram method**

The histogram method can deal with FPSB auction or English auction. The key assumption of the method is that the distribution of the winning price depends on the historical frequencies. It means that the winning probability with price \( z \) in an initial auction is the same as the ratio of number of the times that the agent could win with price \( z \) in the past auctions to the number of all past auctions.
Thus \( F_c(b_z) \) is equal to the number of previous auctions with final price no higher than \( b_z \), divided by the number of all past auctions. Mathematically the winning probability is represented as:

\[
F_c(b_z) = P(x(a) \leq b_z, a \in A) = \frac{\sum_{x(a) \leq b_z} |A|}{|A|} = \frac{\sum_{x(a) \leq b_z}}{|A|} \tag{3.12}
\]

If it is known that the winning price of the auction must be no lower than \( q \), the winning probability should be revised to

\[
F_c(b_z) = \frac{P(x(a) \leq b_z, a \in A)}{P(x(a) \geq q, a \in A)} = \frac{P(x(a) \leq b_z, a \in A)}{1 - P(x(a) < q, a \in A)} = \frac{\sum_{x(a) \leq b_z}}{|A| - \sum_{x(a) \leq q}} \tag{3.13}
\]

where \( b_z \geq q \). Otherwise, \( F_c(b_z) = 0 \).

Similar to above, if it is known that the winning price of the auction must be no higher than \( q \), the winning probability should be revised to

\[
F_c(b_z) = \frac{P(x(a) \leq b_z, a \in A)}{P(x(a) \leq q, a \in A)} = \frac{\sum_{x(a) \leq b_z}}{\sum_{x(a) \leq q}} \tag{3.14}
\]

where \( b_z \leq q \). Otherwise, \( F_c(b_z) = 0 \).

The histogram method is simple to understand. However, it has two disadvantages:

- Winning probability is computationally intensive. That is, each time the prediction is needed, the whole database that stores the historical records must be re-processed to calculate the corresponding winning probability.

- If the minimum bidding price \( q \) is higher than the winning prices of all the past auctions, the histogram method is generally not applicable. It is
because that the denominator of the above formula is equal to zero. In other words, the histogram method is unable to give a prediction if the current quote has never been observed in the past.

Normal method

The normal method assumes that the distribution of winning price follows a normal distribution with mean \( \mu \) and variation \( \sigma \) [19]. Thus the winning probability with price \( b_z \) can be calculated as:

\[
F_c(b_z) = P(x(a) \leq b_z) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\frac{b_z-\mu}{\sigma}} e^{-\frac{t^2}{2}} dt
\]  

(3.15)

The winning probability of an auction with minimum price \( q \) is:

\[
P_c(b_z) = P(x(a) \leq b_z | x(a) \geq q) = \frac{P(q \leq x(a) \leq b_z)}{P(x(a) \geq q)} = \frac{\int_{\frac{q-\mu}{\sigma}}^{\frac{b_z-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2}} dt}{\int_{\frac{q-\mu}{\sigma}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2}} dt}
\]  

(3.16)

Similar to above, given that the winning price of auction is no higher than \( q \), the winning probability with maximum bidding price \( b_z \) can be calculated as:

\[
P_c(b_z) = P(x(a) \leq b_z | x(a) \leq q) = \frac{P(x(a) \leq b_z)}{P(x(a) \leq q)} = \frac{\int_{-\infty}^{\frac{b_z-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2}} dt}{\int_{-\infty}^{\frac{q-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{t^2}{2}} dt}
\]  

(3.17)

Since the winning price in English auctions and Vickrey auctions reflect the maximum bidding price of the second highest bidder instead of the maximum bidding price of the winner, a simulation program is used to find the maximum bidding price of the highest bidder. It firstly inspects all the maximum bidding prices of the failed bidders and figures out the distribution of the maximum
bidding price. For each auction, a series of numbers are produced according to the distribution and the first one that equals to or higher than the final price of that auction is assigned as the maximum bidding price of the winner. After considering all the auction records, the appropriate distribution can be worked out.

Compared to the histogram method, the normal method addresses the above two drawbacks. It only needs to store the number of records, the mean and the variation. The normal method is able to compute a probability of winning an auction with a given bid, even if the value of the specific quote in that auction is greater than that of all the final prices of the past auctions. In fact, the normal distribution considers the whole set of real numbers. This is unlike the histogram method which only considers the winning prices that have occurred.

3.4.2 Method of Moments and Maximum Likelihood Estimation

In fact, the above prediction schemes both consider that the winning prices have a consistent distribution. They employ the winning prices of previous auction records and predict the distribution (in Histogram method the distribution consists of historical frequencies of the winning prices; in Normal method the distribution is normal, and the mean and variation are determined by the historical records). However, the distribution is influenced by other important factors such as the number of agents that participate in an auction. From the analysis of the popular auction market in previous sections, we can get that the distribution of the winning price depends on the number of agents. While the Normal method is grounded on an assumption that the distribution of winning price follows a normal distribution, it is more reasonable to assume that the
distribution of the maximum bidding price of each agent follows a normal distribution. The method of Moments and the Maximum likelihood estimators are based on this assumption and make use of both the winning price and the number of agents in the auction records to perform the prediction [16].

Method of Moments

Let $X$ be a random variable having the probability density function $f(x; Q_1, Q_2, ..., Q_k)$, where $Q_1, Q_2, ..., Q_k$ are parameters that characterize the distribution. The $r$-th moment about the origin, $u_r(Q_1, Q_2, ..., Q_k)$ is defined as

$$u_r(Q_1, Q_2, ..., Q_k) = E[x^r]$$

(3.18)

If the distribution is continuous, the expected value is

$$u_1 = E[x] = \int_{-\infty}^{\infty} x f(x) \, dx$$

(3.19)

and

$$u_2 = E[x^2] = \int_{-\infty}^{\infty} x^2 f(x) \, dx$$

(3.20)

There is another kind of moments, called the moments for center. It can be calculated as follows:

$$v_r(Q_1, Q_2, ..., Q_k) = E[(x - E(x))^r]$$

(3.21)

If the distribution is continuous, we get

$$v_1 = E[x - E(x)] = 0$$

(3.22)

and

$$v_2 = E[(x - E(x))^2] = E(x^2) - E^2(x) = D(x) = \int_{-\infty}^{\infty} (x - E(x))^2 f(x) \, dx$$

(3.23)
Equation (3.23) is the variation of the distribution.

Consider a random sample, \( X_1, X_2, \ldots, X_n \) from the distribution having density function \( f(x; Q_1, Q_2, \ldots, Q_k) \). Then the \( j \)-th sample moment is

\[
M_j = \frac{1}{n} \sum_{i=1}^{n} X_i^j
\]

(3.24)

where \( j = 1, 2, \ldots, k \)

These statistic moments can be used to estimate the parameters \( Q_1, Q_2, \ldots, Q_k \). Thus we can have the following equation set:

\[
M_j = u_j(Q_1, Q_2, \ldots, Q_k)
\]

(3.25)

where \( j = 1, 2, \ldots, k \)

A solution of the equation set, say \( (\hat{Q}_1, \hat{Q}_2, \ldots, \hat{Q}_k) \) is called the estimator of \( (Q_1, Q_2, \ldots, Q_k) \).

In our problem, we need to figure out the distribution of the winning price. The analysis of auction model in section 3.2.1 provides the winning price distribution, given the distribution of agents’ reserve prices and the number of agents. Since the reserve price is assumed to follow a normal distribution, we only need to get the mean and variation of the distribution and the number of agents participating in the auction. Here we set the average number of agents in previous auctions as the number of agents in current auction \( c \). Since the number of agents should be an integer, we adopt its rounded value, which is denoted as \( n_r(c) \). Here we use the 1st moment for origin and 2nd moment for center to determine the mean and variation of the reserve price distribution \( R(b_z) \).
As shown in section 3.2.1, the distribution of the winning price in English auction is

\[
F(b_i) = R^n(b_i) + n(1 - R(b_i)) \sum_{i=1}^{n-1} \frac{1}{i+1} \times \left( R^{n-i}(b_{i+1}) \right) \left( R(b_i) - R(b_{i-1}) \right) \\
= R^n(b_i) + \frac{1 - R(b_i)}{R(b_i) - R(b_{i-1})} \left( R^n(b_i) - R^n(b_{i-1}) \right)
\]  \hspace{1cm} (3.26)

The mean \(E(b)\) and variation \(D(b)\) are

\[
E(b) = \sum_{i=1}^{\infty} b_i \times (F(b_i) - F(b_{i-1})) = \sum_{i=1}^{\infty} (b_0 + ih)(F(b_i) - F(b_{i-1})) \\
D(b) = E(b^2) - E^2(b)
\]  \hspace{1cm} (3.27)

The distribution for Dutch auction is

\[
F(b_i) = R^n(b_i)
\]  \hspace{1cm} (3.29)

The corresponding mean and variation are

\[
E(b) = \sum_{i=0}^{d} b_i \times (F(b_i) - F(b_{i-1})) = \sum_{i=0}^{d} b_i \times (R^n(b_i) - R^n(b_{i-1})) = b_d - h \sum_{i=0}^{d-1} R^n(b_i) \\
D(b) = E(b^2) - E^2(b)
\]  \hspace{1cm} (3.30)

For the past auction records, its 1-st moments for origin and 2-nd moments for center can be calculated as

\[
u_1 = E(x) = \frac{1}{|A|} \sum_{i=1}^{d} x(a_i)
\]  \hspace{1cm} (3.32)

The calculation of 2-nd moments for center is a little different:

\[
u_2 = E(x) = \frac{1}{|A|} \sum_{i=1}^{d} (x(a_i) - E_{n(a_i)}(x))^2
\]  \hspace{1cm} (3.33)
where \( E_i(x) = \frac{1}{|A_i|} \sum_{a_i \in A_i} x(a_i) \)

As time elapses, the price of commodity may fluctuate and thus the older auction records may be less important than the recent auction records. To address this issue, different weights can be added to each auction record. Denote the weight of auction \( a_i \) as \( w_i \), the calculation of moments can be revised as follows:

\[
    u_1 = E(x) = \frac{\sum_{i=1}^{|A|} w_i x(a_i)}{\sum_{i=1}^{|A|} w_i}
\]

(3.34)

The calculation of 2-nd moments for center is:

\[
    v_2 = E(x) = \frac{\sum_{i=1}^{|A|} w_i (x(a_i) - E_{m(a_i)}(x))^2}{\sum_{i=1}^{|A|} w_i}
\]

(3.35)

where \( E_i(x) = \frac{\sum_{a_i \in A_i} w_i x(a_i)}{\sum_{i=1}^{|A|} w_i} \)

Denote the estimation of \( \mu \) and \( \sigma \) as \( \hat{\mu} \) and \( \hat{\sigma} \). They are the solution to the following equation set:

\[
    \begin{cases}
        E(b) = u_1 \\
        D(b) = v_2
    \end{cases}
\]

(3.36)

**Maximum Likelihood Estimation**

Given some random samples \( X_1, X_2, \ldots, X_n \) generated based on a certain distribution. The distribution of \( X \) depends on several unknown parameters \( Q_1, Q_2, \ldots, Q_k \), which are denoted as \( P(X | Q_1, Q_2, \ldots, Q_k) \).
The following likelihood function $S$ is defined as the function that indicates
the probability of generating a series of random numbers. If the distribution is
discrete, the function $S$ is represented as [21]:

$$S(X_1, X_2, ..., X_n \mid Q_1, Q_2, ..., Q_k) = \prod_{i=1}^{n} P(X_i = X_i \mid Q_1, Q_2, ..., Q_k)$$

