Cooperative Filter–and–Forward Beamforming for Frequency–Selective Channels with Equalization

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Abstract

Most of the existing literature on cooperative relay networks has focused on frequency–nonselective channels or frequency–selective channels with multi–carrier transmission. However, several practical systems employ single–carrier transmission over frequency–selective channels and the design of corresponding relaying schemes is a largely under–explored topic. In this paper, we investigate filter–and–forward beamforming (FF–BF) for relay networks employing single–carrier transmission over frequency–selective channels. In contrast to prior work, we assume that the destination node is equipped with a simple linear or decision feedback equalizer. The FF–BF filters at the relays are optimized for maximization of the signal–to–noise ratio at the equalizer output under a joint relay power constraint. For infinite impulse response (IIR) FF–BF filters, we derive a unified expression for the filter frequency response valid for linear equalization, decision feedback equalization, and an idealized matched filter receiver. A numerical algorithm with guaranteed convergence is developed for optimization of the power allocation factor included in the expression for the IIR FF–BF filter frequency response. We also provide an efficient gradient algorithm for recursive calculation of near–optimal finite impulse response (FIR) FF–BF filters. Simulation results show that, in general, short FIR FF–BF filters are sufficient to closely approach the performance of IIR FF–BF filters even in severely frequency–selective channels and that the proposed FF–BF scheme with equalization at the destination achieves substantial performance gains compared to a previously proposed FF–BF scheme without equalization.
1 Introduction

Multiple–antenna processing is a promising approach to improve the capacity and reliability of next generation communication systems. However, in many situations it is impractical to implement multiple antennas at the transmitter and/or receiver due to limitations in size and/or processing power. In this case, it is advantageous to allow network nodes to act as relays forming a virtual antenna array and achieving spatial diversity in a distributed fashion [1].

The two main relay protocols considered in the literature are amplify–and–forward (AF) and decode–and–forward (DF) relaying [1]. Thereby, AF relaying is generally believed to be less complex as the relays only perform a linear processing of the received signals, whereas the relays have to decode and re–encode the received signals in DF relaying. In AF relaying, beamforming (BF) across the relays is a simple yet efficient technique to improve capacity and reliability. BF for AF relays and frequency–flat channels has been extensively studied in the literature, see e.g. [2]–[11]. However, in most practical applications, the channels are frequency selective and techniques optimized for frequency–flat channels are not directly applicable. AF relaying techniques for systems employing orthogonal frequency–division multiplexing (OFDM) to cope with the frequency selectivity of the channel have been investigated in [12]–[14]. However, while OFDM is gaining popularity [15], there are still many applications where single–carrier transmission techniques are preferred because of legacy issues or the disadvantages of OFDM such as a high peak–to–average power ratio. Such applications include the GSM/EDGE mobile communication system, whose standard is still being further extended, and sensor networks, for which the cost and power consumption of the highly linear power amplifiers required for OFDM may be prohibitive.

Relaying schemes for single–carrier transmission over frequency–selective channels have received little attention in the literature so far with [16, 17] being two notable exceptions. Specifically, a cooperative filter–and–forward (FF) BF technique was proposed and optimized under the assumptions that (1) there is no direct link between the source and the destination, (2) an equalizer is not available at the destination, and (3) full channel state information (CSI) of all links is available [16]. We note that FF relaying for frequency–flat channels was also considered in [18]. For the frequency–selective case, distributed space–time block coding at the relays and equalization at the destination has been proposed in [17]. Distributed space–time coding does not require full CSI but has a worse performance than FF–BF.

In this paper, we investigate cooperative FF–BF for frequency–selective channels for the case
where the destination node has enough processing power to perform low-complexity equalization such as linear equalization (LE) or decision feedback equalization (DFE). Similar to [16] we assume that the central node, which computes the optimal FF–BF filters, has full CSI of all links. However, unlike [16], our model also includes a direct link between the source and the destination node. To the best of our knowledge, cooperative FF–BF with equalization at the destination has not been considered before. In particular, this paper makes the following contributions:

- Assuming finite impulse response (FIR) and infinite impulse response (IIR) filters at the relays, we optimize FF–BF for maximization of the signal–to–noise ratio (SNR) at the output of linear and decision–feedback equalizers as well as an idealized matched filter (MF) receiver ignoring any intersymbol interference (ISI) in the filter output. The latter provides a natural performance upper bound for any equalization scheme [19] and allows us to bound possible performance gains achievable with more complex equalization schemes such as maximum–likelihood sequence estimation (MLSE).
- For the IIR case, we show that the frequency response vector of the optimal FF–BF filters can be decomposed into a unit–norm direction vector and a scalar power allocation factor across frequencies. We provide a unified closed–form solution for the direction vector valid for all three considered receiver structures and an efficient numerical method with guaranteed convergence for the power allocation.
- For the FIR case, we show that the FF–BF filter optimization problem is related to a difficult mathematical problem for which an exact solution in closed–form does not seem to exist. Therefore, we provide an efficient numerical method for recursive calculation of the optimum FIR FF–BF filters.
- Our simulation results show that (1) relatively short FIR FF–BF filters suffice to closely approach the performance of IIR FF–BF filters, (2) the gap between FF–BF with LE and DFE, respectively, and the MF receiver is small implying that little can be gained by adopting more complex equalization schemes, and (3) the addition of simple LE and DFE equalizers at the destination node yields large performance gains compared to FF–BF without equalization advocated in [16].

Organization: In Section 2, the adopted system model is presented. The optimization of IIR FF–BF filters for LE, DFE, and the MF receiver is discussed in Section 3, and the FIR case is considered in Section 4. Simulation results are provided in Section 5, and some conclusions are drawn in Section 6.
Notation: In this paper, \((\cdot)^T\), \((\cdot)^H\), \((\cdot)^*\), \(I_X\), \(0_X\), \(\mathcal{E}\{\cdot\}\), and \(*\) denote transpose, Hermitian transpose, complex conjugate, the \(X \times X\) identity matrix, the all–zero column vector of length \(X\), expectation, and discrete–time convolution, respectively. \(Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp(-t^2/2) \, dt\) and \(\delta[\cdot]\) denote the Gaussian \(Q\)–function and the Kronecker delta function, respectively. \(\text{diag}\{X_1, X_2, \ldots, X_N\}\) denotes a block–diagonal matrix with matrices \(X_1, X_2, \ldots, X_N\) on the main diagonal. Furthermore, \(X(f) \triangleq \mathcal{F}\{x[k]\} = \sum_{k=-\infty}^{\infty} x[k] e^{-j2\pi fk}\) is the Fourier transform of discrete–time signal \(x[k]\).

2 System Model

We consider a relay network with one source node, \(N_R\) relays, and one destination node. A block diagram of the discrete–time overall transmission system in equivalent complex baseband representation is shown in Fig. 1. As usual, transmission is organized in two intervals. In the first interval, the source node transmits a data packet which is received by the destination and the relays. In the second interval, the relays filter the received packet and forward it to the destination node. At the destination, the packets received during the first and second intervals are combined, processed, and detected.

In Fig. 1, the discrete–time channel impulse responses (CIRs) between the source and relay \(i\), \(g_i[k], 0 \leq k \leq L_g - 1\), between relay \(i\) and the destination, \(h_i[k], 0 \leq k \leq L_h - 1\), and between the source and the destination, \(f[k], 0 \leq k \leq L_f - 1\), contain the combined effects of transmit pulse shaping, the continuous–time channel, receive filtering, and sampling. Here, \(L_g, L_h,\) and \(L_f\) denote the lengths of the source–relay, the relay–destination, and the source–destination channels, respectively.

