Cooperative Filter–and–Forward Beamforming with Linear Equalization

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Abstract—In this paper, we investigate filter–and–forward beamforming (FF–BF) for relay networks employing single–carrier transmission over frequency–selective channels. In contrast to prior work, we assume that the destination node is equipped with a simple linear equalizer. The FF–BF filters at the relays are optimized for maximization of the signal–to–noise ratio at the equalizer output under a joint relay power constraint. For infinite impulse response (IIR) FF–BF filters, we derive a closed–form expression for the filter frequency response, and a numerical algorithm with guaranteed convergence is developed for optimization of the power allocation factor included in the expression. We also provide an efficient gradient algorithm for recursive calculation of near–optimal finite impulse response (FIR) FF–BF filters. Simulation results show that, in general, short FIR FF–BF filters are sufficient to closely approach the performance of IIR FF–BF filters even in severely frequency–selective channels and that the proposed FF–BF scheme with equalization at the destination achieves substantial performance gains compared to a previously proposed FF–BF scheme without equalization.

I. INTRODUCTION

Relaying is a promising technique to extend the range of wireless communication systems by allowing intermediate nodes to forward packets transmitted by a source to the intended destination [1]. The two main relay protocols considered in the literature are amplify–and–forward (AF) and decode–and–forward (DF) relaying [1]. Thereby, AF relaying is generally believed to be less complex as the relays only perform a linear processing of the received signals, whereas the relays have to decode and re–encode the received signals in DF relaying. In AF relaying, beamforming (BF) across the relays is a simple yet efficient technique to improve capacity and reliability. BF for AF relays and frequency–flat channels has been extensively studied in the literature, cf. e.g. [2] and references therein. However, in most practical applications, the channels are frequency selective and techniques optimized for frequency–flat channels are not directly applicable. AF relaying techniques for systems employing orthogonal frequency–division multiplexing (OFDM) to cope with the frequency selectivity of the channel have been investigated in e.g. [3] and references therein. However, while OFDM is gaining popularity [4], there are still many applications where single–carrier transmission techniques are preferred because of legacy issues or the disadvantages of OFDM such as a high peak–to–average power ratio. Such applications include the GSM/EDGE mobile communication system, whose standard is still being further extended, and sensor networks, for which the cost and power consumption of the highly linear power amplifiers required for OFDM may be prohibitive.

Relaying schemes for single–carrier transmission over frequency–selective channels have received little attention in the literature so far with [5], [6] being two notable exceptions. Specifically, a cooperative filter–and–forward (FF) BF technique was proposed and optimized under the assumptions that (1) there is no direct link between the source and the destination, (2) an equalizer is not available at the destination, and (3) full channel state information (CSI) of all links is available [5]. Distributed space–time block coding at the relays and equalization at the destination has been proposed in [6]. Distributed space–time coding does not require full CSI but has a worse performance than FF–BF.

In this paper, we investigate cooperative FF–BF for frequency–selective channels for the case where the destination node has enough processing power to perform simple linear equalization (LE). Similar to [5], we assume that the central node, which computes the optimal FF–BF filters, has full CSI of all links. However, unlike [5], our model also includes a direct link between the source and the destination node. This paper makes the following contributions:

• Assuming finite impulse response (FIR) and infinite impulse response (IIR) filters at the relays, we optimize FF–BF for maximization of the signal–to–noise ratio (SNR) at the output of a linear equalizer. For the IIR case, we show that the frequency response vector of the optimal FF–BF filters can be decomposed into a unit–norm direction vector and a scalar power allocation factor across frequencies. We provide a closed–form solution for the direction vector and an efficient numerical method with guaranteed convergence for the power allocation.

• For the FIR case, we show that the FF–BF filter optimization problem results in a difficult non–convex optimization problem and provide an efficient numerical method for recursive calculation of near–optimal FIR FF–BF filters.

• Our simulation results show that (1) relatively short FIR FF–BF filters suffice to closely approach the performance of IIR FF–BF filters, (2) the gap between FF–BF with LE and the matched filter (MF) receiver [7] is small implying that little can be gained by adopting more complex equalization schemes, and (3) the addition of a simple linear equalizer at the destination node yields large performance gains compared to FF–BF without equalization advocated in [5].