(3.37)

If the distribution of $X$ is continuous with density function denoted as
$f(x|Q_1, Q_2, ..., Q_k)$, the likelihood function is represented as:

$$S(X_1, X_2, ..., X_n \mid Q_1, Q_2, ..., Q_k) = \prod_{i=1}^{n} f(X_i \mid Q_1, Q_2, ..., Q_k)$$

(3.38)

The maximum likelihood estimation assumes that the best estimation of $Q_1$, $Q_2, ..., Q_k$ is the set that maximizes the probability to generate the samples $X_1, X_2, ..., X_n$. Thus they are the solutions to the equation set:

$$\begin{align*}
\frac{dS(X_1, X_2, ..., X_n \mid Q_1, Q_2, ..., Q_k)}{dQ_1} &= 0 \\
\frac{dS(X_1, X_2, ..., X_n \mid Q_1, Q_2, ..., Q_k)}{dQ_2} &= 0 \\
&\vdots \\
\frac{dS(X_1, X_2, ..., X_n \mid Q_1, Q_2, ..., Q_k)}{dQ_n} &= 0
\end{align*}$$

(3.39)

For this problem, the maximum likelihood function can be calculated as

$$S(X_1, X_2, ..., X_n \mid \mu, \sigma) = \prod_{i=1}^{n} f(X_i \mid \mu, \sigma)$$

(3.40)

The estimation of $\mu$ and $\sigma$, $\hat{\mu}$ and $\hat{\sigma}$ are the solutions to the following equations:

$$\begin{align*}
\frac{dS(X_1, X_2, ..., X_n \mid \mu, \sigma)}{d\mu} &= 0 \\
\frac{dS(X_1, X_2, ..., X_n \mid \mu, \sigma)}{d\sigma} &= 0
\end{align*}$$

(3.41)
Having got the distribution of the agents’ reserve prices, we can get the winning probability for a reserve price \( b_z \), \( F_c(b_z) \) as follows:

\[
P_c(b_z) = R^{n,(c)}(b_z)
\]  \hspace{1cm} (3.42)

where \( R \) is the distribution of \( \Phi(\mu, \sigma) \).

However, the winning probability should be revised slightly according to different bidding states. For an English auction with a current price \( q \), the winning probability should be revised as:

\[
P_c(b_z) = \begin{cases} 
F(b_z) - F(q - h) & \text{if } q \leq b_z \text{ and hold highest bid} \\
1 - F(q - h) - \frac{F(b_z) - F(q)}{1 - F(q)} & \text{if } q \leq b_z - h \text{ and not hold highest bid} \\
0 & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (3.43)

The method of moments and the maximum likelihood estimators both employ the information on the number of agents in the previous auction records. This enhances the accuracy of prediction. As shown in the following section, these two schemes perform better than the histogram method and normal method. The maximum likelihood estimator is well suited to predict goods with high price fluctuation particularly, when limited historical records are available. Compared to the maximum likelihood estimators, the computation of method of moments is almost independent on the size of the number of past auctions. When the number of auction records increases, the new value of moments can be calculated by considering the previous moment values and the number of new auction records. The complexity of calculating the method of moments is generally lower than that of the maximum likelihood estimators. Furthermore, considering the effect of the price fluctuation of the commodity, appropriate weights can be added to
each auction record according to its time of occurrence. However, the accuracy of the method of moments is not as good as that of the maximum likelihood estimators (see the next section).

3.4.3 Evaluation of the prediction methods

In this section, a series of experiments are carried out to evaluate the performance of the moment method and maximum likelihood estimators. Moreover, we also compare the performance of these two estimation methods with the histogram method and normal method.

<table>
<thead>
<tr>
<th>(μ, σ)</th>
<th>(μ, σ)=(250,10) n=[10, 20]</th>
<th>(μ, σ)=(250,20) n=[10, 20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record number</td>
<td>50 100 200 500</td>
<td>50 100 200 500</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>248.4 10.4 249.0 10.6 250.0 9.2</td>
<td>255.2 14.4 248.8 20.6 249.5 19.9 249.6 19.8</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>247.5 11.0 248.3 10.5 248.9 10.2</td>
<td>247.0 21.5 248.8 20.5 248.5 20.7 248.5 20.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(μ, σ)</th>
<th>(μ, σ)=(250,50) n=[10, 20]</th>
<th>(μ, σ)=(250,100) n=[10, 20]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record number</td>
<td>50 100 200 500</td>
<td>50 100 200 500</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>245.0 54.8 253.3 46.9 253.3 47.9 252.7 47.5</td>
<td>255.0 90.0 255.0 95.1 247.0 103.5 250.0 99.9</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>253.5 47.2 251.6 48.7 248.8 51.1 249.0 50.8</td>
<td>248.2 102.7 255.0 98.8 250.2 98.3 250.0 100.9</td>
</tr>
</tbody>
</table>

Table 3.1. Estimated results of the prediction schemes for English/Vickrey auction

As we know, the proxy bidding strategy is a popular strategy, which has been widely used many auction sites (i.e., eBay). As shown in the analytical results, the final winning price depends largely on the number of agents participating in an auction. Although there are many auction records provided in eBay, they only provide the final winning price of the auctions without disclosing the number of bidders involved. Therefore, we adopt the auction
records in a simulated MAGICS environment to estimate the winning probabilities. Based on the assumption that the reservation price of the agents follows a certain normal distribution \( \phi(\mu, \sigma) \), a great deal of auctions has been carried out in MAGICS (as described above) to generate a series of English/Vickrey and Dutch/FPSB auction records. The auction records are divided into a training set and a testing set, which are used to generate the prediction function by using several prediction schemes. The aim is to verify the performance of the prediction methods. In the experiment, the number of agents participating in an auction submits a bid according to a uniform distribution between $10 and $20. The mean of the agent’s reserve price is $250. Totally eight sets of auction records have been generated: English/Vickrey and Dutch/FPSB auction records with variation of reserve prices $10, $20, $50 and $100. Based on the above data, different prediction schemes were set up. The results are shown in Table 3.1.

Table 3.1 shows the estimation results of the method of moments and the maximum likelihood estimation for English/Vickrey auctions. We can see that the results of these two schemes are close to the actual parameters and thus the performance of the two schemes are good. For each auction record, we also conducted the experiment based on different number of records: 50, 100, 200 and 500. It is obvious that with more auction records, the performance is better.

After evaluating the estimation results of the two schemes, the prediction functions were set up to estimate the winning probability of an agent with a certain reserve price. To analyze the prediction schemes, we employed the following method:
Given that the winning probability of an agent with reserve price $b_z$ in an auction $c$ is $P_c(b_z)$ and the actual winning probability is $O_c(b_z)$, the error of the prediction $\varepsilon_c(b_z)$ is defined as

$$
\varepsilon_c(b_z) = \frac{|P_c(b_z) - O_c(b_z)|}{O_c(b_z)}
$$

(3.44)

Moreover, to evaluate the prediction function, the square distance $\varepsilon$ is defined as follows:

$$
\varepsilon = \sqrt{\sum_{i \in N} (P(b_i) - O(b_i))^2}
$$

(3.45)

Table 3.2 shows the error of the four prediction schemes with different reserve prices. The number of past auction records is 500 for all cases. It is obvious that the errors of the method of moments and maximum likelihood estimation are close and the errors are much smaller than those of the other two schemes, especially when the variation of the agent’s reserve price is large. The histogram method gives the worst performance. In the table, the errors for low reserve price are usually larger because of the smaller denominator $O_c(b_z)$. It can also be seen that the method of moments performs better when the variance of the normal distribution is large.

<table>
<thead>
<tr>
<th>Error $\varepsilon (p)$</th>
<th>$(\mu, \sigma) = (250, 10) \ n=[10, 20]$</th>
<th>$(\mu, \sigma) = (250, 20) \ n=[10, 20]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserve Price</td>
<td>270</td>
<td>275</td>
</tr>
<tr>
<td>Histogram method</td>
<td>0.408358</td>
<td>0.097683</td>
</tr>
<tr>
<td>Normal method</td>
<td>0.222384</td>
<td>0.097683</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>0.128615</td>
<td>0.045014</td>
</tr>
<tr>
<td>Maximum Likelihood</td>
<td>0.054479</td>
<td>0.014560</td>
</tr>
</tbody>
</table>
Error $\delta(p)$ \hspace{1cm} $(\mu, \sigma) = (250, 50)$ \hspace{1cm} $n = [10, 20]$ \hspace{1cm} $(\mu, \sigma) = (250, 100)$ \hspace{1cm} $n = [10, 20]$

<table>
<thead>
<tr>
<th>Reserve Price</th>
<th>350</th>
<th>375</th>
<th>400</th>
<th>425</th>
<th>450</th>
<th>475</th>
<th>500</th>
<th>550</th>
</tr>
</thead>
<tbody>
<tr>
<td>Histogram method</td>
<td>0.352024</td>
<td>0.086706</td>
<td>0.020449</td>
<td>0.002052</td>
<td>0.343574</td>
<td>0.175183</td>
<td>0.095488</td>
<td>0.020449</td>
</tr>
<tr>
<td>Normal method</td>
<td>0.163403</td>
<td>0.081850</td>
<td>0.020025</td>
<td>0.003332</td>
<td>0.135021</td>
<td>0.129130</td>
<td>0.079647</td>
<td>0.019953</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>0.038935</td>
<td>0.018169</td>
<td>0.005807</td>
<td>0.001342</td>
<td>0.001655</td>
<td>0.001058</td>
<td>0.000660</td>
<td>0.000199</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>0.009931</td>
<td>0.005328</td>
<td>0.001913</td>
<td>0.000495</td>
<td>0.014990</td>
<td>0.009861</td>
<td>0.006048</td>
<td>0.001856</td>
</tr>
</tbody>
</table>

Table 3.2. Winning probability errors of different prediction schemes for English/Vickrey auctions

Table 3.3 shows the square distance of the four prediction schemes with different number of auction records: 50, 100, 200 and 500. Generally, the square distance of the prediction function becomes smaller when there are more auction records. It is obvious that the square distance of the method of moments and the maximum likelihood estimation are much lower than that of the histogram method and normal method. Thus the former methods achieve a better performance. When the reserve price varies more significantly, the square distance of the histogram method and normal method becomes quite large while that of the moment method and maximum likelihood estimation only increases slightly. Also, when the number of auction records is small, the performance of the maximum likelihood scheme is better than that of the moment method.

<table>
<thead>
<tr>
<th>error</th>
<th>$(\mu, \sigma) = (250, 10)$ \hspace{1cm} $n = [10, 20]$ \hspace{1cm} $(\mu, \sigma) = (250, 20)$ \hspace{1cm} $n = [10, 20]$</th>
</tr>
</thead>
</table>
| Record number | \begin{tabular}{c|cccc|cccc}  
|       | $N=50$ & $n=100$ & $n=200$ & $n=500$ & $n=50$ & $n=100$ & $n=200$ & $n=500$ \\
| Histogram method | 1.535694 & 1.463549 & 1.428478 & 1.388681 & 1.791228 & 1.749224 & 1.736389 & 1.689896 \\
| Normal method | 0.735076 & 0.544424 & 0.497218 & 0.333425 & 0.658226 & 0.629831 & 0.513279 & 0.507945 \\
| Method of Moments | 0.018824 & 0.014669 & 0.019836 & 0.032049 & 0.032919 & 0.003684 & 0.010301 & 0.011567 \\
| Maximum Likelihood | 0.016333 & 0.007502 & 0.017258 & 0.015787 & 0.009116 & 0.005220 & 0.005178 & 0.004389 |
|}
After analyzing the performance of the prediction methods for English/Vickrey auctions, we discuss the experimental result for Dutch/FPSB auctions. Unlike the English/Vickrey auctions, the winning price is the highest reserve price among all agents. Table 3.4 gives the estimation results of the method of moments and the maximum likelihood estimation in Dutch/FPSB auctions with various numbers of records: 50, 100, 200 and 500. It can be seen that the more the auction record, the better the estimation of the two prediction schemes.