In the following, we describe the processing performed at the relays and the destination in detail.

2.1 FF–BF at Relays

The signal received at the \(i\)th relay during the first transmission interval is given by

\[ y_i[k] = g_i[k] * s[k] + n_{iR_i}[k], \quad i = 1, \ldots, N_R, \]

where \(s[k]\) are independent and identically distributed (i.i.d.) symbols taken from a scalar symbol alphabet \(\mathcal{A}\) such as phase–shift keying (PSK) or quadrature amplitude modulation (QAM) with
variance $\sigma_n^2 \triangleq \mathcal{E}\{|s[k]|^2\} = 1$, and $n_{R_i}[k]$ denotes additive white Gaussian noise (AWGN) with variance $\sigma_n^2 \triangleq \mathcal{E}\{|n_{R_i}[k]|^2\}$.

The FF–BF filter impulse response coefficients of relay $i$, $1 \leq i \leq N_R$, are denoted by $a_i[k]$, $-q_i \leq k \leq q_i$. For IIR FF–BF filters $q_i \rightarrow \infty$ and $q_a \rightarrow \infty$ and for FIR FF–BF filters $q_i = 0$ and $q_a = L_a - 1$, where $L_a$ is the FF–BF filter length. The signal transmitted by the $i$th relay during the second transmission interval can be expressed as

$$t_i[k] = a_i[k] * g_i[k] = a_i[k] * g_i[k] * s[k] + a_i[k] * n_{R_i}[k], \quad i = 1, \ldots, N_R. \quad (2)$$

### 2.2 Equalization at Destination Node

The signal received at the destination node during the first transmission interval is given by

$$r_0[k] = f[k] * s[k] + n_0[k], \quad (3)$$

where $n_0[k]$ is AWGN with variance $\sigma_n^2$. The signal received at the destination during the second transmission interval is given by

$$r_1[k] = \sum_{i=1}^{N_R} h_i[k] * t_i[k] + n_1[k]$$

$$= \sum_{i=1}^{N_R} h_i[k] * a_i[k] * g_i[k] * s[k] + \sum_{i=1}^{N_R} h_i[k] * a_i[k] * n_{R_i}[k] + n_1[k]$$

$$= h_{eq}[k] * s[k] + n'_1[k], \quad (4)$$

where $n_1[k]$ is AWGN with variance $\sigma_n^2$. The equivalent CIR $h_{eq}[k]$ between source and destination and the effective noise $n'_1[k]$ are given by

$$h_{eq}[k] \triangleq \sum_{i=1}^{N_R} h_i[k] * a_i[k] * g_i[k] \quad (5)$$

and

$$n'_1[k] \triangleq \sum_{i=1}^{N_R} h_i[k] * a_i[k] * n_{R_i}[k] + n_1[k], \quad (6)$$

respectively. Note that $n'_1[k]$ is colored noise because of the filtering of $n_{R_i}[k]$ by $h_i[k]$ and $a_i[k]$.

Eqs. (3) and (4) show that a cooperative relay network with FF–BF can be modeled as a single–input multiple–output (SIMO) system with two outputs $r_0[k]$ and $r_1[k]$. Therefore, at the destination node the same channel estimation, equalization, and channel tracking techniques as for point–to–point SIMO transmission can be used [20]. Here, we adopt SIMO LE [21] and SIMO DFE [22] optimized under zero–forcing (ZF) and minimum mean–squared error (MMSE) criteria.
2.3 Feedback Channel

We assume that the destination estimates the relay–destination CIRs $h_i[k]$, $0 \leq k \leq L_h - 1$, $1 \leq i \leq N_R$, and the source–destination CIR $f[k]$, $0 \leq k \leq L_f - 1$, during a training phase. Similarly, relay $i$ estimates its own source–relay CIR $g_i[k]$, $0 \leq k \leq L_g - 1$, and forwards the estimate to the destination node. Alternatively, the destination may directly estimate the combined CIR of the source–relay and relay–destination channels, $h_i[k] * g_i[k]$ if relay $i$ retransmits the training signal received from the source. The destination can then extract $g_i[k]$ from $h_i[k] * g_i[k]$ and $h_i[k]$ via deconvolution. Subsequently, the destination node computes the FF–BF filters using the CSI of all links and feeds back the filter coefficients to the relays. Throughout this paper we assume that the CSI and the feedback channel are perfect, which implies that the nodes in the network have limited mobility, and thus, all channels are slowly fading. We note that similar assumptions regarding the availability of CSI and the feedback channel are typically made in the distributed BF literature for both frequency–flat and frequency–selective channels, cf. e.g. [2, 6, 9, 11, 16].

3 Optimal IIR FF–BF Filters

Throughout this paper we assume that the destination node employs LE or DFE with IIR equalization filters. In a practical implementation, FIR equalization filters are used, of course, but sufficiently long FIR filters will approach the performance of IIR filters arbitrarily close. Assuming IIR equalization filters has the advantage that relatively simple and elegant expressions for the SNR at the equalizer output exist [20, 22]. However, in order to make these simple SNR expressions applicable, we first have to whiten the noise impairing the signal received in the second transmission interval at the destination.

The power spectral density of $n'_1[k]$ in (6) can be obtained as

$$
\Phi_{n'_1}(f) = \sigma_n^2 \sum_{i=1}^{N_R} |H_i(f)|^2 |A_i(f)|^2 + \sigma_n^2,
$$

where $H_i(f) \triangleq \mathcal{F}\{h_i[k]\}$ and $A_i(f) \triangleq \mathcal{F}\{a_i[k]\}$ denote the frequency responses of the $i$th relay–destination channel and the corresponding FF–BF filter, respectively. In order to whiten $n'_1[k]$, we pass $r_1[k]$ through a filter with frequency response

$$
W(f) = \left(\sum_{i=1}^{N_R} |H_i(f)|^2 |A_i(f)|^2 + 1\right)^{-1/2} = (a^H(f) \Gamma(f) a(f) + 1)^{-1/2}
$$
and denote the filter output by $r'_1[k]$. In (8), we use the definitions $\Gamma(f) \triangleq \text{diag}\{|H_1(f)|^2, \ldots, |H_{NR}(f)|^2\}$ and $\mathbf{a}(f) \triangleq [A_1(f), \ldots, A_{NR}(f)]^T$. After whitening, the frequency response of the equivalent overall channel is given by

$$H'_{\text{eq}}(f) \triangleq W(f)\mathcal{F}\{h_{\text{eq}}[k]\} = \mathbf{a}^T(f)q(f)(\mathbf{a}^H(f)\Gamma(f)\mathbf{a}(f) + 1)^{-1/2}, \quad (9)$$

where $q(f) \triangleq [Q_1(f), \ldots, Q_{NR}(f)]^T$, $Q_i(f) \triangleq H_i(f)G_i(f)$, and $G_i(f) \triangleq \mathcal{F}\{g_i[k]\}$. The power spectral density of the noise component, $n''_1[k]$, of $r'_1[k]$ is $\Phi_{n''_1}(f) = \sigma^2_n$.

In the remainder of this section, we formulate and solve the IIR FF–BF filter optimization problems for LE, DFE, and an idealized MF receiver in a unified manner.

