Transmit Beamforming for Frequency–Selective Channels with Decision–Feedback Equalization

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Abstract

In this paper, we propose beamforming schemes for frequency–selective channels with decision–feedback equalization (DFE) at the receiver and with, respectively, perfect and quantized channel state information (CSI) at the transmitter. For beamforming with perfect CSI and infinite impulse response (IIR) beamforming filters (BFFs) we derive a closed–form expression for the optimum BFFs. We also provide two efficient numerical methods for recursive calculation of the optimum finite impulse response (FIR) BFFs with perfect CSI. For beamforming with quantized CSI and finite–rate feedback channel, we propose a global vector quantization (GVQ) algorithm for code–book design. This algorithm is deterministic and independent of initial conditions and does not impose any constraints on the number of transmit and receive antennas, the antenna correlation, or the fading statistics. Our simulation results for typical GSM/EDGE channels show that in general short FIR BFFs are sufficient to closely approach the performance of IIR BFFs even in severely frequency–selective channels. Furthermore, finite–rate feedback beamforming with only a few feedback bits achieves significant performance gains over single–antenna transmission, transmit antenna selection, and optimized delay diversity in frequency–selective fading.

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1 Introduction

In recent years, the application of multiple antennas in wireless communication systems has received considerable interest from academia and industry. In particular, beamforming was shown to be a simple yet efficient technique for exploiting the benefits of multiple transmit antennas, cf. e.g. [1] and references therein.

Beamforming generally requires channel state information (CSI) at the transmitter. Since perfect CSI may not be available at the transmitter in practical systems, recent research in this field has focused on beamforming with imperfect CSI, cf. e.g. [2, 3, 4]. From a practical point of view the assumption of quantized CSI is particularly interesting. In this case, the transmitter selects the beamforming vector from a pre–designed codebook based on information received from the receiver over a finite–rate feedback channel [3, 4]. Depending on the adopted performance measure and the statistical properties of the underlying channel, a closed–form solution for the optimum beamforming vector codebook may or may not exist, cf. [3, 4, 5]. For the latter case vector quantization algorithms have been proposed for codebook construction. In particular, Linde–Buzo–Gray (LBG) type algorithms [6] (also referred to as generalized Lloyd–type algorithms) have been adopted in [5] and [7], respectively. However, the LBG algorithm is a local search procedure and its performance heavily depends on the starting conditions [8].

Most of the existing literature on transmit beamforming with perfect or imperfect CSI has assumed frequency–nonselective fading. A notable exception is [9] where it was shown that beamforming with infinite impulse response (IIR) filters is asymptotically capacity achieving in strongly correlated frequency–selective multiple–input multiple–output (MIMO) fading channels. In addition, in [9] jointly optimum IIR beamforming and equalization filters were derived for various optimization criteria. For systems employing orthogonal frequency–division multiplexing (OFDM) to cope with the frequency selectivity of the channel, effective beamforming techniques for the imperfect CSI case were proposed in [10, 11].

In this paper, we consider transmit beamforming with, respectively, perfect and quantized CSI for frequency–selective fading channels which are typically encountered in high–rate transmission. We focus on single–carrier transmission and the developed beamforming schemes may be used to
e.g. upgrade existing communication systems such as the Global System for Mobile Communications (GSM) and the Enhanced Data Rates for GSM Evolution (EDGE) system. Note that the multi–carrier based techniques in [10, 11] are not applicable in this case. Due to the intersymbol interference (ISI) caused by the frequency selectivity of the channel, equalization is necessary at the receiver and the optimum beamformer depends on the equalizer used. Here, we adopt decision–feedback equalization (DFE) because of its low complexity, good performance, and practical relevance [12, 13]. Although the main emphasis of this paper is on the practically relevant case of finite impulse response (FIR) beamforming, for the sake of completeness and since there are many interesting parallels and differences between the FIR and the IIR cases, we also consider IIR beamforming.

Contributions: This paper makes the following contributions.

- For perfect CSI we derive a closed–form solution for the optimum IIR beamforming filters (BFFs) maximizing the signal–to–noise ratio (SNR). Interestingly, although our derivation of the optimum IIR BFFs is much simpler than that presented in [9] as we do not perform a joint optimization of the BFFs and the DFE filters, our final result is identical to that given in [9]. More importantly, our approach can be readily extended to FIR BFFs which does not seem to be easily possible for the approach taken in [9].

- We show that, similar to the optimum IIR BFFs, the optimum FIR BFFs with perfect CSI are the solution to a nonlinear eigenvalue problem. However, in the FIR case a closed–form solution to this problem does not seem to exist, and we provide two efficient numerical methods for calculation of the optimum FIR BFFs.

- We propose a practical finite–rate feedback beamforming scheme for frequency–selective channels. The beamforming vector codebook design is based on the optimum FIR BFFs for perfect CSI. In particular, exploiting the findings in [14], we propose a global vector quantization (GVQ) algorithm for codebook design which performs a deterministic global search. The GVQ algorithm does not depend on starting conditions and employs the LBG algorithm as a local search procedure.

- Our simulation results show that short FIR BFFs can closely approach the performance of
the optimum IIR BFFs. In fact, for quantized CSI with small codebook sizes BFFs of length one (i.e., beamforming weights) are preferable. If the channel is severely frequency selective, longer BFFs become beneficial as the codebook size increases.

- For typical GSM/EDGE channel profiles beamforming with finite-rate feedback enables large performance gains compared to single-antenna transmission, transmit antenna selection, and optimized delay diversity [15].

Organization: In Section 2, the adopted system model is presented. The optimization of IIR and FIR BFFs with perfect CSI is discussed in Sections 3 and 4, respectively. In Section 5, finite-rate feedback beamforming is considered and the proposed GVQ algorithm is introduced. Simulation results are provided in Section 6, and some conclusions are drawn in Section 7.

Notation: In this paper, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, $I_X$, $0_X$, $\mathcal{E}\{\cdot\}$, and $\otimes$ denote transpose, Hermitian transpose, complex conjugate, the $X \times X$ identity matrix, the all-zero column vector of length $X$, expectation, and discrete-time convolution, respectively. Furthermore, $X(f) \triangleq \mathcal{F}\{x[k]\}$ is the discrete-time Fourier transform of $x[k]$ and $[x]^+ \triangleq \max(x, 0)$.