Organization: In Section II, the adopted system model is presented. The optimization of IIR FF–BF filters is discussed in Section III, and the FIR case is considered in Section IV. Simulation results are provided in Section V, and some conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider a relay network with one source node, $N_R$ relays, and one destination node. A block diagram of the discrete–time overall transmission system in equivalent complex baseband representation is shown in Fig. 1. As usual, transmission is organized in two intervals. In the first interval, the source node transmits a data packet which is received by the destination and the relays. In the second interval, the relays filter the received packet and forward it to the destination node. At the destination, the packets received during the first and second intervals are combined, processed, and detected.

In Fig. 1, the discrete–time channel impulse responses (CIRs) between the source and relay $i$, $g_i[k]$, $0 \leq k \leq L_g - 1$, between relay $i$ and the destination, $h_i[k]$, $0 \leq k \leq L_h - 1$, and between the source and the destination, $f[k]$, $0 \leq k \leq L_f - 1$, contain
the combined effects of transmit pulse shaping, the continuous–time channel, receive filtering, and sampling. Here, $L_s$, $L_a$, and $L_f$ denote the lengths of the source–relay, the relay–destination, and the source–destination channels, respectively.

In the following, we describe the processing performed at the relays and the destination in detail.

A. FF–BF at Relays

The signal received at the $i$th relay during the first transmission interval is given by

$$ y_i[k] = g_i[k] * s[k] + n_{R_i}[k], \quad i = 1, \ldots, N_R, $$

where $s[k]$ is independent and identically distributed (i.i.d.) symbols taken from a scalar symbol alphabet $A$ such as phase–shift keying (PSK) or quadrature amplitude modulation (QAM) with variance $\sigma^2_q \triangleq \mathcal{E}\{|s[k]|^2\} = 1$, and $n_{R_i}[k]$ denotes additive white Gaussian noise (AWGN) with variance $\sigma^2_n \triangleq \mathcal{E}\{|n_{R_i}[k]|^2\}$.

The FF–BF filter impulse response coefficients of relay $i$, $1 \leq i \leq N_R$, are denoted by $a_i[k]$, $-q_i \leq k \leq q_u$. For IIR FF–BF filters $q_i \to \infty$ and $q_u \to \infty$ and for FIR FF–BF filters $q_i = 0$ and $q_u = L_a - 1$, where $L_a$ is the FF–BF filter length. The signal transmitted by the $i$th relay during the second transmission interval can be expressed as

$$ t_i[k] = a_i[k] * y_i[k], \quad i = 1, \ldots, N_R. $$

B. Equalization at Destination Node

The signal received at the destination node during the first transmission interval is given by

$$ r_0[k] = f[k] * s[k] + n_0[k], \quad (1) $$

where $n_0[k]$ is AWGN with variance $\sigma^2_n$. The signal received at the destination during the second transmission interval is

$$ r_1[k] = \sum_{i=1}^{N_R} h_i[k] * t_i[k] + n_1[k] = h_{eq}[k] * s[k] + n_1'[k], \quad (2) $$

where $n_1[k]$ is AWGN with variance $\sigma^2_n$. The equivalent CIR $h_{eq}[k]$ between source and destination and the effective noise $n_1'[k]$ are given by

$$ h_{eq}[k] \triangleq \sum_{i=1}^{N_R} h_i[k] * a_i[k] * g_i[k] \quad (3) $$

and

$$ n_1'[k] \triangleq \sum_{i=1}^{N_R} h_i[k] * a_i[k] * n_{R_i}[k] + n_1[k], \quad (4) $$

respectively. Note that $n_1'[k]$ is colored noise because of the filtering of $n_{R_i}[k]$ by $h_i[k]$ and $a_i[k]$.