### 3.1 Problem Formulation

After whitening, we have an equivalent SIMO channel where the sub–channels have frequency responses $H'_{\text{eq}}(f)$ and $F(f) \triangleq \mathcal{F}\{f[k]\}$ and are impaired by AWGN with variance $\sigma^2_n$, respectively. Thus, after introducing

$$Z(\mathbf{a}(f)) \triangleq |H'_{\text{eq}}(f)|^2 = \frac{\mathbf{a}^H(f)q^*(f)q^T(f)\mathbf{a}(f)}{\mathbf{a}^H(f)\Gamma(f)\mathbf{a}(f) + 1}, \quad (10)$$

we can express the SNRs at the outputs of a decision feedback equalizer and a linear equalizer as [20]–[22]

$$\text{SNR}_{\text{DFE}}(\mathbf{a}(f)) = \frac{\sigma^2_s}{\sigma^2_n} \exp \left\{ \frac{1}{2} \int_{-1/2}^{1/2} \ln \left( Z(\mathbf{a}(f)) + |F(f)|^2 + \xi \right) df \right\} - \chi \quad (11)$$

and

$$\text{SNR}_{\text{LE}}(\mathbf{a}(f)) = \frac{\sigma^2_s}{\sigma^2_n} \left( \int_{-1/2}^{1/2} (Z(\mathbf{a}(f)) + |F(f)|^2 + \xi)^{-1} df \right)^{-1} - \chi, \quad (12)$$

respectively. In (11) and (12), we have $\chi = 0$, $\xi = 0$ and $\chi = 1$, $\xi = \sigma^2_s/\sigma^2_n$ if the equalization filters are optimized based on a ZF and an MMSE criterion, respectively. Similarly, if only a single, isolated symbol $s[k]$ is transmitted, the SNR at the output of an MF is given by [19]

$$\text{SNR}_{\text{MF}}(\mathbf{a}(f)) = \frac{\sigma^2_s}{\sigma^2_n} \int_{-1/2}^{1/2} (Z(\mathbf{a}(f)) + |F(f)|^2) df. \quad (13)$$
It is not difficult to show that regardless of how the FF–BF filter frequency responses \(a(f)\) are chosen, we always have \[22\]

\[
\text{SNR}_{\text{MF}}(a(f)) \geq \text{SNR}_{\text{DFE}}(a(f)) \geq \text{SNR}_{\text{LE}}(a(f)).
\] (14)

Thus, the MF receiver constitutes a performance upper bound for DFE and LE with continuous transmission of symbols \(s[k]\). In fact, it can be shown that the MF receiver provides a performance upper bound for any realizable equalization structure including optimal MLSE [19]. Note, however, that the MF receiver generally has a poor performance for continuous symbol transmission since it does not combat ISI.

In this paper, our goal is to optimize the FF–BF filters for maximization of the SNRs at the output of the considered equalization schemes. To make the problem well defined, we constrain the relay transmit power, \(P_R\), which is given by

\[
P_R = \sum_{i=1}^{N_R} \int_{-1/2}^{1/2} \Phi_{t_i}(f) \, df = \int_{-1/2}^{1/2} a^H(f)D(f)a(f) \, df
\] (15)

where \(\Phi_{t_i}(f) \triangleq |A_i(f)|^2(\sigma_s^2|G_i(f)|^2 + \sigma_n^2), i = 1, \ldots, N_R,\) is the power spectral density of the transmit signal \(t_i[k]\) of the \(i\)th relay and \(D(f) \triangleq \text{diag}\{\sigma_s^2|G_1(f)|^2, \ldots, \sigma_s^2|G_{N_R}(f)|^2\} + \sigma_n^2I_{N_R}.

Formally, the IIR FF–BF filter optimization problem can now be stated as

\[
\max_{a(f)} \quad \text{SNR}_X(a(f)) \tag{16a}
\]

s.t. \[
\int_{-1/2}^{1/2} a^H(f)D(f)a(f) \, df \leq P, \tag{16b}
\]

where \(P\) denotes the maximum relay transmit power, and \(X = \text{DFE}, X = \text{LE},\) and \(X = \text{MF}\) for DFE, LE, and an MF receiver, respectively. It is convenient to introduce vector \(v(f) \triangleq D^{1/2}(f)a(f),\) which can be expressed as \(v(f) = \sqrt{p(f)}u(f)\) without loss of generality, where \(p(f)\) denotes the power of \(v(f)\) and \(u(f)\) has unit norm, \(||u(f)||^2 = 1\). Furthermore, we introduce \(\bar{Z}(v(f)) = Z(\sqrt{p(f)}u(f)) \triangleq Z(a(f)),\) which is given by

\[
\bar{Z}(v(f)) = \frac{a^H(f)q^*(f)q^T(f)a(f)}{a^H(f)\Gamma(f)a(f) + 1} = \frac{u^H(f)J(f)u(f)}{u^H(f)X(f)u(f)}
\] (17)

with rank one, positive semi–definite matrix

\[
J(f) = p(f)D^{-1/2}(f)q^*(f)q^T(f)D^{-1/2}(f)
\] (18)
and full rank, positive definite matrix

\[ X(f) = p(f)D^{-1/2}(f)\Gamma(f)D^{-1/2}(f) + I_{NR}. \]  

(19)

Introducing \( \text{SNR}_X(v(f)) = \text{SNR}_X\left(\sqrt{p(f)}u(f)\right) \triangleq \text{SNR}_X(a(f)) \), we can restate problem (16) in equivalent form as

\[
\begin{align*}
\text{maximize} & \quad \text{SNR}_X\left(\sqrt{p(f)}u(f)\right) \\
\text{s.t.} & \quad \int_{-1/2}^{1/2} p(f) \, df \leq P \\
& \quad p(f) \geq 0.
\end{align*}
\]

(20a) (20b) (20c)

In the following, we provide a unified solution to problem (20) valid for all three considered equalization schemes.

### 3.2 Optimum IIR FF–BF Filters

We observe from (20) that the constraints of the considered optimization problem do not depend on \( u(f) \). Thus, without loss of generality, we can find the globally optimal solution of problem (20) by first maximizing the SNR with respect to \( u(f) \) for a given power allocation \( p(f) \) and by subsequently optimizing the resulting SNR expression with respect to \( p(f) \).

Furthermore, for all three considered receiver structures, the SNR \( \text{SNR}_X(v(f)) \) is monotonically increasing in \( \tilde{Z}\left(\sqrt{p(f)}u(f)\right) \). Thus, for any given power allocation \( p(f) \), we can maximize the SNR \( \text{SNR}_X(v(f)) \) by maximizing \( \tilde{Z}\left(\sqrt{p(f)}u(f)\right) \) with respect to \( u(f) \) for all frequencies \( f \). Hence, the optimal FF–BF direction vector, \( u_{opt}(f) \), can be found from the following optimization problem

\[
\begin{align*}
\text{maximize} & \quad \tilde{Z}\left(\sqrt{p(f)}u\right) = \frac{u^H(f)J(f)u(f)}{u^H(f)X(f)u(f)} \\
\text{s.t.} & \quad u^H(f)X(f)u(f). \quad (21)
\end{align*}
\]

Problem (21) is a generalized eigenvalue problem for which a closed–form solution exists since matrix \( J(f) \) has rank one and matrix \( X(f) \) has full rank. The solution of problem (21) can be obtained as

\[
u_{opt}(f) = c(f)X^{-1}(f)D^{-1/2}(f)q^*(f), \quad (22)\]
where \( c(f) \) is a real-valued scaling factor which is given by

\[
c(f) = \frac{1}{\sqrt{q^T(f)D^{-1/2}(f)X^{-2}(f)D^{-1/2}(f)q^*(f)}}.
\]  

(23)

The maximum \( \bar{Z} \left( \sqrt{p(f)}u(f) \right) \) achievable with \( u_{opt}(f) \) is

\[
\bar{Z} \left( \sqrt{p(f)}u_{opt}(f) \right) = p(f)q^T(f)D^{-1/2}(f)X^{-1}(f)D^{-1/2}(f)q^*(f)
\]

\[
= p(f)q^T(f)(p(f)\Gamma(f) + D(f))^{-1}q^*(f).
\]