2 System Model

We consider a MIMO system with $N_T$ transmit and $N_R$ receive antennas. The block diagram of the discrete-time overall transmission model in complex baseband representation is shown in Fig. 1. The independent and identically distributed (i.i.d.) symbols $b[k]$ are taken from a scalar symbol alphabet $\mathcal{A}$ such as phase-shift keying (PSK) or quadrature amplitude modulation (QAM), and have variance $\sigma_b^2 \triangleq \mathcal{E}\{|b[k]|^2\} = 1$. The transmit BFF impulse response coefficients of antenna $n_t$, $1 \leq n_t \leq N_T$, are denoted by $g_{n_t}[k]$, $-q_l \leq k \leq q_u$, and their energy is normalized to $\sum_{n_t=1}^{N_T} \sum_{k=-q_l}^{q_u} |g_{n_t}[k]|^2 = 1$. For IIR BFFs $q_l \to \infty$ and $q_u \to \infty$ and for FIR BFFs $q_l = 0$ and $q_u = L_g - 1$, where $L_g$ is the FIR BFF length. For future reference we define the FIR BFF vector $g \triangleq [g_1[0] \ g_1[1] \ \ldots \ g_1[L_g-1] \ g_2[0] \ \ldots \ g_{N_T}[L_g-1]]^T$ of length $N_T L_g$.

The signal transmitted over antenna $n_t$ at time $k$ is given by

$$s_{n_t}[k] = g_{n_t}[k] \otimes b[k]. \quad (1)$$
The discrete–time received signal at receive antenna $n_r$, $1 \leq n_r \leq N_R$, can be modeled as

$$r_{n_r}[k] = \sum_{n_t=1}^{N_T} h_{n_tn_r}[k] \otimes s_{n_t}[k] + n_{n_r}[k], \quad (2)$$

where $n_{n_r}[k]$ denotes additive (spatially and temporally) white Gaussian noise (AWGN) with variance $\sigma_n^2 \triangleq \mathbb{E}\{|n_{n_r}[k]|^2\} = N_0$, and $N_0$ denotes the single–sided power spectral density of the underlying continuous–time passband noise process. $h_{n_tn_r}[k]$, $0 \leq k \leq L - 1$, denotes the overall channel impulse response (CIR) between transmit antenna $n_t$ and receive antenna $n_r$ of length $L$. In our model, $h_{n_tn_r}[k]$ contains the combined effects of transmit pulse shaping, wireless channel, receive filtering, and sampling. As is typically done for beamforming applications, we assume that the channel is slowly time–variant and can be considered approximately constant for the transmission of a few data packets. In general, the $h_{n_tn_r}[k]$ are spatially and temporally correlated because of insufficient antenna spacing and transmit/receive filtering, respectively. For convenience we define the channel vector $\mathbf{h} \triangleq [h_{11}[0] \ldots h_{11}[L-1] \ h_{12}[0] \ldots h_{N_TN_R}[L-1]]^T$ of length $N_TN_RL$.

By substituting Eq. (1) into Eq. (2) we obtain

$$r_{n_r}[k] = h_{n_r}^{eq}[k] \otimes b[k] + n_{n_r}[k], \quad (3)$$

where the equivalent CIR $h_{n_r}^{eq}[k]$ corresponding to receive antenna $n_r$ is defined as

$$h_{n_r}^{eq}[k] = \sum_{n_t=1}^{N_T} h_{n_tn_r}[k] \otimes g_{n_t}[k]. \quad (4)$$

Eq. (3) shows that a MIMO system with beamforming can be modeled as an equivalent single–input multiple–output (SIMO) system. Therefore, at the receiver the same equalization, channel estimation, and channel tracking techniques as for single–antenna transmission can be used. Here, we adopt receive–diversity zero–forcing (ZF) or minimum mean–squared error (MMSE) DFE [13] for detection and assume perfect channel estimation at the receiver.

Furthermore, we assume that a feedback channel from the receiver to the transmitter is available, cf. Fig. 1. In case of perfect CSI, the receiver sends the optimum BFFs (or equivalently the channel vector $\mathbf{h}$) to the transmitter. If the feedback channel allows only the transmission of $B$
bits per channel update, the receiver and the transmitter have to agree on a pre–designed BFF vector codebook \( G = \{ \hat{g}_1, \hat{g}_2, \ldots, \hat{g}_N \} \) of size \( N = 2^B \), where \( \hat{g}_n \) is a vector of length \( N_L g \) and \( \hat{g}_n^H \hat{g}_n = 1, 1 \leq n \leq N \). For a given channel vector \( h \) the receiver determines the address \( n \) of the codeword (BFF vector) \( \hat{g}_n \in G, 1 \leq n \leq N \), which yields the minimum bit error rate (BER). Subsequently, address \( n \) is sent to the transmitter which then utilizes \( g = \hat{g}_n \) for beamforming.

Since the primary goal of this paper is to investigate the achievable performance of beamforming with, respectively, perfect and quantized CSI at the transmitter, similar to [3, 4, 5] we assume that the feedback channel is error–free and has zero delay.

3 Beamforming with Perfect CSI and IIR Filters

In this section, we assume perfect CSI and IIR BFFs. For this scenario jointly optimum BFFs and equalization filters were derived in [9, Section IV]. Thereby, the BFFs and equalization filters were optimized for a fixed overall CIR (including the BFFs, the channel, and the equalization filters), and subsequently this overall CIR was chosen or optimized. Here, we pursue a much simpler and more straightforward approach. In particular, we assume that the receiver employs the optimum ZF– or MMSE–DFE filters for given MIMO channel and BFFs, and optimize the BFFs for maximization of the SNR under this assumption.

3.1 Optimization Problem

For convenience the frequency responses of the IIR BFFs \( G_n(f) \triangleq \mathcal{F}\{g_n[k]\} \) are collected in vector \( \mathbf{G}(f) \triangleq [G_1(f) \ G_2(f) \ \ldots \ G_{N_T}(f)]^T \). For a given channel vector \( h \) and a given beamforming vector \( G(f) \), the unbiased SNR for DFE with optimum IIR feedforward and corresponding feedback filtering is given by [12, 13]

\[
\text{SNR}(\mathbf{G}(f)) = \frac{\sigma^2_b}{\sigma^2_n} \exp \left\{ \int_{-1/2}^{1/2} \ln \left[ \xi + \sum_{n_r=1}^{N_R} |H_{n_r}^{eq}(f)|^2 \right] df \right\} - \chi, \tag{5}
\]

where \( \chi = 0, \xi = 0 \) and \( \chi = 1, \xi = \sigma^2_n/\sigma^2_b \) for ZF- and MMSE–DFE filter optimization, respectively. In Eq. (5), the equivalent channel frequency response \( H_{n_r}^{eq}(f) \triangleq \mathcal{F}\{h_{n_r}^{eq}[k]\} \) is given
by \( H_{n_r}^{eq}(f) = \sum_{n_t=1}^{N_T} G_{n_t}(f) H_{n_t n_r}(f) \) with \( H_{n_t n_r}(f) \triangleq \mathcal{F}\{h_{n_t n_r}[k]\} \).