<table>
<thead>
<tr>
<th>Number of records</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>500</th>
<th>50</th>
<th>100</th>
<th>200</th>
<th>500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method of Moments</td>
<td>251.6</td>
<td>249.6</td>
<td>249.7</td>
<td>249.5</td>
<td>259.5</td>
<td>253.9</td>
<td>248.8</td>
<td>248.9</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>247.7</td>
<td>250.3</td>
<td>249.4</td>
<td>248.8</td>
<td>251.8</td>
<td>251.5</td>
<td>248.4</td>
<td>249.6</td>
</tr>
</tbody>
</table>

Table 3.3 Square distance of different prediction schemes for English/Vickrey auctions
Table 3.4. Estimation result of the prediction schemes for Dutch/FPSB auction

Table 3.5 shows the winning probability error of the four prediction schemes with different reserve prices. The number of past auction records of all schemes is 500. We can see that the errors of the method of moments and the maximum likelihood estimation are close to each other. In fact, they are much smaller than those of the other two schemes. The winning probability error of the histogram method and the normal method in Dutch/FPSB auctions is much closer than that in English/Vickrey auction. It is because the winning probability of English/Vickrey auction is determined by the second highest of agents’ reserve price. An enhancement has been made by the normal method to simulate the highest price of the auction records while the histogram method has not.

The square distance of the four prediction schemes based on various number of auction records is shown in table 3.6. It is obvious that the minimum square error of the method of moments and the maximum likelihood estimation are much lower than that of the histogram method and normal method. Thus the former schemes give better performances. Similar to above, the square distance of the histogram method and the normal method in Dutch/FPSB auctions is smaller than that in English/Vickrey auctions.

<table>
<thead>
<tr>
<th>Error $\varepsilon(p)$</th>
<th>$(\mu, \sigma)=(250,10) \ n=[10, 20]$</th>
<th>$(\mu, \sigma)=(250,20) \ n=[10, 20]$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reserve Price</strong></td>
<td><strong>270</strong></td>
<td><strong>290</strong></td>
</tr>
<tr>
<td>275</td>
<td>0.052109</td>
<td>0.093499</td>
</tr>
<tr>
<td>280</td>
<td>0.012774</td>
<td>0.016966</td>
</tr>
<tr>
<td>285</td>
<td>0.001964</td>
<td>0.001519</td>
</tr>
<tr>
<td>290</td>
<td></td>
<td>0.001964</td>
</tr>
<tr>
<td>300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>310</td>
<td></td>
<td></td>
</tr>
<tr>
<td>320</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Histogram method</strong></td>
<td>0.183684</td>
<td>0.093499</td>
</tr>
<tr>
<td><strong>Normal method</strong></td>
<td>0.077984</td>
<td>0.008834</td>
</tr>
<tr>
<td></td>
<td>0.035787</td>
<td>0.017685</td>
</tr>
<tr>
<td></td>
<td>0.013440</td>
<td>0.010862</td>
</tr>
<tr>
<td></td>
<td>0.003456</td>
<td>0.003281</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>0.024172</td>
<td>0.006356</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>0.061869</td>
<td>0.016801</td>
</tr>
<tr>
<td>Error $\varepsilon_b(p)$</td>
<td>$(\mu,\sigma)=(250,50)$ $n=[10, 20]$</td>
<td>$(\mu,\sigma)=(250,100)$ $n=[10, 20]$</td>
</tr>
<tr>
<td>Reserve Price</td>
<td>350</td>
<td>375</td>
</tr>
<tr>
<td>Histogram method</td>
<td>0.107590</td>
<td>0.036734</td>
</tr>
<tr>
<td>Normal method</td>
<td>0.034544</td>
<td>0.031686</td>
</tr>
<tr>
<td>Method of Moments</td>
<td>0.084010</td>
<td>0.033904</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>0.032032</td>
<td>0.008804</td>
</tr>
</tbody>
</table>

Table 3.5 Winning probability errors of different prediction schemes in Dutch/FPSB auctions

<table>
<thead>
<tr>
<th>Error</th>
<th>$(\mu,\sigma)=(250,10)$ $n=[10, 20]$</th>
<th>$(\mu,\sigma)=(250,20)$ $n=[10, 20]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Record number</td>
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<tr>
<td>Normal method</td>
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<td>0.176886</td>
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<tr>
<td>Method of Moments</td>
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<td>0.001740</td>
</tr>
<tr>
<td>Maximum Likelihood Estimation</td>
<td>0.013914</td>
<td>0.002692</td>
</tr>
</tbody>
</table>

<table>
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<th>$(\mu,\sigma)=(250,100)$ $n=[10, 20]$</th>
</tr>
</thead>
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<td>$n=100$</td>
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<td>Normal method</td>
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</tr>
<tr>
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<tr>
<td>Maximum Likelihood Estimation</td>
<td>0.082169</td>
<td>0.080185</td>
</tr>
</tbody>
</table>

Table 3.6. Square distance of different prediction schemes in Dutch/FPSB auction
3.5 Summary

In this chapter, we have presented a mobile agent-based auction system, which can be accessed via a Web/WAP interface. Based on the bidding information, the agent will bid for the user up to the specified reserve price. A general mathematical model has been set up to analyze the bidding results. Furthermore, we have studied two distributions for the reserve price namely: linear and normal distribution. Some simulations were conducted to validate the correctness of the mathematical model. The simulation and analytical results indicate that Dutch auctions result in a higher average winning price in general, particularly when the number of agents is small. However, when more agents participate in an auction, the difference between the average winning price for English and Dutch auctions becomes negligible. The generic model can also be applied to analyze the bidding results for other distributions of the reserve price. An important issue is to compute the winning probability for a certain maximum bidding price. This is essential in determining the maximum bidding prices. We have investigated two prediction schemes called “method of moment” and “maximum likelihood estimator” for the proxy bidding strategy. With the previous auction records, the two prediction schemes can be used to estimate the winning probability in an auction given a certain maximum bidding price. Some experiments have been done to evaluate the performance of the two schemes and compare the estimation schemes with two other prediction schemes proposed by [20]. Experimental results indicate that the method of moment and maximum likelihood estimator perform much better than the other two schemes, especially in English/Vickrey auctions. As expected, with more auction records, the estimation schemes achieve a better accuracy.
In this chapter, we assume that the reserve prices of all agents follow the same distribution. As a future work, the analysis can also be extended to consider agents with different reserve price distributions. Furthermore, for the prediction schemes, we only consider the case that the reserve prices of the agents follow a normal distribution. More analysis can be done to evaluate the performance of the prediction schemes by using other distributions (e.g., power-law distributions).
CHAPTER 4

PROBABILITY BIDDING STRATEGY

4.1 Introduction

In chapter 3, we have discussed a proxy bidding strategy. It is based on the popularly used proxy bidding service. Basically, an agent keeps renewing a bid for a user provided that the bidding price is not greater than the user’s price ceiling. Essentially, the agent will bid the price with certainty given it is lower than the specified price. However, sometimes users may not be certain whether to place a bid. They just have a certain degree of willingness to submit a bid at a certain price. The proxy bidding strategy can not address this requirement. In this chapter, we introduce a probabilistic bidding strategy, where each agent submits bids according to a willingness function. In essence, a willingness function specifies the willingness (or technically the bidding probability) to bid at a certain price. Bidders can use different willingness functions to instruct their auction agents to bid based on their preferences. By using an appropriate willingness function, the required bidding strategy can be conveyed to the agent effectively. For example, as shown later, we can convey the following instructions to a bidding agent by using a suitable willingness function:

- “Always bid up to $10 for me.”
- “… I am a bit uncertain so bid for me with a probability of 0.8 between $10 and $20.”
“For the first two minutes and the residual time, keep bidding if the price is below $20 and $30, respectively.”

In fact the proxy bidding strategy can be considered as a probabilistic bidding strategy with a constant willingness function: the willingness stays at one between certain price values.

As an example, we consider the application of the probabilistic bidding strategy for supporting mobile commerce (m-commerce) in general and mobile auction (m-auction) services in particular. M-commerce is the extension of e-commerce, enabled by wireless networking and mobile computing [3, 34]. M-auction services allow users to buy or sell something by using a dynamic pricing mechanism through their mobile terminals. Compared to the Web-based auction services, m-auction services are generally time-critical and location-dependent [33]. For example, a hotel may provide an auction-based reservation service for some of its rooms. Mobile consumers or travelers around the hotel can bid for the rooms within a short period of time. Because of the characteristics of m-commerce, bidding strategies should be simple and efficient in terms of computation so that agents can make decisions quickly.

To support m-auctions, we define a number of bidding strategies by using some willingness functions and formulate a mathematical model to analyze the bidding results for both the English and Dutch auction methods. In some cases, close-form mathematical expressions have been derived. Furthermore, we make use of the mathematical model to study the system behavior. In particular, all the analytical results are validated by simulation results to ensure the correctness.
The model, analysis and results provide valuable insights into the design of agent-based m/e-auction services.

The organization of the rest of this chapter is as follows. Section 4.2 gives the system architecture. Section 4.3 presents the bidding strategies. Section 4.4 formulates a mathematical model for computing the average winning price for the English and Dutch auctions. Section 4.5 discusses the simulation results. Section 4.6 concludes the chapter.

### 4.2 System Architecture

![System overview and key modules](image)

Figure 4.1 shows the basic architecture of the mobile agent-based auction system, which is also part of the Mobile AGent-based Internet Commerce Systems (MAGICS) research project. Actually it is the extension of the system described in chapter 3, where the mobile agent could automatically search for the auction sites based on certain location requirements and then bid sequentially. To
find out the location information, the proxy server is connected to a directory server, which can be co-located.

Figure 4.2: Bidding process

Figure 4.2 shows the basic operation of the system. To participate in an auction, bidders need to access the proxy server through their mobile terminals and provide the bidding information, such as the minimum and maximum bidding prices, and choose one of the four bidding strategies: constant, linearly decreasing, aggressive and conservative. Of course, additional bidding strategies can be supported by using the corresponding willingness functions as explained in the next section. After submitting the bidding information, a mobile agent is created, and then it moves to the corresponding auction server to bid for the required products. The proxy server searches for the auction server(s) with the assistance of a directory server. In an m-auction environment, the auction server can be location-based. As an example, let us consider that a user wants to book a room at the nearest hotel by means of m-auction. Having received the request, the proxy server identifies the potential hotels ranked according to the ascending order of the distance to the user. If required, the proxy server can also send the
list to the user for confirmation before sending off the mobile auction agents. After confirmation, the proxy server sends a mobile agent to bid for a hotel room with other agents. In this chapter, we assume that the mobile auction agent bid in a sequential manner (e.g., try to find the closest hotel within the affordable price range). We are also investigating a parallel bidding scenario, which is outside the scope of this chapter. At each auction server, a seller agent is set up to control the bidding process. Upon arrival, an auction agent registers with the seller agent. The seller agent announces the new price to the auction agents whenever there is an update. Each auction agent submits bids to the seller agent based on its willingness function (see the next section) as defined by its owner. All messages exchanged are XML-based. To ensure the non-repudiation requirement, all messages can be digitally signed with the user’s private key, which is carried by the corresponding agent. To verify the messages, each mobile auction agent can provide a digital certificate to the selling agent for authentication purposes. Finally, if the bidding result is unsuccessful, the mobile agent moves to the next hotel to continue the bidding. If successful, the mobile agent reserves the room accordingly and notifies the user (e.g., through Short Messaging Service).