(24)

Now, we can express the optimal FF–BF filter frequency response vector (for a given power allocation), \( \alpha^*(f) \), as

\[
\alpha^*(f) = \sqrt{p(f)}D^{-1/2}(f)u_{opt}(f) = \sqrt{p(f)c(f)}(p(f)\Gamma(f) + D(f))^{-1}q^*(f).
\]

(25)

Furthermore, using the definitions of matrices \( D(f) \) and \( \Gamma(f) \), we can express the FF–BF filter frequency response at relay \( i \) as

\[
A_i^*(f) = \frac{\sqrt{p(f)c(f)}}{p(f)|H_i(f)|^2 + \sigma_n^2|G_i(f)|^2 + \sigma_n^2}H_i^*(f)G_i^*(f), \quad i = 1, \ldots, N_R.
\]

(26)

Eq. (26) reveals that the optimal IIR FF–BF filters for all considered receiver structures may be interpreted as the concatenation of a filter matched to the source–relay and the relay-destination link with frequency response \( H_i^*(f)G_i^*(f) \) and a second filter whose frequency response \( \sqrt{p(f)c(f)}/(p(f)|H_i(f)|^2 + \sigma_n^2|G_i(f)|^2 + \sigma_n^2) \) depends on the power allocation, and thus on the particular equalizer used at the destination node. Note that \( A_i^*(f) \) of relay \( i \) depends on the CIRs of all source–relay, all relay–destination, and the source–destination channels via power allocation factor \( p(f) \). The power allocation problem will be tackled in the next section.

### 3.3 Optimum Power Allocation

Before we formulate the power allocation problem for the three considered receiver structures in a unified way, we first introduce the following definitions:

\[
S_{DFE}(f) \triangleq \ln(M(f)), \quad S_{LE}(f) \triangleq -1/M(f), \quad \text{and} \quad S_{MF}(f) \triangleq M(f),
\]

(27)

with

\[
M(f) \triangleq q^T(f)(\Gamma(f) + D(f)/p(f))^{-1}q^*(f) + |F(f)|^2 + \xi.
\]

(28)
where for DFE and LE $\xi$ is defined after (12) and $\xi = 0$ for the MF receiver. Based on these definitions, the equalizer output SNRs (11)–(13), the original optimization problem (20), and the optimal frequency response direction in (22), we can formulate the power allocation problem as

$$\begin{align*}
\max_{p(f)} & \quad \int_{-1/2}^{1/2} S_X(f) \, df \\
\text{s.t.} & \quad \int_{-1/2}^{1/2} p(f) \, df \leq P \\
& \quad p(f) \geq 0,
\end{align*}$$

(29a)

(29b)

(29c)

where $X = \text{DFE}$, $X = \text{LE}$, and $X = \text{MF}$ for DFE, LE, and an MF at the receiver, respectively. Since $S''_X(f) \triangleq \frac{\partial^2 S_X(f)}{\partial p^2(f)} < 0$ for $X \in \{\text{DFE, LE, MF}\}$, the power allocation problem is convex for all considered equalizer structures. The Lagrangian of problem (29) is given by

$$\mathcal{L}(p(f), \mu) = \int_{-1/2}^{1/2} S_X(f) \, df - \mu \int_{-1/2}^{1/2} p(f) \, df,$$

(30)

where $\mu \geq 0$ is the Lagrangian multiplier. The corresponding Lagrange dual function is

$$\mathcal{D}(\mu) = \max_{p(f)} \mathcal{L}(p(f), \mu) = \max_{p(f)} \int_{-1/2}^{1/2} (S_X(f) - \mu p(f)) \, df = \int_{-1/2}^{1/2} \max_{p(f)} (S_X(f) - \mu p(f)) \, df.$$  

(31)

The last step in (31) can be established because the total power constraint (29b) is implicitly captured by the dual variable $\mu$ and the maximization over $p(f)$ can be moved inside the integration. Therefore, for a given $\mu$, $p(f)$ can be obtained from

$$\max_{p(f)} S_X(p(f)) = S_X(f) - \mu p(f)$$

(32)

or equivalently

$$S'_X(f) \triangleq \frac{\partial S_X(f)}{\partial p(f)} = \mu.$$  

(33)

$S'_X(f)$ can be easily computed for all considered equalization schemes. In particular, we obtain

$$S'_{\text{DFE}}(f) \triangleq M'(f)/M(f), \quad S'_{\text{LE}}(f) \triangleq M'(f)/M^2(f), \quad \text{and} \quad S'_{\text{MF}}(f) \triangleq M'(f),$$

(34)
where

\[ M'(f) \triangleq \frac{\partial M(f)}{\partial p(f)} = q^T(f) D(f)(p(f)\Gamma(f) + D(f))^{-2} q^*(f). \]  

(35)

Note that constraint (29c), which has been ignored in (31), can be taken into account by evaluating

\[ S'_X(f) \triangleq \frac{\partial S_X(f)}{\partial p(f)} \text{ for } p(f) \to 0^+. \]

In particular, since \( S'_X(f) \) is a monotonic decreasing function of \( p(f) \) for all considered equalization schemes, for a given \( \mu \), \( S'_X(f) = \mu \) does not have a positive solution if \( \lim_{p(f)\to 0^+} S'_X(f) < \mu \), and we set \( p(f) = 0 \) in this case. Otherwise, we find \( p(f) \) from (33) by using e.g. the bisection search method [23]. On the other hand, the optimal value \( \mu = \mu_{opt} \) that ensures the power constraint is satisfied can be found iteratively by another bisection search. More specifically, if the corresponding total power \( P_R = \int_{-1/2}^{1/2} p(f)df \) is less than the maximum power \( P \) for a given \( \mu \), the Lagrange multiplier \( \mu \) has to be decreased, whereas it is increased if \( P_R > P \).

We note that since the frequency axis is real valued, in practice, \( f \) has to be discretized in \(-1/2 \leq f \leq 1/2\) to make the problem computationally tractable. A summary of the numerical algorithm for finding the optimal power allocation, \( p_{opt}(f) \), for discrete frequency points for the three considered equalization schemes is given in Table 1. Applying \( p_{opt}(f) \) found with the algorithm in Table 1 in (26), yields the optimal FF–BF filter frequency response \( A_{i, opt}(f) \) for relay \( i, 1 \leq i \leq N_R \).

Although we concentrate in this paper on the case where the direct source–destination link is exploited for detection, with a minor modification our results are also valid if the source–destination link is not used. In particular, in the latter case, (26) is still valid but for calculation of the optimal power allocation, we have to set \( F(f) = 0 \) in (28).