The optimum BFF vector \( \bar{G}(f) \triangleq [\bar{G}_1(f) \ \bar{G}_2(f) \ \ldots \ \bar{G}_{N_T}(f)]^T \) shall maximize \( \text{SNR}(G(f)) \) subject to the transmit power constraint

\[
\int_{-1/2}^{1/2} \bar{G}(f)^H G(f) \, df = 1.
\] (6)

A convenient approach for calculating \( \bar{G}(f) \) is the classical Calculus of Variations method [16]. Following this method, we model the BFF of antenna \( n_t \) as \( G_{n_t}(f) = \bar{G}_{n_t}(f) + \varepsilon_{n_t} B_{n_t}(f) \), where \( B_{n_t}(f) \) and \( \varepsilon_{n_t} \) denote an arbitrary function of \( f \) and a real–valued variable, respectively. The optimization problem can now be formulated in terms of its Lagrangian

\[
L(\varepsilon) = \text{SNR}(G(f)) + \mu \int_{-1/2}^{1/2} G(f)^H G(f) \, df,
\] (7)

where \( \varepsilon \triangleq [\varepsilon_1 \ \varepsilon_2 \ \ldots \ \varepsilon_{N_T}]^T \) and \( \mu \) is the Lagrange multiplier. The optimum BFF vector \( \bar{G}(f) \) has to fulfill the condition [16]

\[
\frac{\partial L(\varepsilon)}{\partial \varepsilon^*} \bigg|_{\varepsilon=0_{N_T}} = 0_{N_T},
\] (8)

for arbitrary \( B_{n_t}(f), 1 \leq n_t \leq N_T \).

### 3.2 Optimum IIR BFFs

Combining Eqs. (7) and (8), it can be shown that vector \( \bar{G}(f) \) has to fulfill

\[
S(f) \bar{G}(f) = \bar{\mu} [\bar{G}(f)^H S(f) G(f) + \xi] \bar{G}(f),
\] (9)

where \( \bar{\mu} \) is a constant and \( S(f) \) is an \( N_T \times N_T \) matrix given by \( S(f) \triangleq \sum_{n_r=1}^{N_R} H_{n_t n_r}(f) H_{n_r}^H(f) \),

\[
H_{n_r}(f) \triangleq [H_{1n_r}(f) \ H_{2n_r}(f) \ \ldots \ H_{N_T n_r}(f)]^T.
\]

Eq. (9) is a nonlinear eigenvalue problem and \( \bar{G}(f) \) can be expressed as

\[
\bar{G}(f) = X(f) E(f),
\] (10)

where \( E(f) \triangleq [E_1(f) \ E_2(f) \ \ldots \ E_{N_T}(f)]^T \) is that unit–norm eigenvector of \( S(f) \) which corresponds to its largest eigenvalue \( \lambda_{\text{max}}(f) \), and \( X(f) \) is a scalar factor. For example, for \( N_R = 1 \)
we obtain $E(f) = H_1(f)/\sqrt{H_1^H(f)H_1(f)}$. Eq. (10) shows that in general the optimum IIR BFFs can be viewed as concatenation of two filters: A filter $X(f)$ which is common to all transmit antennas and a filter $E_{nt}(f)$ which is transmit antenna dependent. In the following, we derive filter $X(f)$ for ZF–DFE and MMSE–DFE.

1) ZF–DFE: In this case, $\xi = 0$ is valid and combining Eqs. (9) and (10) results in $|X(f)| = 1/\sqrt{\mu}$. Furthermore, from the power constraint in Eq. (6), we obtain $\bar{\mu} = 1$. Therefore, the optimum IIR BFFs for ZF–DFE are given by

$$\bar{G}(f) = E(f) e^{j\varphi(f)}, \quad (11)$$

where $\varphi(f)$ is the phase which can be chosen arbitrarily. Replacing $G(f)$ in Eq. (5) by $\bar{G}(f)$ from Eq. (11) yields the maximum SNR for ZF–DFE

$$\text{SNR}(\bar{G}(f)) = \frac{\sigma_b^2}{\sigma_n^2} \exp \left\{ \frac{1}{2} \int_{-1/2}^{1/2} \ln \left( \lambda_{\max}(f) \right) df \right\}. \quad (12)$$

2) MMSE–DFE: For MMSE–DFE $\xi = \sigma_n^2/\sigma_b^2$ holds. Therefore, it can be shown that in this case the magnitude of the optimum $X(f)$ is given by

$$|X(f)| = \sqrt{\left[ \frac{1}{\bar{\mu}} - \frac{\xi}{\lambda_{\max}(f)} \right]^{+}}, \quad (13)$$

where we took into account that $|X(f)|$ has to be non–negative. Finding the optimum $\bar{\mu}$ is a typical Water Filling problem [17]. In particular, motivated by the power constraint in Eq. (6) we define

$$P(\bar{\mu}) \triangleq \int_{-1/2}^{1/2} \left[ \frac{1}{\bar{\mu}} - \frac{\xi}{\lambda_{\max}(f)} \right]^{+} df. \quad (14)$$

The optimum $\bar{\mu}_{\text{opt}}$ can be found by slowly increasing a very small starting value $\bar{\mu} = \bar{\mu}_0$ which yields $P(\bar{\mu}_0) > 1$ until $P(\bar{\mu} = \bar{\mu}_{\text{opt}}) = 1$ is achieved. The optimum IIR BFFs for MMSE–DFE are given by

$$\bar{G}(f) = \sqrt{\left[ \frac{1}{\bar{\mu}_{\text{opt}}} - \frac{\xi}{\lambda_{\max}(f)} \right]^{+}} E(f) e^{j\varphi(f)}, \quad (15)$$
where $\varphi(f)$ is again the phase which can be chosen arbitrarily. The corresponding maximum SNR is

$$\text{SNR}(\bar{G}(f)) = \frac{\sigma_b^2}{\sigma_n^2} \exp \left\{ \int_{-1/2}^{1/2} \ln \left( \left[ \frac{\lambda_{\max}(f)}{\mu_{\text{opt}}} - \xi \right]^+ + \xi \right) \, df \right\} - 1. \quad (16)$$

Regarding the BFFs and the maximum SNRs for ZF–DFE and MMSE–DFE, we make the following interesting observations.

a) Although we have optimized the BFFs for fixed DFE filters, the optimum IIR BFF vectors given by Eqs. (11) and (15) are identical to the jointly optimum solution given in [9].

b) As expected, for $\sigma_n^2 \to 0$ (i.e., $\xi \to 0$) the optimum IIR BFFs for MMSE–DFE approach those for ZF–DFE. Furthermore, for arbitrary $\sigma_n^2$ and $L = 1$ (i.e., frequency–nonselective channel) the optimum BFFs for ZF–DFE and MMSE–DFE, respectively, have only one non–zero tap and are identical to the well–known beamforming weights, cf. e.g. [1].