![Figure 4.3: Demonstration of the auction process](image)

(1) Input the bidding information  (2) Create a bidding agent
(3) Bid with other agents
(4) Notify the bidding result
A prototype system has been built to show the basic function of the mobile auction system (see Figure 4.3). Basically a user accesses the system through a WAP phone. A mobile agent is then created and sent to the required auction server. The mobile agent then participates in the auction with other agents by using a predefined bidding strategy. Finally, the auction result is returned to the user.

4.3 Willingness Function and Bidding Strategy

As mentioned before, to support agent-based auction services, it is essential to develop an effective mechanism to convey the bidding strategies to the agents. We propose using a willingness function for this purpose. Basically a willingness function reflects the degree of willingness to submit a bid at a certain price. Bidders can use different willingness functions to instruct their auction agents to bid based on their preferences. For instance, the popular proxy bidding service can be represented by a willingness function with a value of one within a certain price range. This means that the agent will bid with certainty (i.e., with a probability of one) within the specified prices. Sometimes, a user may not be so certain whether he/she will bid at a certain price. In this case, he/she can specify a willingness value of less than one to reflect the “fuzzy or uncertain nature”. For example, a willingness of 0.5 tells the agent to bid at the specified price with probability 0.5. In general, the willingness function can be time-dependent too. This can then be used to model the price-time function used in Kasbah [2]. For instance, before and after 12:00 pm, the willingness value for a buying agent remains at one up to $10 and $12, respectively. This means that the buying agent will accept a selling price not exceeding $10 and $12 before and after 12:00 pm,
respectively. Compared to the price-time function used in Kasbah, the willingness function is more flexible and general because it can also be used to model some “fuzzy situations” (i.e., the user wants to tell the agent to bid with a certain probability rather than with certainty). However, to facilitate the explanation, we focus on a non-time dependent willingness function in this chapter.

In this section, we introduce four bidding strategies or willingness functions for the mobile auction service, namely the constant, linear, aggressive, conservative bidding strategies. Before explaining the strategies in details, the following assumptions and notations are defined:

- $b_1$ is the minimum bidding price set by a bidder
- $b_z$ is the maximum bidding price set by a bidder
- $h$ is the incremental bidding price or the difference between $b_j$ and $b_{j+1}$. Hence, we have $b_j = b_1 + (j-1)h$.
- $W(b_j)$ is the willingness value (i.e., bidding probability) at the bidding price $b_j$, where $0 \leq W(b_j) \leq 1$ and $b_1 \leq b_j \leq b_z$

**Constant Willingness Function**

![Figure 4.4: The constant willingness bidding strategy](image-url)

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The proxy bidding service can be represented by a constant willingness function, as shown in Figure 4.4. In this case, if the current bidding price is no higher than a user’s minimum bidding price \((b_1)\), the agent will always submit a bid. If the current bidding price is between the user’s minimum bidding price \((b_1)\) and maximum bidding price \((b_2)\), user will always bid with probability \(p\). Note that for the proxy bidding strategy, \(p\) is set to one. If the current bidding price is higher than the user’s maximum bidding price, user won’t bid it at all. Mathematically, the willingness function is as shown in (4.1).

\[
W(b) = \begin{cases} 
1 & \text{if } b \leq b_1 \\
p & \text{if } b_1 < b \leq b_2 \\
0 & \text{if } b > b_2 
\end{cases}
\]  

(4.1)

**Linearly Decreasing Willingness Function**

![Linearly Decreasing Willingness Function](image)

Figure 4.5: The linearly decreasing willingness approach

In general, a user may be less willing to submit a bid as the bidding price increases. To model this situation, the linearly decreasing willingness bidding strategy is introduced. As shown in Figure 4.5, the degree of willingness to bid for an item decreases linearly as the bidding price increases. Mathematically, the willingness function is shown in (4.2).
Aggressive Bidding Strategy

\[
W(b) = \begin{cases} 
1 & \text{if } b \leq b_1 \\
\frac{b - b}{b_2 - b_1} & \text{if } b_1 < b \leq b_2 \\
0 & \text{if } b > b_2 
\end{cases}
\]  

(4.2)

The aggressive bidding approach is similar to the linearly decreasing willingness bidding strategy in that the willingness value decreases as the bidding price increases. However, the rate of decrease in the willingness value for the aggressive bidding strategy is less than that of the linearly decreasing willingness bidding strategy. Therefore, this bidding strategy is suitable for users who want to submit bids more aggressively. As shown in Figure 4.6, the degree of aggressiveness can be adjusted by the index value \( n \). Equation (4.3) shows the willingness function for the aggressive bidding strategy where \( n \) is the value of the degree of aggressiveness.

\[
W(b) = \begin{cases} 
1 & \text{if } b \leq b_1 \\
1 - \left( \frac{b - b_1}{b_2 - b_1} \right)^n & \text{if } b_1 < b \leq b_2 \\
0 & \text{if } b > b_2 
\end{cases}
\]  

(4.3)
It can be seen from Figure 4.6 that when the value of $n$ increases, the slope of the curve becomes flattened. By using a higher index value, a user can participate more actively in an auction.

**Conservative Bidding Strategy**

$$W(b) = \begin{cases} 
1 & \text{if } b \leq b_1 \\
\left(1 - \frac{b - b_1}{b_z - b_1}\right)^n & \text{if } b_1 < b \leq b_z \\
0 & \text{if } b > b_z 
\end{cases} \quad (4.4)$$

![Figure 4.7: The conservative bidding approach](image)

In contrast to the aggressive bidding strategy, the conservative bidding strategy is for users with a conservative attitude. In this case, the willingness value decreases sharply as the bidding price increases. Mathematically, the willingness function is defined as follows:

Similar to the aggressive bidding strategy, the index $n$ controls the degree of conservativeness. When $n$ increases, the slope of the curve is steeper, indicating that the bidder is less willing to place a bid as the price increases (see Figure 4.7).

Of course, other bidding strategies can be defined as well by specifying the corresponding willingness function. In the following section, we will analyze the
bidding results of the English and Dutch auctions for the above bidding strategies (i.e., willingness functions).

4.4 Mathematical Model

In the previous sections, we discussed different bidding strategies and the implementation of the auction system. In this section, we present a mathematical model to compute the average winning price of an auction. First we define the following notations:

- $b_j$ is the bidding price of the item where $b_1 \leq b_j \leq b_z$
- $m_j$ is the probability that at least one bidder bids at $b_j$
- $x$ is the number of bidders
- $P(b_k)$ is the probability that the price will stop at $b_k$
- $E(b)$ is the expectation of the winning price

4.4.1 English Auction

Figure 4.8: Model of the English auction

Figure 4.8 shows the model for computing the average winning price for the English auction. Consider that the highest bid price at the present moment is $b_j$. If any agent bids at $b_{j+1}$, the highest bid price will obviously increase to $b_{j+1}$.
This situation occurs with probability $m_{j+1}$. In other words, if no agent bids at $b_{j+1}$, the auction will stop at $b_j$ with a probability of $1-m_{j+1}$. Note that the bid price will not increase above $b_z$.

Hence, $m_{j+1}$ is the probability that the bid price will change from $b_j$ to $b_{j+1}$. It depends on the bidding strategies of the bidders. Assuming that all bidding agents use the same bidding strategy (i.e., have the same willingness function), $[1 - W(b_{j+1})]^{x-1}$ is the probability that no bid is received at $b_{j+1}$. Note that the current winning agent does not bid. Hence, $1 - [1 - W(b_{j+1})]^{x-1}$ is the probability that at least one agent places a bid at $b_{j+1}$. As a result, the bid price increases to $b_{j+1}$ with probability

$$m_{j+1} = 1 - [1 - W(b_{j+1})]^{x-1}$$  \hspace{1cm} (4.5)

<table>
<thead>
<tr>
<th>Stop at</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>$1 - m_2$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$m_2 \times (1 - m_3)$</td>
</tr>
<tr>
<td>$b_j$</td>
<td>$m_2 \times m_3 \times m_4 \ldots \times m_j \times (1 - m_{j+1})$</td>
</tr>
<tr>
<td>$b_z$</td>
<td>$m_2 \times m_3 \times m_4 \ldots \times m_{z-1} \times m_z \times 1$</td>
</tr>
</tbody>
</table>

Table 4.1: Probability of an auction stopping at different bidding prices

Based on the above discussion, Table 4.1 gives the probability that an auction stops at $b_j$.

Thus we can easily get the distribution of the winning price as:

$$P(b_k) = (1 - W(b_{k+1}))^{x-1} \prod_{i=2}^{k} \left(1 - (1 - W(b_i))^{x-1}\right)$$  \hspace{1cm} (4.6)

In other words, (4.6) gives the probability that the winning price is $b_k$. According to (4.6), we can compute the average winning price of the English auction as follows:
\[
E(b) = b_0 + h \sum_{k=1}^{z} k \times (1 - W(b_{k+1}))^{x-1} \prod_{i=2}^{k} \left(1 - (1 - W(b_i))^{x-1}\right) \quad (4.7)
\]

where \(b_0 = b_1 - h\).

We now evaluate (4.6), (4.7) based on the different willingness functions (or bidding approaches) as follows.

**Constant Willingness Function**

Substituting (4.1) into (4.6), we get:

\[
P(b_k) = \left(\frac{1-p}{(1-p)^{x-1}} \right)^{k-1} \quad if \quad 1 \leq k < z
\]
\[
P(b_k) = \frac{1}{(1-p)^{x-1}} \quad if \quad k = z
\]

(4.8)

Putting (4.1) into (4.7), the average winning price can be found to be:

\[
E(b) = b_0 + h \frac{1 - \left(1 - \left(\frac{1-p}{(1-p)^{x-1}}\right)^{x-1}\right) + \left(1 - \left(\frac{1-p}{(1-p)^{x-1}}\right)^{x-1}\right) + \left(1 - \left(\frac{1-p}{(1-p)^{x-1}}\right)^{x-1}\right)}{(1-p)^{x-1} + h \times \left(1 - \left(\frac{1-p}{(1-p)^{x-1}}\right)^{x-1}\right)}
\]

(4.9)

Detailed calculation of (4.9) is shown in Appendix I.