### 4 Optimal FIR FF–BF Filters

In practice, it is not possible to implement the IIR FF–BF filters discussed in the previous section since they would require the feedback of an infinite number of filter coefficients. However, the performance achievable with these IIR FF–BF filters provides a useful upper bound for the FIR FF–BF filters considered in this section. In particular, the performance of the IIR solution can be used for optimizing the FIR BF–FF length to achieve a desired trade–off between the amount of feedback and performance. We note that although FIR FF–BF filters are considered in this section, in order to be able to exploit the simple SNR expressions in (11)–(13), we still assume that the equalizers at the destination employ IIR filters.
With FIR FF–BF filters of length $L_a$ at the relays the length of the equivalent CIR $h_{eq}[k]$ (5) is given by $L_{eq} = L_a + L_g + L_h - 2$. In this case, the Fourier transform of $h_{eq}[k]$ can be expressed as

$$H_{eq}(f) = d^H(f)Qa$$

(36)

with $d(f) \triangleq [1 \ e^{j2\pi f} \ \ldots \ e^{j2\pi f(L_{eq}-1)}]^T$, FIR FF–BF coefficient vector $a \triangleq [a_1[0] \ a_1[1] \ \ldots \ a_1[L_a-1] \ a_2[0] \ \ldots \ a_{N_R}[L_a-1]]^T$, and $L_{eq} \times N_R L_a$ matrix $Q \triangleq [Q_1 \ \ldots \ Q_{N_R}]$, where $Q_i$ is an $L_{eq} \times L_a$ column–circulant matrix with vector $[(H_i g_i)^T \ 0_{L_a-1}^T]T$ in the first column. Here, $H_i$ is an $(L_h + L_g - 1) \times L_g$ column–circulant matrix with vector $[h_i[0] \ h_i[L_h - 1] \ 0_{L_g-1}^T]T$ in the first column and $g_i \triangleq [g_i[0] \ \ldots \ g_i[L_g - 1]]^T$.

The noise whitening filter in the FIR case is given by

$$W(f) = \left(a^H \bar{\Gamma}(f) a + 1\right)^{-1/2}$$

(37)

with $L_a N_R \times L_a N_R$ block diagonal matrix $\bar{\Gamma}(f) \triangleq \text{diag} \{\bar{\Gamma}_1(f), \ldots, \bar{\Gamma}_{N_R}(f)\}$ of rank $N_R$, where $\bar{\Gamma}_i(f) \triangleq \bar{H}_i^H d(f) d^H(f) \bar{H}_i$ is an $L_a \times L_a$ matrix of rank 1. Here, $\bar{H}_i$ is an $(L_h + L_a - 1) \times L_a$ column–circulant matrix with vector $[h_i[0] \ h_i[L_h - 1] \ 0_{L_a-1}^T]T$ in the first column and $\bar{d}(f) \triangleq [1 \ e^{j2\pi f} \ \ldots \ e^{j2\pi f(L_a+L_h-2)}]^T$. Therefore, after noise whitening, the frequency response of the overall channel is

$$H'_{eq}(f) = d^H(f)Qa \left(a^H \bar{\Gamma}(f) a + 1\right)^{-1/2}.$$  

(38)

We note that for a practical implementation, the noise whitening filter does not have to be implemented. Instead, the noise correlation can be directly taken into account for equalizer filter design [20]. However, in order to be able to exploit the simple existing expressions for the SNR of the equalizer output given in [20, 22], it is advantageous to assume the presence of a whitening filter for FIR BF–FF filter design.

### 4.1 Problem Formulation

Similar to the IIR case in (10), also for the FIR case it is convenient to introduce the definition

$$Z(a) \triangleq |H'_{eq}(f)|^2 = \frac{a^H Q^H d(f) d^H(f) Qa}{a^H \bar{\Gamma}(f) a + 1}.$$  

(39)

Note, however, that this is a slight abuse of notation since while the argument of $Z(a(f))$ in (10) is a vector containing all frequency responses of the IIR FF–BF filters, the argument of $Z(a)$ in (39) is a vector containing all FIR FF–BF coefficients. Replacing $Z(a(f))$ now in the SNR expressions
in (11)–(13) by \( Z(\alpha) \) from (39), we obtain the SNRs \( \text{SNR}_X(\alpha) \), where \( X = \text{DFE} \), \( X = \text{LE} \), and \( X = \text{MF} \) for DFE, LE, and an MF receiver, respectively. This allows us to formulate the FIR FF–BF filter optimization problem in a unified manner:

\[
\begin{align*}
\text{maximize} & \quad \text{SNR}_X(\alpha) \\
\text{s.t.} & \quad \alpha^H \bar{D} \alpha \leq P ,
\end{align*}
\]

where \( \bar{D} = \text{diag}\{\sigma_s^2 G_i^H G_1, \ldots, \sigma_s^2 G_{N_R}^H G_{N_R}\} + \sigma_n^2 I_{N_R L_a} \) with \((L_g + L_a - 1) \times L_a\) column circulant matrix \( G_i \) which has vector \([g_i[0] \ldots g_i[L_g - 1] 0^T_{L_a-1}]^T\) in the first column. Although problem (40) formally looks very similar to problem (16), it is substantially more difficult to solve. The main reason for this lies in the fact that the optimization variable \( \alpha(f) \) in (16) can be chosen freely for each frequency \( f \), whereas the coefficient vector \( \alpha \) in (40) is fixed for all frequencies.

To simplify the power constraint, we introduce \( \nu \triangleq \bar{D}^{1/2} \alpha \). Furthermore, it is not difficult to see that at optimality, the power constraint in (40b) is fulfilled with equality, i.e., \( \alpha^H \bar{D} \alpha = \nu^H \nu = P \). With this identity, we obtain

\[
M(\nu, f) \triangleq Z(\alpha) + |F(f)|^2 + \xi \\
\triangleq \frac{\nu^H \bar{J}(f) \nu}{\nu^H \bar{X}(f) \nu}
\]

where

\[
\begin{align*}
\bar{J}(f) & \triangleq D^{-1/2} \Phi(f) D^{-1/2} + \frac{|F(f)|^2 + \xi}{P} I_{N_R L_a}, \\
\bar{X}(f) & \triangleq \bar{D}^{-1/2} \bar{\Gamma}(f) \bar{D}^{-1/2} + \frac{1}{P} I_{N_R L_a}, \\
\bar{\Phi}(f) & \triangleq Q^H d(f) d^H(f) Q + (|F(f)|^2 + \xi) \bar{\Gamma}(f).
\end{align*}
\]

Now, we can rewrite optimization problem (40) in equivalent form as

\[
\begin{align*}
\text{maximize} & \quad \int_{-1/2}^{1/2} S_X(\nu, f) \, df \\
\text{s.t.} & \quad \nu^H \nu = P ,
\end{align*}
\]

where

\[
S_{\text{DFE}}(\nu, f) \triangleq \ln(M(\nu, f)), \quad S_{\text{LE}}(\nu, f) \triangleq -1/M(\nu, f), \quad \text{and} \quad S_{\text{MF}}(\nu, f) \triangleq M(\nu, f).
\]

\[
(46)
\]
4.2 Optimum FIR FF–BF Filters

The FIR FF–BF optimization problem in (45) is a difficult non-convex optimization problem. To substantiate this claim, we consider the special case of DFE and discretize the integral in (45a). This leads to the new equivalent problem

\[
\underset{\mathbf{v}^H \mathbf{v} = P}{\text{maximize}} \prod_{i=1}^{N} \frac{\mathbf{v}^H \mathbf{J}(f_i) \mathbf{v}}{\mathbf{v}^H \mathbf{X}(f_i) \mathbf{v}},
\]

where \( f_i \triangleq -1/2 + (i - 1)/N \) and \( N \) denotes the number of sampling points. The objective function in (47) is a product of generalized Rayleigh quotients. Unfortunately, it is well known that the maximization of a product of generalized Rayleigh quotients is a difficult mathematical problem which is not well understood and a solution is not known except for the trivial case \( N = 1 \), cf. e.g. [24, 25]. Therefore, we also do not expect to find a simple solution for optimization problem (45). Similar statements apply for the optimization problems resulting for LE and an MF receiver.