c) In case of ZF–DFE, the optimum BFF frequency response vector $\bar{G}(f)$ at frequency $f = f_0$ is just the dominant eigenvector of matrix $S(f)$ at frequency $f = f_0$. Since $S(f_0)$ only depends on the channel frequency responses $H_{n_{tn_r}}(f)$ at frequency $f = f_0$, $\bar{G}(f_0)$ is independent of the $H_{n_{tn_r}}(f)$, $f \neq f_0$. This is not true for MMSE–DFE, where the optimum frequency response vector $\bar{G}(f)$ at frequency $f = f_0$ also depends on the channel frequency responses $H_{n_{tn_r}}(f)$ at frequencies $f \neq f_0$, because of the constraint $P(\tilde{\mu}) = 1$, cf. Eq. (14). In fact, for MMSE–DFE $X(f)$ may be interpreted as a power allocation filter which allocates more power to frequencies with large eigenvalues $\lambda_{\max}(f)$.

The SNRs achievable with IIR BFFs given by Eqs. (12) and (16) will serve as upper bounds for the SNRs achievable with the FIR BFFs considered in the next section.

## 4 Beamforming with Perfect CSI and FIR Filters

The IIR BFFs derived in the previous section are not suitable for practical implementation. Therefore, in this section, we derive the optimum FIR BFFs for perfect CSI using the same approach as for the IIR case in Section 3. We note that although FIR BFFs are adopted in this section, the DFE
is still assumed to employ an IIR feedforward filter. This is not a major restriction as reasonably long FIR DFE feedforward filters yield practically the same performance as IIR feedforward filters.

4.1 Optimum FIR BFFs

Since the FIR BFFs have length $L_g$, the resulting equivalent overall CIR $h_{eq}[k]$ has length $L_{eq} = L + L_g - 1$. The frequency response of the equivalent channel can now be expressed as

$$H_{eq}^r(f) = d^H(f)H_{nr}^r,$$

where $d(f) \triangleq [1 \ e^{j2\pi f} \ldots e^{j2\pi f(L_{eq}-1)}]^T$, $H_{nr}^r \triangleq [H_{1nr} \ H_{2nr} \ldots H_{N_Tnr}]$, and $H_{nrnr}$ is an $L_{eq} \times L_g$ column–circulant matrix with vector $[h_{nr0} \ldots h_{nrl}[L-1] 0_{L_g-1}^T$ in the first column.

Combining Eqs. (5) and (17) the SNR of ZF- and MMSE–DFE with FIR BFFs is obtained as

$$\text{SNR}(g) = \frac{\sigma_b^2}{\sigma_n^2} \exp \left\{ \frac{1}{2} \int_{-1/2}^{1/2} \ln(g^H B(f)g + \xi) \, df \right\} - \chi,$$

with $N_TL_g \times N_TL_g$ matrix $B(f) \triangleq \sum_{n=1}^{N_T} H_{nr}^H d(f)d^H(f)H_{nr}$. The optimum BFF vector $\bar{g}$ shall maximize $\text{SNR}(g)$ subject to the power constraint $g^Hg = 1$. Thus, the Lagrangian of the optimization problem can be formulated as

$$L(g) = \text{SNR}(g) + \mu g^Hg,$$

where $\mu$ denotes again the Lagrange multiplier. The optimum vector $\bar{g}$ has to fulfill $\partial L(g)/\partial g^* = 0_{N_TL_g}$, which leads to the nonlinear eigenvalue problem

$$\begin{bmatrix} \int_{-1/2}^{1/2} \frac{B(f) + \xi I_{N_TL_g}}{g^H(B(f) + \xi I_{N_TL_g})} \, df \\ \int_{-1/2}^{1/2} \frac{B(f) + \xi I_{N_TL_g}}{\bar{g}^H(B(f) + \xi I_{N_TL_g})} \, df \end{bmatrix} \bar{g} = \bar{g},$$

where $\mu$ does not appear since Eq. (20) already includes the constraint $\bar{g}^H\bar{g} = 1$. However, in contrast to the IIR eigenvalue problem in Eq. (9), Eq. (20) does not seem to have a closed–form solution. To substantiate this claim, we discretize the integral in Eq. (18) and rewrite the
optimization problem in Eq. (19) as
\[
\tilde{L}(g) = \prod_{i=1}^{S} \frac{g^H (B(f_i) + \xi I_{N_T L_g}) g}{g^H g}, \tag{21}
\]
where \( f_i = -1/2 + (i-1)/(S-1) \) and \( S \) is a large integer. Note that Eq. (21) is scale invariant (i.e., \( \tilde{L}(g) \) does not depend on the magnitude of \( g \)) and the resulting solution has to be scaled to meet \( g^H g = 1 \). Eq. (21) is a product of Rayleigh quotients. Unfortunately, it is well–known that the maximization of a product of Rayleigh quotients is a difficult mathematical problem which is not well understood and a closed–form solution is not known except for the trivial case \( S = 1 \), cf. e.g. [18, 19]. Therefore, we also do not expect to find a closed–form solution for the nonlinear eigenvalue problem in Eq. (20). Instead, we provide two efficient numerical methods for recursive calculation of the optimum FIR BFF vector \( \bar{g} \) in the next section.

4.2 Calculation of the Optimum FIR BFFs

For calculation of the optimum FIR BFFs we propose two different algorithms. Both algorithms recursively improve an initial BFF vector \( g_0 \). Since the cost function in Eq. (18) is not concave, we cannot guarantee that the algorithms converge to the global maximum. However, adopting the initialization procedure explained below, the solutions found by both algorithms seem to be close–to–optimum, i.e., for \( L_g \gg 1 \) the obtained FIR BFFs approach the performance of the optimum IIR BFFs derived in Section 3.

1) Gradient Algorithm (GA): The first algorithm is a typical GA where in iteration \( i + 1 \) the BFF vector \( g_i \) from iteration \( i \) is improved in the direction of the steepest ascent [17]. The GA is summarized in Table 1. The main drawback of the GA is that its speed of convergence critically depends on the adaptation step size \( \delta_i \), which has to be empirically optimized.