**Linear Willingness Function**

Similarly, substituting (4.2) into (4.6), we have:

\[
P(b_k) = \left(\frac{k}{z-1}\right)^{x-1} \prod_{i=2}^{k} \left(1 - \left(\frac{i-1}{z-1}\right)^{x-1}\right) \quad if \quad 1 \leq k < z
\]
\[
P(b_k) = \prod_{i=2}^{z} \left(1 - \left(\frac{i-1}{z-1}\right)^{x-1}\right) \quad if \quad k = z
\]

(4.10)

Also, the average winning price can be found to be:

\[
E(b) = b_0 + h \sum_{k=1}^{z} k \times \left(\frac{k}{z-1}\right)^{x-1} \prod_{i=2}^{k} \left(1 - \left(\frac{i-1}{z-1}\right)^{x-1}\right) + h \times \prod_{i=2}^{z} \left(1 - \left(\frac{i-1}{z-1}\right)^{x-1}\right)
\]

(4.11)
4.4.2 Dutch Auction

Figure 4.9: Model of the Dutch auction

Figure 4.9 shows the mathematical model of the Dutch auction. In this case, $b_j$ moves to $b_{j-1}$ with probability $1 - m_j$ (i.e., no agent wants to submit a bid at the price $b_j$). If one of the agents bids at $b_j$, which occurs with probability $m_j$, the auction ends. Note that the bid price will not decrease below $b_1$ (i.e., $m_1 = 1$). For the Dutch auction, $m_j$ can be found as follows:

$$m_j = 1 - [1 - W(b_j)]^x$$  \hspace{1cm} (4.12)

Table 4.2: Probability of auction stopping at different bidding prices

<table>
<thead>
<tr>
<th>Stop at</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_z$</td>
<td>$m_z$</td>
</tr>
<tr>
<td>$b_{z-1}$</td>
<td>$(1 - m_z) \times m_{z-1}$</td>
</tr>
<tr>
<td>$b_j$</td>
<td>$(1 - m_z) \times (1 - m_{z-1}) \times \ldots \times (1 - m_{j+1}) \times m_j$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$(1 - m_z) \times (1 - m_{z-1}) \times \ldots \times (1 - m_2) \times 1$</td>
</tr>
</tbody>
</table>

Note that there are $x$ bidding agents. According to Table 4.2, we can easily compute the distribution of the winning price to be:
The average winning price can then be worked out as:

\[ P(b_k) = \left(1 - (1 - W(b_k))^x\right) \times \prod_{i=k+1}^{z} (1 - W(b_i))^x \]  
(4.13)

The average winning price can be determined as follows:

\[ E(b) = b_0 + h \sum_{k=1}^{z} k \times \left(1 - (1 - W(b_k))^x\right) \times \prod_{i=k+1}^{z} (1 - W(b_i))^x \]  
(4.14)

where \( b_0 = b_1 - h \).

We now evaluate (4.13), (4.14) by using different willingness functions.

**Constant Willingness Function**

Substituting (4.1) into (4.13), we obtain:

\[ P(b_k) = \left(1 - \left(\frac{k - 1}{z - 1}\right)^x\right) \times \prod_{i=k+1}^{z} \left(\frac{i - 1}{z - 1}\right)^x \]  
(4.15)

The average winning price is

\[ E(b) = b_0 + h \sum_{k=1}^{z} k \times \left(1 - \left(\frac{k - 1}{z - 1}\right)^x\right) \times \prod_{i=k+1}^{z} \left(\frac{i - 1}{z - 1}\right)^x \]  
(4.16)

Detailed calculation of (4.16) is shown in Appendix II.

**Linear Willingness Function**

According to formula (4.2), distribution of auction winning price given all \( x \) agents apply linear willingness approach is given out as follows:

\[ P(b_k) = \left(1 - \left(\frac{k - 1}{z - 1}\right)^x\right) \times \prod_{i=k+1}^{z} \left(\frac{i - 1}{z - 1}\right)^x \]  
(4.17)

The average winning price is

\[ E(b) = b_0 + h \sum_{k=1}^{z} k \times \left(1 - \left(\frac{k - 1}{z - 1}\right)^x\right) \times \prod_{i=k+1}^{z} \left(\frac{i - 1}{z - 1}\right)^x \]  
(4.18)
4.5 Results and Discussions

Based on the above bidding strategies, we present some simulation, analytical and experimental results in this section. As an example, the minimum and maximum bidding prices are set to be $300 and $500, respectively. For the analytical results, they were calculated by the respective equations given in the last section. The simulation model was written in Java to simulate a bidding process. Each simulation is repeated 10,000 times to obtain the average results. To validate the simulation/analytical models, we have also run some experiments using the aforementioned prototype system (i.e., using IBM Aglet). In each experiment, the auction process was repeated 50 times to compute the average winning price. Two representative examples of the experiments for the linearly decreasing willingness approach are shown in Figures 4.10 and 4.11. Figure 4.10 shows the average winning price for the English auction when different numbers of agents are used. It can be seen that the experimental, simulation and analytical results are almost the same. Figure 4.11 shows the simulation, experimental and
analytical results for the Dutch auction. Again, the average winning price as found by the three approaches are almost the same, thus confirming the correctness of simulation/analytical/experimental models. Unfortunately, the experimental approach was time consuming because many agents were created in each auction. Having validated the simulation and analytical models, we will use them to obtain the results in the following sub-sections.

![Dutch auction graph](image)

Figure 4.11: Comparison of the experimental, simulation and analytical results of Dutch auction

Results for the English Auction

In this sub-section, we present the results for the English auction method. First, we evaluate the relationship between the average winning price and different bidding strategies. As shown in Figure 4.12, we can see that the average winning price is higher if the aggressive bidding strategy is used. This is because the agents bid more actively, so the average winning price becomes higher. In contrast, if the conservative bidding strategy is used, the average winning price becomes lower. Figure 4.12 also shows that when the number of agents
increases, the average winning price also increases slightly. This is because the overall chance for submitting a bid is higher when the number of agents is larger.

Figure 4.12: Average winning price of the English auction with different bidding strategies

Figure 4.13: Average winning price of the English auction with different degree of aggressiveness values
Figure 4.14: Average winning price of the English auction with different degree of conserveness values

Figure 4.15: Average winning price of the English auction with different bidding probabilities

Figure 4.13 shows that when the number of agents is small, the average winning price is more sensitive to the change in the aggressiveness value.
However when the number of agents is large, the effect is much smaller. Figure 4.14 shows a similar trend with the conservativeness bidding approach (i.e., the conservative values have a larger effect when the number of agents is small). Figure 4.15 shows the average winning price for the constant willingness function with different values of $p$. It can be seen that for each curve, the average winning price increases dramatically when the number of agents reaches a certain threshold. The threshold value is smaller when $p$ is larger.

Results for the Dutch Auction

In this sub-section, we present the results for the Dutch auction. Similar to above, we first evaluate the average winning price of different bidding strategies, namely the aggressive, conservative and linearly decreasing bidding strategies. Then, we discuss the effect on the average winning price if the degree of aggressiveness and conservativeness are varied.

![Figure 4.16: Average winning price of the Dutch auction with different bidding strategies](image-url)
Figure 4.17: Average winning price of the Dutch auction with different degree of aggressiveness values

Figure 4.18: Average winning price of the Dutch auction with different degree of conservativeness values
Figure 4.19: Average winning price of the Dutch auction with different bidding probabilities

Figure 4.16 shows the average winning price when the number of bidding agents is varied. As expected, the average winning price for the aggressive bidding strategy is higher than that of the other bidding strategies. Furthermore, the average winning price for the conservative bidding strategy is the lowest. This is because the bidding probability of the aggressive bidding approach is higher than that of the other bidding strategies, especially the conservative bidding strategy. Due to the use of a higher bidding probability, the auction stops at a higher price. As a result, the average winning price for the aggressive bidding strategy is higher than that of the conservative or linearly decreasing strategy. Furthermore, as the number of agents increases, the average winning price increases as expected. This is because as more agents participate in an auction, the chance for having a higher bid is larger. Hence the auction is likely to stop at a higher price. Referring to (4.3), it can be seen that as \( n \) increases, the degree of aggressiveness increases (i.e., the probability that an agent places a bid
is higher). As shown in Figure 4.17, as $n$ increases, the average winning price is higher because the agents bid more actively. In other words, with a higher degree of aggressiveness, an agent is more likely to bid at a higher price. Figure 4.18 shows the average winning price for the Dutch auction when the conservative bidding strategy is used. It shows that as $n$ increases, the average winning price decreases. The reason is similar to that discussed above. As $n$ increases, an agent is less likely to place a bid, thus resulting in a lower average winning price. Hence we can use $n$ to control the level of conservativeness. The average winning price with constant willingness approach is shown in Figure 4.19. As expected, with a higher bidding probability, the final winning price increases.

![Graph](image-url)

**Figure 4.20: Comparison of the average winning price of the English/Dutch auction**

Finally, we compare the bidding results for the English and Dutch auction methods. As shown in Figure 4.20, the average winning price of the English auction is generally lower than that of the Dutch auction. This is related to the
auction protocol as shown in Figure 4.8 and Figure 4.9. From Figure 4.20, it can be seen that the average winning price of the Dutch auction is higher than that of the English auction if the conservative or linearly decreasing bidding strategy is used. However, if the aggressive bidding strategy is adopted or the number of agents is large, the average winning prices for the English and Dutch auctions are close to each other.

4.6 Summary

In conclusion, we have presented an auction system based on mobile agents. The system can be accessed via a mobile terminal. A user can generate a mobile auction agent that uses one of four different bidding strategies: constant, linearly decreasing, aggressive and conservative. For each bidding strategy, a willingness function is used to define the bidding probability at a particular price. According to the specified bidding strategy (or willingness function) and other necessary information, each mobile agent moves to the respective auction server to bid with other agents. In the context of m-commerce, the auction server can be location-based (e.g., closest to the user) and the bidding results can be conveyed to the user through SMS. A mathematical model has been presented to evaluate the bidding results for both the English and Dutch auction methods. In particular, some close form mathematical expressions have been derived for analysis purposes. Simulation and analytical results have been presented to study the bidding strategies. As shown above, all the simulation results match closely with the analytical results, thus validating the correctness of both the simulation and analytical models. In general, the analytical model can be applied and extended to analyze other bidding strategies. The study indicates that the Dutch auction
results in a higher average winning price in general, particularly if the number of agents is small. However, if the number of agents is large, the average winning price of the English auction is close to that of the Dutch auction in many cases. Furthermore, as expected, the average winning price is generally higher if a more aggressive bidding strategy is adopted.
CHAPTER 5

BACKWARD INDUCTION BIDDING STRATEGY (BIDS)

5.1 Introduction

The above two chapters have analyzed several bidding strategies for single-site auctions. However, with the development of online auctions, it is possible for agents to participate in many concurrent auctions selling the same item. For example, when a user wants to buy a certain digital camera, he/she may find that there are many auctions selling the same camera, probably using different auction types. In general, we can achieve better results by participating in multiple auctions rather than a single auction. To enable an agent to participate in multiple auctions, we need to develop a special bidding strategy. In this chapter, we introduce a backward induction strategy for users to bid in multiple concurrent heterogeneous auctions. In particular, two contributions are made namely: the auction chain concept, and the backward induction auction algorithm. In general, basic concept of the backward induction bidding strategy is given as follows:

Given that it is not possible to participate in all auctions due to overlapping deadlines, we first identify all the possible sets of auctions (the auction chains) in which an agent can bid. The “attractiveness” of each auction chain is determined by a utility function based on the past winning probability.
After getting the expected utility of all the auction chains, we can easily select the right auctions to participate and determine the corresponding bidding prices. Compared with the two bidding strategies discussed in Chapters 3 and 4, BIDS is more adaptive as it can change the bidding behaviour based on the market situation.

Details of BIDS and the performance analysis are illustrated in the following sections. Section 3.2 describes the system model and the bidding strategy. Section 3.3 explains the concept of auction chains. Section 3.4 presents the operation of BIDS. Section 3.5 discusses the experimental results. Section 3.6 summaries this chapter.

## 5.2 System model

![Figure 5.1. Structure of the auction site](image)

Before presenting the bidding strategy, we present the system model first. As shown in Figure 5.1, there are totally three kinds of auctions in the market: English auction, FPSB auction and Vickrey auction. Essential information for running the auctions is stored in the database and an access control mechanism is
employed to ensure that only appropriate information can be accessed. The auction site provides two kinds of interfaces for the users: user interface and agent interface. A user can access the system through the user interface. The agent interface provides several methods, based on which, users could create their own agents. For simplicity, we assume that all auctions in the market sell the same item and only one item is on sale in each auction. Each auction may begin and end at different times and the active period of English auctions and sealed-bid auctions are announced before the closing time. The number of bidders in each auction and its current price (if any) is available by sending an inquiry message to the access control subsystem. The system elements are described as follows:

**Agent**

Before bidding, the agent is assigned two parameters:

- $M$ is the maximum price that the agent can bid.
- $T$ denotes the latest time to obtain the item.