In order to obtain a practical and simple method for finding a locally optimal solution for the FIR BF–FF coefficient vectors, we propose a gradient algorithm (GA). In iteration \( i + 1 \), the GA improves vector \( \mathbf{v}_i \) from iteration \( i \) in the direction of the steepest ascent [23]

\[
\int_{-1/2}^{1/2} \frac{\partial S_{\mathbf{X}}(\mathbf{v}, f)}{\partial \mathbf{v}} \, df
\]

of the objective function in (45a). The GA for the three considered equalization schemes is summarized in Table 2. Although, in principle, the GA may not be able to find the globally optimal solution, extensive simulations have shown that for the problem at hand the performance achievable with GA is practically independent of the initialization \( \mathbf{v}_0 \). More importantly, for sufficiently large FIR filter lengths \( L_a \), the solution found with the GA closely approaches the performance of the optimal IIR FF–BF filter. This suggests that the solution found by the GA is at least near optimal. Exemplary simulation results confirming these claims are provided and discussed in the next section.

We note that we can again accommodate the case where the source–destination channel is not exploited for detection by simply setting \( F(f) = 0 \) in (42) and (44).

5 Simulation Results

In this section, we present simulation results for the SNR and the bit error rate (BER) of a cooperative network with FF–BF. Throughout this section we assume \( \sigma_n^2 = 1 \) and \( P = 1 \). This
allows us to decompose the CIRs as $h_i[k] = \sqrt{\gamma_h h_i[k]}$, $g_i[k] = \sqrt{\gamma_g g_i[k]}$, and $f[k] = \sqrt{\gamma_f f[k]}$, where $\gamma_h$, $\gamma_g$, and $\gamma_f$ denote the transmitter SNRs of the relay–destination, the source–relay, and the source–destination links, respectively. The normalized CIRs $\bar{h}_i[k]$, $\bar{g}_i[k]$, and $\bar{f}[k]$ include the effects of multipath fading and path–loss. All IIR and FIR FF–BF filters were obtained using the algorithms outlined in Tables 1 and 2, respectively.

5.1 Convergence of the GA

We first investigate the convergence of the proposed GA for optimization of the FIR FF–BF filters. We assume MMSE–DFE at the destination and $N_R = 5$ relays. The CIRs of all involved channels are given by $\bar{g}_i[k] = \bar{h}_i[k] = \bar{f}[k] = 1/\sqrt{5}$, $0 \leq k < 5$, $1 \leq i \leq 5$, with $L_g = L_h = L_f = 5$ and $\gamma_g = \gamma_h = \gamma_f = 10$ dB. Fig. 2 shows the achievable SNR vs. iteration number $i$ for initialization of the GA with a normalized random vector and a normalized all–one vector for different FIR filter lengths $L_a$, respectively. Note that the adaptation step size, $\delta_i$, is obtained from a backtracking line search, cf. Table 2. After a sufficiently large number of iterations, the SNR converges to the same constant value for both initializations. The steady–state SNR increases with increasing $L_a$ and for sufficiently large FIR filter lengths $L_a$, the steady–state SNR approaches the SNR of IIR FF–BF. Similar observations were made for other random and deterministic initializations of the proposed GA. Thus, for all results shown in the remaining figures, the GA in Table 2 was initialized with a normalized all–one vector.

5.2 Filter Design for a Fixed Test Channel

In order to get some insight into the effect that different equalization schemes have on the IIR and FIR FF–BF filter design, we consider next a cooperative network with $N_R = 1$ relay and assume a simplified test channel with $L_g = L_h = L_f = 2$ and $\bar{g}_1[k] = \bar{h}_1[k] = \bar{f}[k] = 1/\sqrt{2}$, $k \in \{0, 1\}$, i.e., all involved channels are identical and their frequency response has a zero at frequency $f = 1/2$, cf. Fig. 3. We also choose identical transmitter SNRs $\gamma_g = \gamma_h = \gamma_f = 10$ dB for all channels.

In Fig. 3, we show the magnitude of the optimal IIR FF–BF filter frequency response $|A_{1,\text{opt}}(f)|$ vs. frequency $f$. We consider the cases where the destination is equipped with ZF–DFE, MMSE–DFE, ZF–LE, MMSE–LE, and an MF receiver. Interestingly, although the frequency responses for all equalization schemes have the same structure, cf. (26), due to differences in the optimal relay power allocation, $p(f)$, the FF–BF filter frequency response for the ZF–LE case exhibits a
completely different behavior than the frequency responses for the other equalization schemes. In particular, since a zero in the frequency response of the overall channel, consisting of the source–relay channel, the FF–BF filter, and the relay–destination channel, would lead to infinite noise enhancement in a linear zero–forcing equalizer at the destination, the FF–BF filter design tries to avoid this problem by enhancing frequencies around $f = 1/2$. Note that the resulting scheme would still have a very poor performance since most of the relay power is allocated to frequencies where the overall channel is poor. In contrast, the other considered equalization strategies inherently avoid infinite noise enhancement at the destination even if the overall channel has zeros. Thus, in these cases, the optimal FF–BF filters avoid allocating significant amounts of power to frequencies around $f = 1/2$. This is particularly true for the MMSE equalizers and the MF receiver. The former allocate the power such that there is an optimal tradeoff between residual ISI and noise enhancement in the equalizer output signal, whereas the latter, idealized receiver is not affected by residual ISI.

Fig. 4 compares the frequency responses of the IIR FF–BF filter and FIR FF–BF filters of various lengths assuming MMSE–DFE at the receiver. As expected, as the FIR FF–BF filter length $L_a$ increases, the degree to which the FIR frequency response approximates the IIR frequency response increases. Although Fig. 4 suggests that relatively long FIR FF–BF filters are required to closely approximate the IIR filters, the results in the next section will show that short FIR FF–BF filters suffice to closely approach the SNR performance of IIR FF–BF filters.

### 5.3 SNR Performance for Fading Channels

In this section, unless stated otherwise, we consider the cooperative relay network shown in Fig. 5 with $N_R = 5$ relays at locations (a)–(e). The normalized distance between the source and the destination is equal to 2 and the normalized horizontal distance between the source and the relays is $d$. A path–loss exponent of 4 with reference distance $d_{\text{ref}} = 1$ is assumed. The CIR coefficients of all links are modeled as independent quasi–static Rayleigh fading with $L_g = L_h = L_f = 5$ and following an exponential power delay profile [26]

$$p[k] = \frac{1}{\sigma_t} \sum_{l=0}^{L_x-1} e^{-k/\sigma_t} \delta[k - l],$$

(49)

where $L_x \in \{L_g, L_h, L_f\}$ and $\sigma_t$ characterizes the delay spread. All SNR results were obtained from (11)–(13) and averaged over 100,000 independent realizations of the fading channels.
In Figs. 6 and 7 we show the average SNR vs. distance $d$ for various FF–BF filter and equalization designs. Here, $\sigma_t = 2$ and $\gamma_g = \gamma_h = \gamma_f = 10$ dB. In Fig. 6, we compare the performance of the proposed FF–BF filter design with MMSE–LE at the destination with the FF–BF filter design in [16] which assumed a slicer at the receiver and does not exploit the source–destination link. Clearly, by adding a simple linear equalizer at the destination and by exploiting the source–destination link, performance gains of several dB can be achieved for all considered distances $d$. For very small and very large distances, the scheme in [16] may even be outperformed by direct transmission without relay. It should be noted that for a given filter length $L_a$ the feedback requirements and the relay complexity for the proposed FF–BF scheme with equalization and the scheme in [16] without equalization are identical. Fig. 6 also shows that as $L_a$ increases, for the proposed scheme with FIR FF–BF, the performance of IIR FF–BF with MMSE–LE at the destination is approached, which validates the GA in Table 2 used for FIR FF–BF filter optimization. Performance can be further improved by replacing MMSE–LE by MMSE–DFE at the destination. However, for IIR–BF filters Fig. 6 shows that the additional gain achievable is less than 0.3 dB for all considered distances $d$, and the loss compared to an idealized MF receiver, which is the ultimate performance bound for any equalizer architecture, exceeds 1 dB only for $d < 0.7$.