2) Modified Power Method (MPM): It is interesting to observe that if the denominator under the integral in Eq. (20) was absent, \( \bar{g} \) would simply be the maximum eigenvalue of matrix \( \int_{-1/2}^{1/2} (B(f) + \xi I_{N_T L_g}) \, df \), which could be calculated efficiently using the so–called Power Method [17]. Motivated by this observation, we propose an MPM for recursive calculation of \( \bar{g} \). The corresponding algorithm is also given in Table 1 and does not involve an adaptation step size.
As mentioned earlier, global convergence of the GA and the MPM to the maximum SNR solution cannot be guaranteed. However, we found empirically that convergence to the optimum or a close-to-optimum solution is achieved if the BFF length is gradually increased. If the desired BFF length is $L_g$, the GA or the MPM are executed $L_g$ times. For the $\nu$th, $2 \leq \nu \leq L_g$, execution of the algorithm, the first $\nu - 1$ BFF coefficients of each antenna are initialized with the optimum BFF coefficients for that antenna obtained from the $(\nu - 1)$th execution and the $\nu$th coefficients are initialized with zero. For the first ($\nu = 1$) execution, the BFF vector is initialized with the normalized all-ones vector of length $N_T$.

If this initialization procedure is used to calculate the optimum BFFs of length $L_g$, the optimum or close-to-optimum FIR BFFs of lengths 1, 2, $\ldots$, $L_g - 1$ are also obtained as a by-product. This property may be useful when comparing FIR BFFs of different length. Of course, this comes at the cost of increased computational complexity. However, computational complexity is not a major concern here, since in practice beamforming with perfect CSI is not possible anyway. Nevertheless, FIR beamforming with perfect CSI is of interest because it (a) constitutes a benchmark for beamforming with quantized CSI and (b) the optimum FIR BFFs are the input to the (off-line) codebook design for beamforming with quantized CSI, cf. Section 5.

Extensive simulations have shown that the FIR BFFs obtained by respectively the GA and the MPM with the proposed initialization procedure approach the performance of the optimum IIR BFFs as the FIR filter length $L_g$ increases. Exemplary simulation results for both algorithms are shown and discussed in Section 6.

## 5 FIR Beamforming with Quantized CSI

In this section, we consider FIR beamforming with quantized CSI. For this purpose, we first introduce vector quantization in the context of finite-rate feedback beamforming for frequency-selective channels. Subsequently, we discuss the mean quantization error and the distortion measure adopted for quantizer design. Then, we adapt the basic LBG algorithm to the problem at hand and present a global vector quantization (GVQ) algorithm for calculation of the BFF vector codebook $\mathcal{G}$. 
5.1 Vector Quantization – Preliminaries

We assume that a training set $\mathcal{H} \triangleq \{ h_1, h_2, \ldots, h_T \}$ of $T$ channel vectors $h_n$ is available. Thereby, channel vector $h_n$ has length $N_T N_R L$ and contains the CIR coefficients of all $N_T N_R$ CIRs of the $n$th realization of the MIMO channel. In practice, the $h_n$ may be obtained either from measurements or by simulating the MIMO channel. Using the GA or MPM summarized in Table 1 we calculate the optimum BFF vector $\bar{g}_n$ for each channel $h_n$, $1 \leq n \leq T$. The resulting BFF vector training set is denoted by $\mathcal{G}_T \triangleq \{ \bar{g}_1, \bar{g}_2, \ldots, \bar{g}_T \}$.

A vector quantizer $Q$ is a mapping of the BFF vector training set $\mathcal{G}_T$ with $T$ entries to the BFF vector codebook $\mathcal{G} \triangleq \{ \hat{g}_1, \hat{g}_2, \ldots, \hat{g}_N \}$ with $N$ entries, where $N \ll T$ [6]. Therefore, the vector quantizer can be represented as a function $Q : \mathcal{G}_T \rightarrow \mathcal{G}$. The elements $\hat{g}_n$ of the codebook $\mathcal{G}$ are also referred to as codewords. Once $Q$ is determined, we can define partition regions $\mathcal{R}_n$ constituted by subsets of the original training set

$$\mathcal{R}_n \triangleq \{ \bar{g} \in \mathcal{G}_T | Q(\bar{g}) = \hat{g}_n \}, \quad 1 \leq n \leq N, \quad (22)$$

i.e., if $\bar{g}$ falls into $\mathcal{R}_n$ it is quantized to $\hat{g}_n$.

5.2 Mean Quantization Error (MQE) and Distortion Measure

In general, a vector quantizer is said to be optimum if it minimizes the mean quantization error (MQE) for a given codebook size $N$. The MQE is defined as

$$\text{MQE} \triangleq \frac{1}{T} \sum_{n=1}^{T} d(Q(\bar{g}_n), \hat{g}_n), \quad (23)$$

where $d(\hat{g}_m, \bar{g}_n)$ is the so-called distortion measure and denotes the distortion caused by quantizing $\bar{g}_n \in \mathcal{G}_T$ to $\hat{g}_m \in \mathcal{G}$.

Here, our aim is to design a codebook $\mathcal{G}$ which minimizes the average BER, i.e., the average BER is the MQE. Therefore, the distortion measure $d(\hat{g}_m, \bar{g}_n)$ is the BER $P_e(\hat{g}_m, h_n)$ caused by $\hat{g}_m \in \mathcal{G}$ for channel $h_n \in \mathcal{H}$ with optimum BFF vector $\bar{g}_n \in \mathcal{G}_T$, i.e.,

$$d(\hat{g}_m, \bar{g}_n) \triangleq P_e(\hat{g}_m, h_n). \quad (24)$$
In order to obtain a tractable expression for the BER, we assume Gray mapping, DFE with error-free feedback, and that the residual error at the input of the DFE decision device is a Gaussian random variable independent of the data symbol \( b[k] \). The latter assumption is true for ZF–DFE and is a good approximation for MMSE–DFE. With these assumptions the BER of DFE can be approximated as

\[
P_e(\hat{g}_m, h_n) \triangleq C \cdot Q \left( \sqrt{\frac{d_{\text{min}}^2 \text{SNR}(\hat{g}_m, h_n)}{2}} \right)
\]

where \( Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} \, dt \), and both the unimportant constant \( C \) and the minimum Euclidean distance \( d_{\text{min}} \) depend on signal constellation \( \mathcal{A} \). For example, \( C = 2/\log_2 M \) and \( d_{\text{min}} = 2 \sin(\pi/M) \) for \( M \)-ary PSK (MPSK). \( \text{SNR}(\hat{g}_m, h_n) \) can be obtained from Eq. (18).

5.3 LBG Algorithm

The LBG algorithm [6] can be used to improve a given initial codebook. This algorithm exploits two necessary conditions that an optimal vector quantizer satisfies (Lloyd–Max conditions):

1. **Nearest Neighborhood Condition (NNC):** For a given codebook \( \mathcal{G} \) the partition regions \( \mathcal{R}_n \), \( 1 \leq n \leq N \), satisfy

\[
\mathcal{R}_n = \{ \bar{g} \in \mathcal{G}_T | d(\hat{g}_n, \bar{g}) \leq d(\hat{g}_m, \bar{g}), \forall m \neq n \}
\]

i.e., \( \mathcal{R}_n \) is the Voronoi region of codeword \( \hat{g}_n \), \( 1 \leq n \leq N \).