**Auction**

Here we denote $A$ as the set of auctions that were presently active or have not started and the agent should bid in set $A$. Set $A$ includes various kinds of auctions, specifically $A_e$, $A_f$ and $A_v$ denote the set of English auctions, FPSB auctions and Vickrey auctions in $A$, respectively. Therefore, we have

$$A_e \cap A_f = A_e \cap A_v = A_f \cap A_v = \Phi$$

and

$$A_e \cup A_f \cup A_v = A.$$
For an auction $c$, its beginning time and ending time are denoted as $B(c)$ and $D(c)$, respectively. Given that the present system time is $t$, we can easily get that $A = \{ c | D(c) \geq t \}$. Let $h(c)$ be the minimum incremental price for an auction $c$ and its present highest bid be $x(c)$, we have $v(c) = x(c) + h(c)$, where $v(c)$ indicates the minimum bidding price of auction $c$. Furthermore, we assume that in the market, it takes $L$ time units to process a bid so the smallest time gap between two consecutive bids is $L$.

For each bid in an auction, it should be either active or inactive. In an English auction, a bid remains “active” after it is submitted until a higher bid outbids it. When the auction ends, all bids become the “inactive”. All bids in FPSB or Vickrey auctions are active until the auction is over.

After the agent finishes the bidding, the bidding result will be evaluated based on a utility function $U(x)$ as defined by the user. The utility function may be different according to the user requirements. In this section, we adopt the commonly used utility function as given in (5.1) for the later analysis [8]. Note that other utility functions can also be used for BIDS.

$$U(x) = \begin{cases} 
M - x & \text{if the agent can win at price } x \\
0 & \text{otherwise}
\end{cases}$$

(5.1)

Given the above system model, our objective is to design a bidding strategy, with such that an agent can maximize the utility value as computed by (5.1).

### 5.3 Concept of auction chain

In this section, we first introduce the concept of the auction chain. An auction chain is actually an auction sequence where an agent could bid one by
one. We assume that the auction agent only requires one item so that it should stop bidding before the previous bidding result is known. Otherwise, if the agent participates in two active auctions and happens to be the winner in both auctions, it would obtain two items, which is called “accidental purchase”. The agent should avoid the occurrence of such accidental purchase. Before calculating the future expected utility, we should first find out all the possible auction chains between the system time $t$ and the ending time $T$. Denote an auction chain between $t$ and $T$ as $C_{t,T}$ and suppose that $C_{t,T}$ comprises $n$ auctions: $c_1, c_2, \ldots, c_n$. We should fulfill the following conditions:

- All the auctions in $C_{t,T}$ should not be closed at time $t$, i.e., $c_i \in A$, where $1 \leq i \leq n$.
- The respective auctions should be listed according to their deadlines. The deadline of two consecutive auctions should be separated by at least $L$, i.e.,
  \[ D(c_{i+1}) - D(c_i) \geq L \]  
  \[(5.2)\]
  where $1 \leq i \leq n - 1$
- The deadline of auction $c_n$ should not be later than $T-L$, i.e., $D(c_n) \leq T - L$.

Figure 5.2 shows which auctions the agent can participate in. They are represented by the short lines with length $L$. As shown in the figure, an auction chain is defined by a set of non-overlapping lines.
5.4 BIDS

In this section, we present the Backward InDuction bidding Strategy (BIDS) for the aforementioned multi-auction scenario. Using BIDS, an auction agent can decide whether to submit a bid and to decide the appropriate bidding price. Basically, BIDS is based on the following:

- *It is better to stop bidding in an auction if the utility value of the current bid is lower than the expected utility value of all the subsequent auctions.*

- *If not, the auction agent should bid at a specific price.*

In section 5.4.1 to 5.4.3, we introduce the bidding strategy for a single type of auctions.

5.4.1 Bidding in pure English auctions

The operation of BIDS in English auctions is as follows:

- After the deadline ($t > T$) or the agent is waiting for the previous bidding results, it should stop bidding.
- If bidding is allowed, the agent should first find out all the auction chains and compute the respective expected utility.

- Let \( c \) be the first auction in the auction sequence with the maximum expected utility and \( r(c) \) be the maximum bidding price of the auction \( c \). If \( r(c) \geq v_i(c) \), the agent should bid in auction \( c \) with price \( v_i(c) \). If not, the agent should wait because it is better to participate in the upcoming auctions.

**Calculation of future expected utility \( U \)**

Before calculating the future expected utility, we should first find out all auction chains between the current system time \( t \) and the deadline \( T \). For each auction chain, the expected utility is computed. Here we adopt the backward induction method to recursively compute the expected utility value of an auction chain. Given an auction chain \( C \) with \( n \) auctions listed in the ascending order of their deadlines: \( c_1, c_2, \ldots, c_i, \ldots, c_n \). In order to compute the expected utility value of an auction chain \( C \), we first determine whether an agent should bid in an auction. However, the action for \( c_i \) depends on the expected utility between \( c_{i+1} \) and \( c_n \). Denote the expected utility between \( c_i \) and \( c_n \) as \( U_i \) and the expected utility value of auction \( c_i \) as \( u_i \). It can be seen that \( U_i \) is dependent on \( U_{i+1} \). Hence the expected utility of the auction chain can be calculated recursively as follows:

**Step 1: Calculate \( U_n \)**

The agent should only bid in the final auction \( c_n \) if the minimum bidding price is less than its maximum bidding price \( M \). Note that if the agent submits a bid and wins, the final utility value will be greater than zero. However, if the bidding agent does not bid, the utility value will be zero. Denote the maximum bidding price in auction \( c_i \) as \( r(c_i) \). We have \( r(c_n) = M \). Because the bidding agent...
(with the maximum bidding price \( r(c_n) \)) may not buy the item at price \( r(c_n) \), two outcomes are possible:

1. The bidding agent bids at price \( y \), where \( y \leq r(c_n) \) and where the other agents do not submit a higher bid. In this case, the bidding agent wins at price \( x \) and the resultant utility value is \( M - y \).

2. The bidding agent bids up to \( r(c_n) \) but some agents submit a higher bid. The bidding agent loses.

Let the expected utility value of auction \( c_i \) be \( u_i \) and the conditional winning probability for a bidding price \( y \) in the \( i \)-th auction \( c_i \) be \( F_i(y) \) on the condition that the winning price will be lower than \( y \). The expected utility of auction \( c_i \) can be computed as follows:

\[
 u_i = \int_{r(c_i)}^{r(c_n)} (M - y) dF_i(y) \quad (5.3)
\]

For auction \( c_n \), we can get

\[
 U_n = u_n = \int_{r(c_n)}^{M} (M - y) dF_i(y) \quad (5.4)
\]

Step 2: Calculate \( U_i \) based on \( U_{i+1} \)

Assume that if \( U_{i+1} \) is known, the maximum bidding price for auction \( c_i \) can be calculated as:

\[
 r(c_i) = M - U_{i+1} \quad (5.5)
\]

When the auction agent submits a bid \( y \) (\( y > M - U_{i+1} \)) and wins, the utility value is \( M - y \), which is lower than \( U_{i+1} \). In this case, the agent should not bid in auction \( c_i \).
Based on (5.3) and the value of \( r(c_i) \), we can compute \( u_i \). The average utility value that is obtained from auction \( c_i \) is \( u_i \). If the agent fails, it will keep on bidding in the subsequent auctions. In this case, the expected utility value is \( U_{i+1} \). Hence, the bidding agent with a maximum bidding price \( r(c_i) \) will win the auction \( c_i \) with probability \( F_{<c_i}(r(c_i)) \). The average utility value between \( c_i \) and \( c_n \) can be computed as follows:

\[
U_i = u_i + (1 - F_{<c_i}(r(c_i))) \times U_{i+1}
\] (5.6)

The expected utility value for the auction chain \( C \) can then be determined using the recursive equation (5.6).

**A Simple Example**

In this subsection, we present a simple example to explain how BIDS works. Given an agent uses BIDS with a maximum bidding price \( M \) of $100. Before the deadline \( T \), there are a total of five available English auctions as shown in the following table:

<table>
<thead>
<tr>
<th>Auction</th>
<th>Current price</th>
<th>Minimum incremental price</th>
<th>Deadline</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$20</td>
<td>$1</td>
<td>50</td>
</tr>
<tr>
<td>B</td>
<td>$40</td>
<td>$1</td>
<td>60</td>
</tr>
<tr>
<td>C</td>
<td>$40</td>
<td>$1</td>
<td>75</td>
</tr>
<tr>
<td>D</td>
<td>$50</td>
<td>$1</td>
<td>80</td>
</tr>
<tr>
<td>E</td>
<td>$60</td>
<td>$1</td>
<td>110</td>
</tr>
</tbody>
</table>

Assume that the minimum bidding interval between two consecutive bids is 20 and the winning probability is as follows:
Before making decision, the agent first finds out all the auction chains in the market. It is obvious that there are three auction chains as follows:

Chain I: [A C E]

Chain II: [A D E]

Chain III: [B D E]

The expected utility of each auction chain should then be computed. For Chain I, we get \( r(E) = M = 100 \). Since the current price of auction E is $60, the conditional winning probability is:

\[
F_E(x) = \begin{cases} 
0 & x \leq 60 \\
\frac{x - 60}{40} & 60 < x \leq 100 \\
1 & x > 100 
\end{cases}
\]  
(5.8)

Therefore, the expected utility of auction E is:

\[
U_1 = u_3 = \int_{60}^{100} (M - x) dF_E(x) = 19.01
\]  
(5.9)

The maximum bidding price that the agent should use for auction C is \( r(C) = M - U_3 = 100 - 19.01 = 80.99 \). The expected utility for auction C is:

\[
u_2 = \int_{41}^{80.99} (M - x) dF_C(x) = 26.00
\]  
(5.10)

Thus we get

\[
U_2 = u_2 + (1 - F_C(r_2)) \times U_3 = 32.02
\]  
(5.11)
Also, we have \( r(A) = M - U_2 = 100 - 32.02 = 67.98 \). Hence the expected utility for auction A is:

\[
U_i = \int_{21}^{67.98} (M - x) dF_A(x) = 32.60
\]  

(5.12)

Thus

\[
U_1 = u_i + (1 - F_A(r(A))) \times U_2 = 45.42
\]  

(5.13)

Therefore, the expected utility for Chain I is $45.42. Similarly we can compute the expected utility of Chain II is $43.78 and that of Chain III is $35.37. As a result, Chain I has the highest expected utility. Since the current price of auction A is $20, much lower than $67.98, the agent should submit a bid for auction A at a price of $21.

5.4.2 Bidding in pure FPSB auctions

The operation of BIDS for FPSB auctions is similar to that for English auctions and the operation is as follows:

- After the deadline \( t > T \) or the agent is waiting for the previous bidding results, it should stop bidding.

- If bidding is allowed, the agent should first find out all the auction chains and compute the respective expected utility.