The performance gap between different equalization strategies and for different FF–BF filter lengths is further investigated in Fig. 7. This figure shows that the performance gaps between LE and DFE and between DFE and the MF receiver decrease as $L_a$ increases. This can be explained by the fact that longer FF–BF filters have more degrees of freedom and can partially compensate the short–comings of LE and DFE at the destination by favorably shaping the transmit signals at the relays, cf. Fig. 3. Note that for all three considered receiver structures FIR FF–BF filters of length $L_a = 7$ incur a performance loss of less than 0.5 dB compared to IIR filters for all considered values of $d$, which again confirms the effectiveness of the proposed GA in Table 2.

For the remaining figures (Figs. 8–10), the source and the destination are in the same positions as in Fig. 5 but the relays are uniformly distributed in a circle with unit radius centered at (c) for $d = 1$. All results are averaged over 100,000 random relay positions.

In Fig. 8, we investigate the impact of decay parameter $\sigma_t$ on the performance of FF–BF for $\gamma_g = \gamma_h = \gamma_f = 10$ dB. From (49) we observe that the CIR coefficients decay the faster (i.e., the channel is less frequency selective), the smaller $\sigma_t$ is. As a special case, the channel becomes frequency flat when $\sigma_t = 0$. Fig. 8 shows that the performance of sufficiently long FF–BF filters is practically not affected by the frequency selectivity of the channel if MMSE–LE or MMSE–
DFE are employed at the destination. The idealized MF receiver with IIR FF–BF benefits even slightly from more frequency selectivity (larger $\sigma_t$) because of the additional diversity offered by the channel. In contrast, FF–BF without equalization at the receiver [16] is adversely affected by increased frequency selectivity and is even outperformed by direct transmission without relay (but with equalization at the destination) for $\sigma_t > 4$. The small performance gain of the proposed FF–BF scheme with equalization compared to the FF–BF scheme in [16] for $\sigma_t = 0$ (i.e., frequency–flat channel) is due to the fact the latter does not exploit the signal received during the first time interval at the destination (source–destination link). The fact that the performance difference between both schemes increases with increasing frequency selectivity suggests that the performance gain of the proposed scheme compared to [16] can be mostly attributed to the equalizer at the destination and not to the exploitation of the signal received at the destination during the first transmission interval.

In Fig. 9, we investigate the impact of the number of relays $N_R$ on the performance of various FF–BF and equalizer designs for $\sigma_t = 2$ and $\gamma_g = \gamma_h = \gamma_f = 10$ dB. As $N_R$ increases, the performance difference between IIR FF–BF with MMSE–LE, MMSE–DFE, and the MF receiver decreases since the FF–BF filters can use the additional degrees of freedom introduced by more relays to better compensate for the short–comings of the weaker equalization schemes. However, for short FIR BF–FF lengths the gap to the IIR filter increases. For example, for MMSE–DFE the SNR gap between a one tap FIR filter ($L_a = 1$) and an IIR filter is 0.5 dB and 4.4 dB for $N_R = 1$ and $N_R = 15$ relays, respectively. Nevertheless, for all values of $L_a$, increasing $N_R$ is always beneficial. The performance gap between the proposed FF–BF scheme with equalization and the FF–BF scheme without equalization in [16] decreases as $N_R$ increases. However, even for $N_R = 15$ relays the SNR gap between both schemes is still about 5 dB.

5.4 BER Performance for Fading Channels

In this section, we adopt the same channel model as in Section 5.3 and assume $\sigma_t = 2$ and $N_R = 5$. Fig. 10 shows the BER of BPSK modulation vs. transmit SNR, $\gamma = \gamma_g = \gamma_h = \gamma_f$, for FIR and IIR FF–BF filters. The BERs for FIR FF–BF filters were simulated by implementing MMSE–DFE with FIR equalization filters of length $4 \times \max\{L_{eq}, L_f\}$, which caused negligible performance degradation compared to IIR equalization filters. The BERs for IIR FF–BF were obtained by
approximating the BER of BPSK transmission by [22, 19]

$$\text{BER}_X = Q\left(\sqrt{2\text{SNR}_X}\right),$$

(50)

where $X = \text{DFE}$ and $X = \text{MF}$ for DFE and an MF receiver at the destination, respectively. Here, \(\text{SNR}_{\text{DFE}}\) and \(\text{SNR}_{\text{MF}}\) are given by (11) and (13), respectively, and the BER is averaged over 100,000 channel realizations.

Fig. 10 shows that equalization at the destination is very beneficial in terms of the achievable BER and large performance gains are realized compared to FF–BF without equalization [16]. Also, for sufficiently long FF–BF filters ZF–DFE and MMSE–DFE receivers achieve practically identical BERs and the gap to the idealized MF receiver is less than 0.7 dB. This gap could potentially be closed by trellis–based equalizers at the expense of an increase in complexity. Fig. 10 also shows that for MMSE–DFE at the destination, FIR FF–BF filters of length $L_a = 7$ achieve a performance close to that of IIR filters.

6 Conclusions

In this paper, we have considered FF–BF for frequency–selective cooperative relay networks with one source, multiple relays, and one destination. In contrast to prior work, we have assumed that the destination is equipped with a simple equalizer such as a linear or a decision feedback equalizer. The FF–BF filters at the relays were optimized for maximization of the SNR at the equalizer output under a joint relay power constraint. For IIR FF–BF filters, we found a unified expression for the frequency response of the optimal filters valid for LE, DFE, and an idealized MF receiver. We proposed a simple algorithm with guaranteed convergence for optimization of the power allocation factor included in the optimal frequency response. For FIR FF–BF filters, we showed that a difficult non–convex optimization problem results and proposed a simple and efficient gradient algorithm to find near–optimal filter coefficients. Our simulation results confirmed that (1) the performance gap between FF–BF filters with LE/DFE and FF–BF filters with an idealized MF receiver is relatively small implying that little can be gained by employing more complex trellis–based equalization schemes at the destination, (2) relatively short FIR FF–BF filters closely approach the performance of IIR FF–BF filters for all considered receiver structures confirming the near–optimal performance of the proposed gradient algorithm for FIR filter optimization, and (3) the proposed FF–BF scheme with simple LE or DFE at the destination achieves large performance gains compared to a previously
proposed scheme without equalization at the destination.