2. **Centroid Condition (CC):** For given partitions \( \mathcal{R}_n \), \( 1 \leq n \leq N \), the optimum codewords satisfy

\[
\hat{g}_n = \underset{\bar{g} \in \mathcal{R}_n}{\text{argmin}} \{ \text{MQE}_n(\bar{g}) \},
\]

where the MQE for region \( \mathcal{R}_n \) and candidate codeword \( \bar{g} \) is defined as

\[
\text{MQE}_n(\bar{g}) \triangleq \frac{1}{|\mathcal{R}_n|} \sum_{\bar{g}' \in \mathcal{R}_n} d(\bar{g}, \bar{g}')
\]
codebook $G$. In the second part of the iteration, the CC is used to find a new codebook based on the partitions found in part one of the iteration. We note that, in contrast to the typically used Euclidean distance distortion measure [6, 8], for the distortion measure in (24) it is not possible to find a closed-form solution for the centroid in Eq. (27). Therefore, $\text{MQE}_n(\bar{g})$ is computed for all training vectors $\bar{g} \in \mathcal{R}_n$, and that training vector which minimizes $\text{MQE}_n(\bar{g})$ is chosen as the new codeword for region $\mathcal{R}_n$. The complexity of this operation can be considerably reduced by pre-computing and storing all $T^2$ possible $d(\bar{g}_m, \bar{g}_n), 1 \leq m, n \leq T$. The LBG algorithm terminates if the reduction in the global MQE given by Eq. (23) from one iteration to the next iteration becomes negligible.

Unfortunately, a codebook $G$ satisfying the NCC and the CC may be a local optimum, and therefore, the final codebook obtained by the LBG algorithm may be a local optimum as well. The most common approach to mitigate the effects of the local optimum problem is to run the LBG algorithm for many different random initial codebooks [8]. The final codebook which yields the lowest global MQE is then used for quantization. Recently, more systematic approaches to overcome the local optimum problem have been reported in the neural network and pattern recognition literature [8]. Two prominent examples are the enhanced LBG algorithm [20] and the adaptive incremental LBG algorithm [21]. These algorithms try to optimize the codeword placement using the assumption that for the globally optimum vector quantizer the MQEs of all partition regions are approximately equal [22]. However, while this assumption is valid for large codebooks, it is questionable for codebooks with a small number of codewords [22]. Since for beamforming small codebooks are both desirable and sufficient to achieve close-to-perfect-CSI performance, we do not further pursue the enhanced and the adaptive incremental LBG algorithms here. Instead, we adapt the so-called global $k$–means clustering algorithm [14] to our problem, since it is particularly well suited for small codebooks [8].

5.4 Global Vector Quantization (GVQ) Algorithm

The global $k$–means clustering algorithm [14] is based on the assumption that the optimum codebook with $i$ codewords can be obtained by initializing the LBG algorithm with the optimal codebook
with $i - 1$ codewords. Although this assumption is difficult to prove theoretically, the excellent performance of the global $k$–means clustering algorithm has been shown experimentally in [14]. Therefore, we adapt the global $k$–means clustering algorithm to the finite–rate feedback beamforming problem and refer to the resulting algorithm as GVQ algorithm in the following. The GVQ algorithm can be used to either design the optimum codebook for a given number of codewords $N$ or to find the codebook with the minimum number of codewords for a given target MQE.

The proposed GVQ algorithm is summarized in Table 2. It is interesting to note that in order to find the optimum codebook of size $N$, the GVQ algorithm computes all intermediate codebooks of size 1, 2, $\ldots$, $N - 1$. This property is useful when comparing the performance of codebooks of different size as they can be obtained by executing the GVQ algorithm only once.

Note that the proposed GVQ algorithm is completely deterministic, which is a major advantage over related algorithms such as the enhanced and the adaptive incremental LBG algorithms [8]. On the other hand, for applications with large codebooks (e.g. $N > 500$) such as image compression the main drawback of the underlying $k$–means clustering algorithm is its high complexity. In particular, the $k$–means clustering algorithm (and therefore also the proposed GVQ algorithm) requires $O(TN)$ executions of the LBG algorithm. However, for finite–rate feedback beamforming problems complexity is not a major concern as typically only a few thousand training BFF vectors are required to capture the statistical behavior of the channel and codebooks with $N < 200$ are usually sufficient to achieve close–to–perfect–CSI performance. Furthermore, it is important to note that codebooks for finite–rate feedback beamforming are designed off–line. Therefore, the proposed GVQ algorithm is an attractive and feasible solution.

6 Simulation and Numerical Results

In this section, we present simulation and numerical results for DFE with transmit beamforming with perfect and quantized CSI, respectively. For all results shown we assume $N_R = 1$ receive antenna and $N_T = 3$ equally mutually correlated transmit antennas with correlation coefficient $\rho = 0.5$. As relevant practical examples we consider the severely frequency–selective equalizer test (EQ) and the moderately frequency–selective typical urban (TU) channel profiles of the GSM/EDGE
system [23]. For modulation we consider Gaussian minimum–shift keying (GMSK) and 8–PSK used in GSM and EDGE, respectively. Thereby, GMSK is modeled as filtered binary PSK (BPSK) as it is usually done in GSM, and a linearized GMSK pulse is assumed for 8–PSK pulse shaping, which is typical for EDGE [24].

6.1 Beamforming with Perfect CSI

In this subsection, we compare FIR and IIR beamforming assuming perfect CSI at the transmitter. However, first we illustrate the convergence behavior of the proposed GA and MPM for calculation of the FIR BFF vector \( g \). We assume \( L_g = 1 \) and for both algorithms the BFF vector is initialized with \( g_0 = \frac{1}{\sqrt{3}}[1 \ 1 \ 1]^T \) as discussed in Section 4.2. Fig. 2 shows the SNR of MMSE–DFE [calculated from Eq. (18)] vs. iteration number \( i \) for one random realization of the EQ channel and \( 10 \log_{10}(E_s/N_0) = 10 \) dB, where \( E_s \) is the average received energy per symbol. While both algorithms converge to the same SNR value, the convergence speed of the MPM is much higher than that of the GA.

Therefore, for all FIR beamforming results shown in the following, the BFFs were calculated for each channel realization using the MPM with \( \epsilon = 10^{-4} \). The BFF vectors were initialized using the procedure proposed in Section 4.2.