Eqs. (1) and (2) show that a cooperative relay network with FF–BF can be modeled as a single–input multiple–output (SIMO) system with two outputs $r_0[k]$ and $r_1[k]$. Therefore, at the destination node the same channel estimation, equalization, and channel tracking techniques as for point–to–point SIMO transmission can be used [8]. Here, we adopt SIMO LE optimized under zero–forcing (ZF) and minimum mean–squared error (MMSE) criteria.

1) In this paper, $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, $I_X$, $0_X$, $\mathbb{E}\{\cdot\}$, and $*$ denote transpose, Hermitian transpose, complex conjugate, the $X \times X$ identity matrix, the all–zero column vector of length $X$, expectation, and discrete–time convolution, respectively. $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2} dt \triangleq \mathcal{G}(x)$ denote the Gaussian $Q$–function and the Kronecker delta function, respectively. $\text{diag}(X_1, X_2, \ldots, X_N)$ denotes a block–diagonal matrix with matrices $X_1, X_2, \ldots, X_N$ on the main diagonal. Furthermore, $X(f) \triangleq \mathcal{F}\{x[k]\} = \sum_{k=-\infty}^{\infty} x[k] e^{-2\pi j f k}$ is the Fourier transform of discrete–time signal $x[k]$. A. Problem Formulation

After whitening, we have an equivalent SIMO channel where the sub–channels have frequency responses $H_{eq}(f)$ and $F(f) \triangleq \mathcal{F}\{h_{eq}[k]\}$.
\( \mathcal{F} \{ f[k] \} \) and are impaired by AWGN with variance \( \sigma^2_n \), respectively. Thus, introducing

\[
Z(\alpha(f)) \triangleq |H_{eq}(f)|^2 = \frac{a^H(f)q^*(f)q^T(f)\alpha(f)}{a^H(f)\Gamma(f)\alpha(f) + 1},
\]

we can express the SNR at the output of the linear equalizer as

\[
\text{SNR}(\alpha(f)) = \frac{\sigma^2}{\sigma^2_n} \left( \int_{-1/2}^{1/2} (Z(\alpha(f)) + |F(f)|^2 + \xi)^{-1} \, df \right)^{-1} - \chi.
\]

In (8), we have \( \chi = 0, \xi = 0 \) and \( \chi = 1, \xi = \sigma^2_n/\sigma^2 \) if the equalization filters are optimized based on a ZF and an MMSE criterion, respectively.

In this paper, our goal is to optimize the FF–BF filters for maximization of the SNR at the output of the equalizer. To make the problem well defined, we constrain the relay transmit power, \( P_R \), which is given by

\[
P_R = \sum_{i=1}^{N_T} \int_{-1/2}^{1/2} \Phi_{i}(f) \, df = \int_{-1/2}^{1/2} a^H(f)D(f)\alpha(f) \, df,
\]

where \( \Phi_{i}(f) \triangleq |A_i(f)|^2(\sigma^2_n|G_i(f)|^2 + \sigma^2_n) \), \( i = 1, \ldots, N_T \), is the power spectral density of the transmit signal \( t_i[k] \) of the \( i \)th relay and \( D(f) \triangleq \text{diag}\{\sigma^2_n|G_1(f)|^2, \ldots, \sigma^2_n|G_{N_R}(f)|^2\} + \sigma^2_nI_{N_R} \).

Formally, the IIR FF–BF filter optimization problem can now be stated as

\[
\begin{align*}
\max_{\alpha(f)} \quad & \text{SNR}(\alpha(f)) \\
\text{s.t.} \quad & \int_{-1/2}^{1/2} a^H(f)D(f)\alpha(f) \, df \leq P, \\
\end{align*}
\]

where \( P \) denotes the maximum relay transmit power. It is convenient to introduce vector \( \upsilon(f) \triangleq D^{1/2}(f)\alpha(f) \), which can be expressed as \( \upsilon(f) = \sqrt{p(f)}u(f) \) without loss of generality, where \( p(f) \) denotes the power of \( u(f) \) and \( u(f) \) has unit norm \( ||u(f)||^2 = 1 \). Furthermore, we introduce \( Z(\upsilon(f)) = Z(\sqrt{p(f)}u(f)) \triangleq Z(\alpha(f)) \), which is given by