References


Tables and Figures:

Figure 1: Cooperative network with one source, multiple relays, and one destination. Equalization (EQ) is used at the destination. $\hat{s}[k]$ are estimated symbols after equalization.
Table 1: Numerical algorithm for finding the optimum power allocation $p(f)$ for IIR FF–BF filters at the relays. $X = \text{DFE}$, $X = \text{LE}$, and $X = \text{MF}$ for DFE, LE, and an MF receiver, respectively. Termination constant $\epsilon$ and frequency spacing $\Delta f$ have small values (e.g. $\epsilon = 10^{-5}$, $\Delta f = 10^{-5}$). $i$ denotes the iteration index.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Let $i = 0$, $N = \lceil 1/\Delta f \rceil$, and $f_n = -1/2 + (n-1) \Delta f$, $1 \leq n \leq N$. Initialize $l = 0$ and $u = \max_f \lim_{p(f) \to 0^+} S'_X(f)$.</td>
</tr>
<tr>
<td>2</td>
<td>Update $\mu$ by $\mu = (l + u)/2$.</td>
</tr>
</tbody>
</table>
| 3    | For $n = 1$ to $N$  
If $\lim_{p(f_n) \to 0^+} (S'_X(f_n) - \mu) < 0$, set $p(f_n) = 0$,  
otherwise compute $p(f_n)$ by solving $S'_X(f_n) = \mu$  
with the bisectional search method [23]. |
| 4    | If $\sum_{n=1}^{N} p(f_n) \Delta f > P$, $l = \mu$, else $u = \mu$. |
| 5    | If $u - l > \epsilon$, goto Step 2; else $p(f_n)$, $1 \leq n \leq N$, are the optimal power allocation parameters, and $\mu$ is the optimum Lagrange multiplier $\mu_{opt}$. |
Table 2: Gradient algorithm (GA) for calculation of near-optimal FIR FF–BF filter vector \( \mathbf{a} \). Termination constant \( \epsilon \) has a small value (e.g., \( \epsilon = 10^{-5} \)). \( i \) denotes the iteration index and \( \delta_i \) is the adaptation step size chosen through a backtracking line search [23].

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Let ( i = 0 ) and initialize vector ( \mathbf{v} ) with some ( \mathbf{v}_0 ) fulfilling ( \mathbf{v}_0^H \mathbf{v}_0 = P ).</td>
</tr>
<tr>
<td>2</td>
<td>Update the vector ( \mathbf{v} ):</td>
</tr>
<tr>
<td>DFE:</td>
<td>( \mathbf{v}_{i+1} = \mathbf{v}<em>i + \delta_i \left[ \int</em>{-1/2}^{1/2} \left( \frac{\bar{\mathbf{J}}(f)}{\mathbf{v}_i^H \bar{\mathbf{J}}(f) \mathbf{v}_i} - \frac{\bar{\mathbf{X}}(f)}{\mathbf{v}_i^H \bar{\mathbf{X}}(f) \mathbf{v}_i} \right) , df \right] \mathbf{v}_i );</td>
</tr>
<tr>
<td>LE:</td>
<td>( \mathbf{v}_{i+1} = \mathbf{v}<em>i - \delta_i \left[ \int</em>{-1/2}^{1/2} \left( \frac{\mathbf{v}_i^H \bar{\mathbf{J}}(f) \mathbf{v}_i \bar{\mathbf{X}}(f) - \mathbf{v}_i^H \bar{\mathbf{X}}(f) \mathbf{v}_i \bar{\mathbf{J}}(f)}{\left( \mathbf{v}_i^H \bar{\mathbf{J}}(f) \mathbf{v}_i \right)^2} \right) , df \right] \mathbf{v}_i );</td>
</tr>
<tr>
<td>MF:</td>
<td>( \mathbf{v}_{i+1} = \mathbf{v}<em>i + \delta_i \left[ \int</em>{-1/2}^{1/2} \left( \frac{\mathbf{v}_i^H \bar{\mathbf{X}}(f) \mathbf{v}_i \bar{\mathbf{J}}(f) - \mathbf{v}_i^H \bar{\mathbf{J}}(f) \mathbf{v}_i \bar{\mathbf{X}}(f)}{\left( \mathbf{v}_i^H \bar{\mathbf{X}}(f) \mathbf{v}_i \right)^2} \right) , df \right] \mathbf{v}_i );</td>
</tr>
<tr>
<td></td>
<td>(Note that normalization of vector ( \mathbf{v}<em>{i+1} ) is not necessary since ( \mathbf{v}</em>{i+1}^H \mathbf{v}_i = P ).)</td>
</tr>
<tr>
<td>3</td>
<td>Compute ( \text{SNR}<em>X(\mathbf{v}</em>{i+1}) ) based on (11)–(13).</td>
</tr>
<tr>
<td>4</td>
<td>If (</td>
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<td>5</td>
<td>( \mathbf{v}<em>{i+1} ) is the desired vector, and the corresponding optimum FF–BF filter is ( \mathbf{a} = D^{-1/2} \mathbf{v}</em>{i+1} ).</td>
</tr>
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</table>
Figure 2: SNR vs. iteration number $i$ of GA given in Table 2 for FIR FF–BF with MMSE–DFE at the destination. $N_R = 5$ relays, $\gamma_g = \gamma_h = \gamma_f = 10$ dB, $L_g = L_h = L_f = 5$, and $\tilde{g}_i[k] = \tilde{h}_i[k] = \tilde{f}[k] = 1/\sqrt{5}$, $0 \leq k < 5$, $1 \leq i \leq 5$. For comparison the SNR for IIR FF–BF is also shown.
Figure 3: Frequency responses of IIR FF–BF filters for $\gamma_g = \gamma_h = \gamma_f = 10$ dB, $N_R = 1$ relay, $L_g = L_h = L_f = 2$, and $\bar{g}_1[k] = \bar{h}_1[k] = \bar{f}[k] = 1/\sqrt{2}$, $k \in \{0, 1\}$. For comparison the frequency response of the test channel is also shown.
Figure 4: Frequency responses of IIR FF–BF filter and FIR FF–BF filters of various lengths for MMSE–DFE at the receiver. All channel parameters are identical to those in Fig. 3.
Figure 5: Locations of source, destination, and relays in simulation.
Figure 6: Average SNR vs. distance $d$ for FF-BF with MMSE-LE, MMSE-DFE, and an MF receiver at the destination. $N_R = 5$ relays, exponentially decaying power delay profile with $\sigma_t = 2$ and $L_g = L_h = L_f = 5$, and $\gamma_g = \gamma_h = \gamma_f = 10$ dB. For comparison the SNRs of FF-BF without (w/o) equalization (EQ) [16] at the destination and without relaying are also shown, respectively.
Figure 7: Average SNR vs. distance $d$ for IIR and FIR FF–BF with MMSE–LE, MMSE–DFE, and an MF receiver at the destination. $N_R = 5$ relays, exponentially decaying power delay profile with $\sigma_t = 2$ and $L_g = L_h = L_f = 5$, and $\gamma_g = \gamma_h = \gamma_f = 10$ dB.
Figure 8: Average SNR vs. decay parameter $\sigma_t$ for FF–BF with MMSE–LE, MMSE–DFE, and an MF receiver at the destination. $N_R = 5$ relays uniformly distributed in unit radius circle centered at (c) for $d = 1$, exponentially decaying power delay profile with $L_g = L_h = L_f = 5$, and $\gamma_g = \gamma_h = \gamma_f = 10$ dB. For comparison the SNRs of FF–BF w/o EQ [16] and without relaying are also shown, respectively.
Figure 9: Average SNR vs. number of relays $N_R$ for FF–BF with MMSE–LE, MMSE–DFE, and an MF receiver at the destination. $N_R$ relays are uniformly distributed in unit radius circle centered at (c) for $d = 1$, exponentially decaying power delay profile with $\sigma_t = 2$ and $L_g = L_h = L_f = 5$, and $\gamma_g = \gamma_h = \gamma_f = 10$ dB. For comparison the SNRs of FF–BF w/o EQ [16] and without relaying are also shown, respectively.
Figure 10: Average BER of BPSK vs. transmit SNR $\gamma$ for FF–BF with MMSE–DFE, ZF–DFE, and an MF receiver at the destination. $N_R = 5$ relays uniformly distributed in unit radius circle centered at (c) for $d = 1$, and exponentially decaying power delay profile with $\sigma_t = 2$ and $L_g = L_h = L_f = 5$. For comparison the BER of FF–BF w/o EQ [16] is also shown.