- Let \( c \) be the first auction in the auction sequence with the maximum expected utility and \( r(c) \) be the bidding price of the auction \( c \). The agent should submit a bid at price \( r(c) \) for the auction \( c \).
Calculation of future expected utility $U$

The expected utility of the FPSB auction chain is similar to the English auction chain. However, each auction in the auction chain is assigned a bidding price instead of the highest bidding price because of the nature of the sealed-bid auction. Assume that an auction chain $C$ has $n$ auctions, which are listed in ascending order of their deadlines: $c_1, c_2, \ldots, c_i, \ldots, c_n$. In order to compute the expected utility value of the auction chain $C$, we first determine the bidding price in each auction. Denote the bidding price of auction $c_i$ as $x_i$, we can see that $x_i$ depends on the expected utility between $c_{i+1}$ and $c_n$. Denote the expected utility between $c_i$ and $c_n$ as $U_i$ and the expected utility value of auction $c_i$ as $u_i$. It can be seen that $U_i$ is dependent on $U_{i+1}$. Hence the expected utility of the auction chain can be calculated recursively as follows:

**Step 1: Calculate $U_n$**

In FPSB auction, a risk-neutral agent only submits a bid that can maximize the expected utility. Given that the agent submits a bid with price $y$, it will win the auction with probability $F_{c_n}(y)$ and the corresponding utility is $M - y$. Otherwise, the agent should leave the market with utility zero. Thus the expected utility of the last auction with bidding price $y$ is $F_{c_n}(y) \times (M - y)$. Therefore, the expected utility of auction $c_n$ should be:

$$U_n = \max_{0 \leq y \leq M} F_{c_n}(y) \times (M - y) \quad (5.14)$$

Also, $r(c_n)$ should be the corresponding bidding price of $U_n$.

**Step 2: Calculate $U_i$ based on $U_{i+1}$**
Assume that if \( U_{i+1} \) is known, \( U_i \) can be worked out easily. If the agent submits a bid with price \( y \) in auction \( c_i \), it will win the auction with probability \( F_{c_i}(y) \) with utility \( M-y \). Otherwise, it should bid in future auctions with utility \( U_{i+1} \). Thus \( U_i \) should be

\[
U_i = \max_{0 \leq y \leq M} \left( F_{c_i}(y) \times (M - y) + (1 - F_{c_i}(y)) \times U_{i+1} \right)
\]

(5.15)

and \( r(c_i) \) is the corresponding bidding price of \( U_i \).

As mentioned above, the expected utility value for the auction chain \( C \) can be determined using the above recursive equation.

### 5.4.3 Bidding in pure Vickrey auctions

The operation of BIDS for Vickrey auctions can be worked out in a similar manner. The only difference is the calculation of the expected utility. In Vickrey auctions, the buying price is the second highest price instead of the highest price as in FPSB. Therefore, the calculation of the auction chain \( C \) should be revised as follows:

**Step 1: Calculate \( U_n \)**

In Vickrey auctions, a risk-neutral agent only submits the bid that can maximize the expected utility. Given that the agent submits a bid with price \( y \), it can win the auction with probability \( F_{c_n}(y) \). However it should buy the item with the second highest price. Denote \( E(y) \) as the expectation of the second highest price given the highest price \( y \), we can get the expected utility of auction \( c_n \) as

\[
U_n = \max_{0 \leq y \leq M} \left( F_{c_n}(y) \times (M - E(y)) \right)
\]

(5.16)

\( r_n \) should be the corresponding bidding price of \( U_n \).
Step 2: Calculate $U_i$ based on $U_{i+1}$

If the agent submits a bid with price $y$ in auction $c_i$, it can win the auction with probability $F_{c_i}(y)$ and the expected utility is $M-E(y)$. Otherwise, it should bid in future auctions with utility $U_{i+1}$. Thus $U_i$ should be

$$U_i = \max_{0 \leq y \leq M} F_{c_i}(y) \times (M - E(y)) + (1 - F_{c_i}(y)) \times U_{i+1}$$  \hspace{1cm} (5.17)

$r(c_i)$ is the corresponding bidding price of $U_i$.

The expected utility value for the auction chain $C$ can then be determined using the recursive equation.

5.5 Experimental results

In this section, we set up a simulation model to evaluate the performance of BIDS. The simulations assume multiple English auctions with randomly chosen start and end times. For comparison purposes, we use five different types of bidding agents:

- A Simple agent: Participates in a randomly chosen auction using a simple proxy bidding strategy, that is, keeps bidding within a maximum bidding price

- A BIDS agent: Submits bids using BIDS.

- A Greedy agent: Uses the Greedy strategy as described in [1].

- A Historian agent: Uses the Historian strategy as described in [1].
- An Ideal agent: Bids based on the ideal scenario that it knows the winning price of each auction. This is not realistic but it can provide the best performance for comparison purposes. Note that even though the ideal agent knows all the final winning prices, it may still lose if its maximum bidding price is lower.

Each of the following simulations was repeated 1,000 times to obtain the average result.

Firstly, we have conducted a simulation with 50 English auctions and 600 background agents. Each English auction lasted for 55 time units. The background agents are all simple agents with the maximum bids randomly selected between $50 and $100. For each simulation, a BIDS, ideal, Greedy, Historian or simple agent was generated. The agent competed with the background agents using a particular maximum bidding price. To ensure a fair comparison between the different bidding agents, the same simulation environment was used on each occasion.

Figure 5.3. Winning probability of the BIDS agent at different maximum bids
Figure 5.4. Average utility of the BIDS agent when the maximum bid changes

Figures 5.3 and Figure 5.4 compare the winning probability and the average utility value of the different agents. The figures show that the BIDS agents perform much better than the simple agents. It can be seen in Figure 5.3 that the BIDS are more likely to win than the Greedy or Historian agents. When the maximum bidding price is $100, which is the maximum possible bidding price for all agents, the winning probability converges to 1 since no agent can submit a higher bid. Note that the aim of BIDS is not to maximize the winning probability but to maximize the expected utility.

Figure 5.4 shows the expected utility of the agents with different bidding strategies. It is obvious that the average utility of the agent increases with larger maximum bidding prices. Note that, the ideal agents achieve the highest average utility as expected. Among the other four strategies, BIDS agents outperform the others. The simple agents achieve the lowest expected utility. The expected utility of the Greedy and Historian agents are very close.
Figure 5.5. Variation of the maximum bidding price

Figure 5.5 shows the variation of the maximum bidding price of the BIDS agents when there are 800 agents in the market, participating in 70 auctions. The two curves on the graph correspond to two different simulations. Both of them indicate the maximum bidding price at a specific system time. It can be seen that the maximum bid responds to the market situation, which involves factors such as remaining time, number of remaining auctions and the bidding prices of all the active auctions. It can be seen that the current reserve price of the agent is generally increasing and the value will reach $M$ at time $T$. However, sometimes the current reserve price may decrease slightly in response to the market condition (i.e., how the other agents bid).

Furthermore, we have also conducted several simulations with different market sizes: 30 auctions and 400 background agents; 70 auctions and 900 background agents and 100 auctions and 1,100 background agents. Although these markets have different number of auctions and agents, the ratios of number of agents to number of auctions are quite close. Other simulation parameters remain the same and the maximum bidding price for the BIDS agents is $80.$
Figure 5.6 and Figure 5.7 show the winning probability and the expected utility of the BIDS agent with different market sizes, respectively. It can be seen that both the winning probability and the average utility of the agent increase with the market size.

Figure 5.6 Winning probability for different auction durations

Figure 5.7 Expected utility for different auction durations

All the above figures consider a market with pure English auctions. In the following, we consider other types of auctions. FPSB auctions and Vickrey auctions is shown as follows, respectively. Figure 5.8 and Figure 5.9 show the
winning probability and the expected utility of agents in a market with 80 FPSB auctions and 1200 simple agents. It can be seen that the BIDS agents outperform the Historian agents and simple agents in both the winning price and expected utility. Note that the winning probabilities of the BIDS agents and Historian agents are lower than those of the simple agents when the maximum bidding price is $100. It is because the simple agents always bid at the maximum bidding prices and therefore always win when the maximum bidding price exceeds the bids of all other agents. However, the goal of the other two agents is to maximize the expected utility, thus they may submit a lower bid even if their maximum bidding price is very large.

Figure 5.8 Winning probabilities of the agents in a market with FPSB auctions
Figure 5.9 Expected utilities of the agents in a market with FPSB auctions

Figure 5.10 and Figure 5.11 show the winning probability and the expected utility of the agents in a market with 80 Vickrey auctions and 1200 simple agents. Like the market with FPSB auctions, the BIDS agents outperform the Historian agents and simple agents in both the winning price and expected utility.

Figure 5.10 Winning probabilities of the agents in a market with Vickrey auctions
Figure 5.11 Average utilities of the agents in a market with Vickrey auctions

Figure 5.12 and Figure 5.13 show the winning probability and expected utility of agents in a market with three kinds of auctions. These include 30 English auctions, 30 FPSB auctions and 30 Vickrey auctions. There are totally 1200 agents in the market. Again, the BIDS agents outperform the Historian agents and simple agents in both the winning price and expected utility.

Figure 5.12 Winning probabilities of the agents in a market with various auctions
Figure 5.13 Expected utilities of the agents in a market with various auctions

As mentioned above, the behavior of the agents can be changed according to various user preferences, which can be represented by different utility functions. Here we define a new utility function as follows:

\[
U(x) = \begin{cases} 
1 - \left( \frac{x}{M} \right)^\beta & \text{if the agent can win at price } x \\
0 & \text{otherwise}
\end{cases} 
\]  

(5.18)

When \( \beta = 1 \), the utility function is

\[
U(x) = \begin{cases} 
1 - \frac{x}{M} & \text{if the agent can win at price } x \\
0 & \text{otherwise}
\end{cases} 
\]  

(5.19)

Since the maximum bidding price is fixed, the agent is essentially risk-neutral. When \( \beta > 1 \), the value of the final buying price \( x \) has a smaller impact on the final utility value. The agent is essentially risk-averse, which tends to achieve a higher winning probability. The larger the \( \beta \), the larger the degree to try winning the auction. When \( \beta < 1 \), the final buying price \( x \) has a larger impact on the final utility value and the agent tends to achieve a lower buying price rather than a higher winning probability. Figure 5.14 shows the winning probability of
the agents with different utility functions. It can be seen that the winning probability of the agent increases with larger values of $\beta$. Moreover, BIDS is so flexible that a user can even change the evaluation utilities when the agent is in the bidding process. After receiving the new utility function, the agent can automatically adjust its bidding behavior according to the new user preferences.

![Winning Probability of the agents in a market with different utility functions](image)

**Figure 5.14** Winning Probability of the agents in a market with different utility functions

### 5.6 Summary

In this chapter, we have proposed a novel bidding strategy called Backward InDuction bidding Strategy (BIDS), which enables agents to bid in markets with multiple heterogeneous auctions: English auctions, FPSB auctions and Vickrey auctions. BIDS seeks to maximize the value of a utility function by solving a backward induction equation recursively according to different utility functions. The key concept of BIDS is the following: Given that it is not possible to participate in all auctions due to overlapping deadlines, we first identify all possible sets of auctions (the auction chains) in which an agent can bid. The “attractiveness” of each auction chain is then determined by a utility function
based on the past winning probability. Compared with other bidding strategies, BIDS is very flexible that the agent can automatically change its behaviours according to user requirements, which can be represented by various utility functions. Furthermore, the utility functions can even be changed during the bidding process.