In Figs. 3 and 4 we show for, respectively, the EQ and the TU channel the average SNR (\( \text{SNR} \)) of ZF–DFE with IIR BFFs and MMSE–DFE with FIR and IIR BFFs. Thereby, \( \text{SNR} \) was obtained by averaging the respective SNRs in Eqs. (12), (16), and (18) over 500 independent realizations of the EQ and TU channels. For comparison we also show the average SNR of ZF–DFE and MMSE–DFE for single–antenna transmission \( (N_T = 1, N_R = 1) \) in Figs. 3 and 4. For both channel profiles transmit beamforming with IIR filters leads to performance gains of 5 dB or more compared to single–antenna transmission. However, while for the moderately frequency–selective TU channel an FIR BFF length of \( L_g = 1 \) achieves practically the same performance as IIR beamforming, especially at high \( E_s/N_0 \), for the severely frequency–selective EQ channel increasing the FIR BFF length beyond \( L_g = 1 \) is highly beneficial.

The fact that the FIR BFFs with large enough \( L_g \) approach the performance of IIR beamforming
in Figs. 3 and 4 also confirms the effectiveness of the MPM for FIR BFF computation.

### 6.2 Finite–Rate Feedback Beamforming

In this section, we present simulation and numerical results for the bit error rate (BER) of MMSE–DFE with finite–rate feedback beamforming. For codebook design the proposed GVQ algorithm is applied to training sets of $T = 5000$ independent MIMO channel realizations generated for the EQ and the TU channel profiles, respectively, assuming an SNR of $10 \log_{10}(E_b/N_0) = 10$ dB, where $E_b \triangleq E_s/\log_2 M$ denotes the received energy per bit.

Figs. 5 and 6 show the BER of MMSE–DFE for finite–rate feedback beamforming (solid lines) as a function of the number of feedback bits $B$ for, respectively, BPSK transmission over the EQ channel and 8–PSK transmission over the TU channel. The BER is identical to the global MQE and is obtained by evaluating Eq. (23) for the 5000 training channels. For comparison Figs. 5 and 6 also contain the respective BERs of MMSE–DFE for beamforming with perfect CSI (dashed lines). For $B = 0$ the codebook has just one entry and no feedback is required. In this case, beamforming degenerates to delay diversity. As can be observed from Figs. 5 and 6 finite–rate feedback beamforming approaches the performance of the perfect CSI case as $B$ increases. For the severely frequency–selective EQ channel increasing the BFF length from $L_g = 1$ to $L_g = 2$ or $L_g = 3$ results in a performance gain for both quantized and perfect CSI, cf. Fig. 5. In contrast, as already expected from Fig. 4, for the moderately frequency–selective TU channel $L_g = 1$ is near optimum and while small gains are possible with $L_g = 2$, 3 for perfect CSI, these gains cannot be realized with quantized CSI and $B \leq 7$ feedback bits, cf. Fig. 6.

Figs. 7 and 8 show the simulated BERs (averaged over 100,000 channel realizations) for BPSK modulation with MMSE–DFE and the EQ channel assuming finite–rate feedback beamforming with BFFs of lengths $L_g = 1$ and $L_g = 3$, respectively. For the simulations we implemented MMSE–DFE with FIR feedforward filters of length $4L_{eq}$ which caused negligible performance degradation compared to IIR feedforward filters. The DFE feedback filter had optimum length $L_{eq} - 1 = L + L_g - 2$.

Fig. 7 shows that even finite–rate feedback beamforming with BFFs of length $L_g = 1$ and a
small number of feedback bits can achieve substantial performance gains over single-antenna transmission ($N_T = 1$, $N_R = 1$). Furthermore, finite-rate feedback beamforming with $B = 1$ feedback bit outperforms antenna selection which employs the codebook $G \triangleq \{[1 0 0]^T, [0 1 0]^T, [0 0 1]^T\}$ and requires $B = 2$ feedback bits. Finite-rate feedback beamforming with $B = 7$ bits closely approaches the performance of beamforming with perfect CSI.

Fig. 8 shows that for the degenerate case of $B = 0$ feedback bits the proposed method with $L_g = 3$ achieves a slightly better performance than the optimized delay diversity (ODD) scheme in [15]. Note that both schemes employ fixed transmit filters. However, the numerical methods used for filter calculation are completely different leading to small performance differences. Significant performance gains over ODD are possible even with few feedback bits. For example, for BER = $10^{-4}$ finite-rate feedback beamforming with 1, 3, and 5 feedback bits yields a performance gain of 1.0 dB, 1.8 dB, and 2.4 dB over ODD, respectively. We also observe from Fig. 8 that finite-rate feedback beamforming with $L_g = 3$ and $B = 7$ feedback bits outperforms beamforming with $L_g = 1$ and perfect CSI.

We note that the BERs at $10 \log_{10}(E_b/N_0) = 10$ dB in Figs. 7 and 8 are somewhat higher than the corresponding BERs in Fig. 5. This can be attributed to the fact that the BER in Fig. 5 has been obtained by evaluating Eq. (23) which does not take into account the effect of error propagation in the DFE feedback filter, whereas the simulation results shown in Figs. 7 and 8 include this effect, of course. Thereby, the performance for $L_g = 3$ is slightly more affected by error propagation than that for $L_g = 1$ since $L_g = 3$ results in a longer overall channel requiring a longer DFE feedback filter for equalization. This explains why the small performance gain promised by Fig. 5 when increasing $L_g$ from 1 to 3 cannot be observed in Figs. 7 and 8 for $B < 7$.

Fig. 9 contains the same BER curves as Fig. 7. However, now 8-PSK transmission over the TU channel is considered instead of BPSK transmission over the EQ channel. Fig. 9 shows that also for 8-PSK and the TU channel the performance loss incurred by quantized CSI becomes negligible for $B = 7$ feedback bits. Furthermore, for high $E_b/N_0$ finite-rate feedback beamforming with a sufficiently large number of feedback bits can achieve a performance gain of up to 2.2 dB compared to antenna selection.
7 Conclusion

In this paper, we have considered beamforming with perfect and quantized CSI for single-carrier transmission over frequency-selective fading channels with ZF–DFE and MMSE–DFE at the receiver. For the case of perfect CSI we have provided a simple approach for derivation of closed-form expressions for the optimum IIR BFFs and we have developed two efficient numerical methods for calculation of the optimum FIR BFFs. For beamforming with finite-rate feedback channel we have proposed a GVQ algorithm for codebook design. The GVQ algorithm performs a deterministic global search and is therefore independent of the starting conditions. This algorithm is applicable for any number of transmit and receive antennas, arbitrary antenna correlation, and arbitrary fading statistics. Simulation results for typical GSM/EDGE channels have shown that short FIR BFFs can approach the performance of IIR BFFs. Furthermore, for finite-rate feedback beamforming with BFFs of length \( L_g = 1 \) few feedback bits are sufficient to approach the performance of beamforming with perfect CSI. In severely frequency-selective channels longer BFFs can further improve performance if a sufficient number of feedback bits can be afforded.