A series of experiments/simulations have been conducted to evaluate the performance of BIDS. The results demonstrate that the advantages of BIDS over other strategies, particularly in the ability of BIDS to achieve a higher winning probability and greater expected utility. The bidding behavior of agents can be changed by using a different utility function. We have also done some experiments to compare the bidding results with different user preferences. The results show that risk-averse agents always achieve higher winning probabilities than risk-neutral agents.
CHAPTER 6
CONCLUSIONS

In this thesis, we have contributed to the development of a mobile-agent-based auction systems with a focus on investigating different bidding strategies to facilitate the design of auction agents. In chapter 1, we have introduced the background of this project. Compared to physical auctions, online auctions are more economical and flexible. In particular, the transaction cost for online auctions is negligible for they are not constrained by the geographical distance. It is generally expected that agents will be used to complement the existing online auction systems.

In chapter 2, we have described several bidding strategies for agent-based online auctions. Generally, the bidding strategies can be divided into two categories: single-site bidding strategies and multiple-site bidding strategies. Unlike single-site bidding strategies, the multiple-site bidding strategies can be used to handle multiple heterogeneous auctions. Since many bidding strategies rely on the prediction of winning probabilities, several simple prediction schemes have also been described.

In chapter 3, we have investigated a deterministic bidding strategy. In essence, it is based on the popularly used proxy bidding services. A general mathematical model has been set up to evaluate the bidding results. Two distributions for the reserve price have been considered, namely: linear and normal distribution. Some simulations have been conducted to validate the
correctness of the mathematical model. In general, the generic model can also be applied to analyze the bidding results for other distributions of the reserve price. Furthermore, two prediction schemes called “method of moment” and “maximum likelihood estimator” have also been investigated. According to the previous auction records, they can estimate the winning probability in an auction given a certain maximum bidding price. Some experiments have been conducted to evaluate the performance of the two schemes and to verify their effectiveness.

In chapter 4, we have proposed a probabilistic bidding strategy. For example, it can be used to support m-auction services. A series of bidding strategies have been introduced. For each bidding strategy, a willingness function is used to define the bidding probability at a particular price. A mathematical model has been presented to evaluate the bidding results for both the English and Dutch auction methods. Some simulation results have been presented to validate the analytical results and to analyze the system behavior.

In chapter 5, we have proposed a bidding strategy called BIDS to handle multiple heterogeneous auctions. Based on an auction chain concept and by solving a backward induction equation recursively, BIDS can be used to determine the auctions to participate and the respective bidding prices. Simulation results demonstrate the advantages of BIDS over other strategies, particularly in the ability of BIDS to achieve a higher winning probability and greater expected utility.
REFERENCES


LIST OF SYMBOLS

Chapter 2

$M$  Maximum bidding price of the user

$G$  User’s eagerness to trade, expected probability of obtaining the item

$S_t$  Number of seller agents at time $t$

$B_t$  Number of buyer agents at time $t$

$I_t$  Number of agents interested in the agent’s current offer at time $t$

$T$  The deadline of obtaining the item

$T(t)$  Remaining market time ratio at time $t$

$C(t)$  Competition measurement at time $t$

$A(t)$  Attractiveness of the agent

$b_t$  Prices that the agent submits at time $t$

$O_i$  $i$-th bidding price of the agent

$C(i+1)$  Concession in $i+1$-th episode

$f_{ri}$  Bidding price of the remaining time tactic

$\alpha_r(t)$  Remaining time ratio parameter

$k_{rt}$  A parameter of the remaining time ratio function

$\beta$  A parameter of the remaining time ratio function

$N_T$  Number of items that the agent wants to buy

$N_a$  Number of active bids that the agent has submitted
\( B(x,q) \) Probability that \( x \) bidders value the good with a valuation greater than \( q \) in a given auction

\( L \) Time for the market to process a bid

\( A_s \) Set of auctions that could be participated sequentially

\( P_a(z) \) Winning probability in an auction \( a \) with maximum bidding price \( z \)

\( (x_i, y_i) \) Previous auction record, where each \( x_i \) belongs to a space \( X \) and each \( y_i \) is in \( R \). The \( x_i \)'s are the auction-specific feature vectors and for some \( n \), \( X \subseteq (R \cup \{\bot\})^n \)

\( p_i \) Probability that winning price is no higher than \( b_i \)

**Chapter 3**

\( b_i \) Sequence of bidding prices in an auction

\( h \) Incremental price in English auctions or decremental price in Dutch auction

\( R(b_i) \) Distribution of bidders’ reserve prices

\( F_E(b_i) \) Distribution of winning prices in English auction

\( F_D(b_i) \) Distribution of winning prices in Dutch auction

\( A_E \) Average winning price in English auction

\( A_D \) Average winning price in Dutch auction

\( n \) Number of agents participating in the auction

\( c \) An auction \( c \)

\( n(c) \) Number of agents in the auction \( c \)

\( b_l \) Lower bound of the reserve prices

\( b_r \) Upper bound of the reserve prices
\( H(b) \) Density function of the reserve prices
\( c_1 \) Density value of \( b_i, H(b_i) \)
\( c_2 \) Density value of \( b_r, H(b_r) \)
\( A \) Set of previous auction records
\( |A| \) Number of auction records
\( A_i \) Set of auction records with \( i \) agents participating in the auction
\( a_i \) \( i \)-th auction in \( A \)
\( x \) A function that indicates the winning prices in an auction,
\( x: A \rightarrow N_R \)
\( N \) A function that indicates number of agents in an auction, \( N: A \rightarrow N_R \)
\( F_c(b_z) \) Winning probability in auction \( c \) with reserve price \( b_z \)
\( D(b) \) Deviation of the winning price
\( q \) Current price of an auction
\( E_t(x) \) Average prices of previous auction records with \( t \) agents
\( u_1 \) Average prices of the previous auction records
\( v_2 \) Deviation of previous auction records
\( f(x; Q_1, Q_2, \ldots, Q_k) \) Function on \( x \), with parameters \( Q_1, Q_2, \ldots, Q_k \)
\( u_r(Q_1, Q_2, \ldots, Q_k) \) \( r \)-th origin moment of function \( f(x; Q_1, Q_2, \ldots, Q_k) \)
\( v_r(Q_1, Q_2, \ldots, Q_k) \) \( r \)-th center moment of function \( f(x; Q_1, Q_2, \ldots, Q_k) \)
\( S \) Likelihood function
\( M_j \) \( j \)-th sample moment of the previous auction records
\( O_c(b_z) \) Actual winning probability in auction \( c \) with reserve price \( b_z \)
\( \varepsilon_c(b_z) \) Errors of the prediction method in auction \( c \) with reserve price \( b_z \).

\( \varepsilon \) Square error, used to evaluate the correctness of a prediction function

Chapter 4

\( b_1 \) Minimum bidding price

\( b_z \) Maximum bidding price set by a bidder

\( h \) Incremental bidding price/decremental bidding price

\( W(b_j) \) Willingness value at the bidding price \( b_j \)

\( m_j \) Probability that at least one bidder bids at \( b_j \)

\( x \) Number of bidders in the auction

\( P(b_k) \) Probability that the price will stop at \( b_k \)

Chapter 5

\( M \) Maximum bidding price of the user

\( T \) Deadline to obtain the item

\( A \) Set of auctions in the market that have not ended

\( A_c \) Set of English auctions in the market that have not ended.

\( A_f \) Set of FPSB auctions in the market that have not ended.

\( A_v \) Set of Vickrey auctions in the market that have not ended.

\( B(c) \) Beginning time of auction \( c \)

\( D(c) \) Deadline of auction \( c \)

\( h(c) \) Minimum incremental price for auction \( c \)

\( x(c) \) Present highest bid for auction \( c \)

\( v(c) \) Minimum bidding price for auction \( c \)
<table>
<thead>
<tr>
<th>$L$</th>
<th>Time for the market to process a bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U(x)$</td>
<td>Utility function with final buying price $x$</td>
</tr>
<tr>
<td>$t$</td>
<td>Current system time</td>
</tr>
<tr>
<td>$C_{t,T}$</td>
<td>An auction chain between $t$ and $T$</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of auctions in the auction chain $C$</td>
</tr>
<tr>
<td>$c_i$</td>
<td>$i$-th auction in the auction chain</td>
</tr>
<tr>
<td>$c$</td>
<td>First auction in an auction sequence</td>
</tr>
<tr>
<td>$r(c)$</td>
<td>Maximum bidding price of auction $c$</td>
</tr>
<tr>
<td>$U_i$</td>
<td>Expected utility between $c_i$ and $c_n$</td>
</tr>
<tr>
<td>$u_i$</td>
<td>Expected utility value of auction $c_i$</td>
</tr>
<tr>
<td>$E(y)$</td>
<td>Expectation of second highest price given the highest price</td>
</tr>
<tr>
<td>$y$</td>
<td></td>
</tr>
</tbody>
</table>
APPENDICES

Appendix I

According to the $W(b_k)$ and $P(b_k)$, the average winning price can then be found as follows:

$$E(b) = b_0 + a \sum_{k=1}^{z} k \times (1 - W(b_{k+1}))^{x-1} \prod_{i=2}^{k} (1 - (1 - W(b_i))^{x-1})$$

$$= b_0 + a \sum_{k=1}^{z} k \times (1 - W(b_{k+1}))^{x-1} \prod_{j=2}^{k} (1 - (1 - W(b_j))^{x-1}) + a \times (1 - W(b_{z+1}))^{x-1} \prod_{i=2}^{z} (1 - (1 - W(b_i))^{x-1})$$

$$= b_0 + a(1-p)^{x-1} \sum_{k=1}^{z} k \times (1 - (1 - p)^{x-1})^{k-1} + a \times (1 - (1 - p)^{x-1})^{z-1}\sum_{k=1}^{z} k \times (1 - (1 - p)^{x-1})^{k-1} + a \times (1 - (1 - p)^{x-1})^{z-1}$$

If $p < 1$, we get

$$E(b) = b_0 + a \frac{1 - (z - 1)(1 - (1 - p)^{x-1})^{z-2} + (z - 2)(1 - (1 - p)^{x-1})^{z-1}}{(1 - p)^{x-1}} + a \times (1 - (1 - p)^{x-1})^{z-1}$$

$$= b_0 + a \frac{1 - (z - 1)(1 - (1 - p)^{x-1})^{z-2} + (z - 2)(1 - (1 - p)^{x-1})^{z-1}}{(1 - p)^{x-1}} + a \times (1 - (1 - p)^{x-1})^{z-1}$$

If $p = 1$, $E(b) = b_z$.

Appendix II

According to the bidding strategy defined in section 4.3 and formula 4.20, we get the average winning price as follows:

\[ \sum \sum \prod \]
\[
E(b) = b_0 + a \sum_{k=1}^{z} k \times (1 - (1 - W(b_k)))^x \times \prod_{i=k+1}^{z} (1 - W(b_i))^\gamma
\]
\[
= b_0 + a \sum_{k=2}^{z} k \times (1 - (1 - p)^x) \times (1 - p)^{\gamma(z-k)} \times a(1 - p)^{\gamma(z-1)}
\]
\[
= b_0 + a(1 - (1 - p)^x) \sum_{k=2}^{z} k \times (1 - p)^{\gamma(z-k)} + a(1 - p)^{\gamma(z-1)}
\]

If \( p > 0 \), we get
\[
E(b) = b_0 + a(1 - (1 - p)^x) \frac{z - 2}{(z - 1)(1 - p)^\gamma - (1 - p)^{\gamma(z-3)} + 2(1 - p)^{\gamma(z-2)}} + a(1 - p)^{\gamma(z-1)}
\]
\[
= b_0 + a \frac{(z - 2)(1 - p)^{2x} - (z - 1)(1 - p)^{3x} + (1 - p)^{2z}}{1 - (1 - p)^\gamma}
\]

If \( p = 0 \), we get \( E(b) = b_1 \).