References


[23] *GSM Recommendation 05.05: “Propagation Conditions”, Vers. 5.3.0, Release 1996.*

Tables and Figures:

Table 1: Gradient algorithm (GA) and Modified Power Method (MPM) for calculation of the optimum FIR BFF vector \( \tilde{g} \). Termination constant \( \epsilon \) has a small value (e.g. \( \epsilon = 10^{-4} \)). \( i \) denotes the iteration and \( \delta_i \) is the adaptation step size necessary for the GA.

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<tr>
<td>1</td>
<td>Let ( i = 0 ) and initialize the BFF vector with some ( g_0 ) fulfilling ( g_0^H g_0 = 1 ).</td>
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| 2 | Update the BFF vector:  
     GA: \( \tilde{g}_{i+1} = g_i + \delta_i \left[ \int_{-1/2}^{1/2} \frac{B(f) + \xi I_{NTLg}}{g_i^H (B(f) + \xi I_{NTLg}) g_i} df \right] g_i \).  
     MPM: \( \tilde{g}_{i+1} = \left[ \int_{-1/2}^{1/2} \frac{B(f) + \xi I_{NTLg}}{g_i^H (B(f) + \xi I_{NTLg}) g_i} df \right] g_i \). |
| 3 | Normalize the BFF vector: \( g_{i+1} = \frac{\tilde{g}_{i+1}}{\sqrt{\tilde{g}_{i+1}^H \tilde{g}_{i+1}}} \). |
| 4 | If \( 1 - |g_{i+1}^H g_i| < \epsilon \), goto Step 5, otherwise increment \( i \rightarrow i + 1 \) and goto Step 2. |
| 5 | \( g_{i+1} \) is the desired BFF vector \( \tilde{g} \). |
Table 2: This table summarizes the proposed GVQ algorithm. $\mathcal{G}[i]$, $\hat{g}_n[i]$, and MQE$_i$ denote the optimum codebook, the $n$th codeword, and the minimum MQE in iteration $i$. Note that in iteration $i$ the codebook size is also $i$.

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<td>1</td>
<td>Pre–define the total number of codewords as $N$ or pre–define the target MQE as MQE$_{\text{tar}}$.</td>
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<td>2</td>
<td>Initialize the number of codewords with $i = 1$.</td>
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<td>3</td>
<td>Calculate the optimum codeword $\hat{g}_1[1]$ by searching the entire training set $\mathcal{G}_T$ for that $\bar{g}_n$ which minimizes the MQE in Eq. (28), where $\mathcal{R}_1 = \mathcal{G}_T$. Set $\mathcal{G}[1] = \hat{g}<em>1[1]$ and record the corresponding MQE[1]. If $N = 1$ or MQE[1] $\leq$ MQE$</em>{\text{tar}}$ goto Step 7, otherwise goto Step 4.</td>
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<tr>
<td>4</td>
<td>Increment the iteration number $i \rightarrow i + 1$.</td>
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<tr>
<td>5</td>
<td>Execute the LBG algorithm described in Section 5.3 for all $T - (i - 1)$ initial codebooks given by ${\hat{g}_1[i - 1], \hat{g}<em>2[i - 1], \ldots, \hat{g}</em>{i-1}[i - 1], \bar{g}_n}$, where $\bar{g}_n \in \mathcal{G}_T$, $\bar{g}_n \not\in \mathcal{G}[i - 1]$. Retain the final codebook delivered by the LBG algorithm with minimum MQE and record it as $\mathcal{G}[i]$. Record the corresponding MQE[i].</td>
</tr>
<tr>
<td>6</td>
<td>If $i &lt; N$ or if MQE[i] $&gt;$ MQE$_{\text{tar}}$ goto Step 4, otherwise goto Step 7.</td>
</tr>
<tr>
<td>7</td>
<td>$\mathcal{G}[i]$ is the desired codebook.</td>
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Figure 1: Block diagram of the overall transmission system. $\hat{b}[k]$ are estimated symbols at the receiver.
Figure 2: SNR of MMSE–DFE vs. iteration $i$ of the proposed GA and MPM for one realization of the EQ channel with $L = 7$, $N_T = 3$, $N_R = 1$, equal antenna correlation $\rho = 0.5$, $L_g = 1$, and $10 \log_{10}(E_s/N_0) = 10$ dB.
Figure 3: Average SNR of DFE for beamforming (BF) with perfect CSI and different BFFs. EQ channel with $L = 7$, $N_T = 3$, $N_R = 1$, and equal antenna correlation $\rho = 0.5$. Results for single-antenna transmission are also included.
Figure 4: Average SNR of DFE for beamforming (BF) with perfect CSI and different BFFs. TU channel with $L = 5$, $N_T = 3$, $N_R = 1$, and equal antenna correlation $\rho = 0.5$. Results for single-antenna transmission are also included.
Figure 5: BER of MMSE–DFE vs. number of feedback bits $B$ per channel update. BPSK transmission over EQ channel with $L = 7$, $N_T = 3$, $N_R = 1$, equal antenna correlation $\rho = 0.5$, and $10 \log_{10}(E_b/N_0) = 10$ dB. The BER is obtained from Eq. (23).
Figure 6: BER of MMSE–DFE vs. number of feedback bits $B$ per channel update. 8–PSK transmission over TU channel with $L = 5$, $N_T = 3$, $N_R = 1$, equal antenna correlation $\rho = 0.5$, and $10 \log_{10}(E_b/N_0) = 10$ dB. The BER is obtained from Eq. (23).
Figure 7: Simulated BER of MMSE–DFE for finite–rate feedback beamforming with BFFs of length $L_g = 1$. BPSK transmission over EQ channel with $L = 7$, $N_T = 3$, $N_R = 1$, and equal antenna correlation $\rho = 0.5$. Results for single–antenna transmission ($N_T = 1$, $N_R = 1$), antenna selection, and beamforming with perfect CSI are also included.
Figure 8: Simulated BER of MMSE–DFE for finite–rate feedback beamforming with BFFs of length $L_g = 3$. BPSK transmission over EQ channel with $L = 7$, $N_T = 3$, $N_R = 1$, and equal antenna correlation $\rho = 0.5$. Results for single–antenna transmission ($N_T = 1$, $N_R = 1$), ODD [15], and beamforming with perfect CSI are also included.
Figure 9: Simulated BER of MMSE–DFE for finite–rate feedback beamforming with BFFs of length $L_g = 1$. 8–PSK transmission over TU channel with $L = 5$, $N_T = 3$, $N_R = 1$, and equal antenna correlation $\rho = 0.5$. Results for single–antenna transmission ($N_T = 1$, $N_R = 1$), antenna selection, and beamforming with perfect CSI are also included.