\[
Z(\upsilon(f)) = \frac{a^H(f)q^*(f)q^T(f)\alpha(f)}{a^H(f)\Gamma(f)\alpha(f) + 1} = \frac{u^H(f)J(f)u(f)}{u^H(f)X(f)u(f)}
\]

with rank one, positive semi–definite matrix \( J(f) = p(f)D^{-1/2}(f)q^*(f)q^T(f)D^{-1/2}(f) \) and full rank, positive definite matrix \( X(f) = p(f)D^{-1/2}(f)\Gamma(f)D^{-1/2}(f) + I_{N_R} \). Introducing \( \text{SNR}(\upsilon(f)) = \text{SNR}(\sqrt{p(f)}u(f)) \triangleq \text{SNR}(\alpha(f)) \), we can restate problem (10) in equivalent form as

\[
\begin{align*}
\max_{p(f),u(f)} \quad & \text{SNR}(\sqrt{p(f)}u(f)) \\
\text{s.t.} \quad & \int_{-1/2}^{1/2} p(f) \, df \leq P, \\
\end{align*}
\]

B. Optimal IIR FF–BF Filters

We observe from (12) that the constraints of the considered optimization problem do not depend on \( u(f) \). Thus, without loss of generality, we can find the globally optimal solution of problem (12) by first maximizing the SNR with respect to \( u(f) \) for a given power allocation \( p(f) \) and by subsequently optimizing the resulting SNR expression with respect to \( p(f) \).

Furthermore, \( \text{SNR}(u(f)) \) is monotonically increasing in \( Z(\sqrt{p(f)}u(f)) \). Thus, for any given power allocation \( p(f) \), we can maximize \( \text{SNR}(u(f)) \) by maximizing \( Z(\sqrt{p(f)}u(f)) \) with respect to \( u(f) \) for all frequencies \( f \). Hence, the optimal FF–BF direction vector, \( u_{\text{opt}}(f) \), can be found from the following optimization problem

\[
\max_{u(f)} Z(\sqrt{p(f)}u(f)) = \frac{u^H(f)J(f)u(f)}{u^H(f)X(f)u(f)}
\]

Problem (13) is a generalized eigenvalue problem for which a closed–form solution exists:

\[
u_{\text{opt}}(f) = c(f)X^{-1/2}(f)D^{-1/2}(f)q^*(f),
\]

where \( c(f) \) is a real–valued scaling factor which is given by

\[
c(f) = (q^T(f)D^{-1/2}(f)X^{-2}(f)D^{-1/2}(f)q^*(f))^{-1/2}.
\]

The maximum \( Z(\sqrt{p(f)}u(f)) \) achievable with \( u_{\text{opt}}(f) \) is

\[
Z(\sqrt{p(f)}u_{\text{opt}}(f)) = p(f)q^T(f)p(f)\Gamma(f)+D(f)^{-1}q^*(f).
\]

Now, we can express the optimal FF–BF filter frequency response vector (for a given power allocation), \( \alpha^*(f) \), as

\[
\alpha^*(f) = \sqrt{p(f)}D^{-1/2}(f)u_{\text{opt}}(f) = p(f)c(f)\left(p(f)\Gamma(f)+D(f)\right)^{-1}q^*(f).
\]

Furthermore, using the definitions of matrices \( D(f) \) and \( \Gamma(f) \), we can express the FF–BF filter frequency response at relay \( i, i = 1, \ldots, N_R \), as

\[
A_i^*(f) = \frac{\sqrt{p(f)}c(f)}{p(f)|H_i(f)|^2+\sigma^2_n|G_i(f)|^2+\sigma^2_n}H_i^*(f)G_i^*(f).
\]

Eq. (17) reveals that the optimal IIR FF–BF filters may be interpreted as the concatenation of a filter matched to the source–relay and the relay–destination link with frequency response \( H_i^*(f)G_i^*(f) \) and a second filter whose frequency response \( \sqrt{p(f)}c(f)/(p(f)|H_i(f)|^2+\sigma^2_n|G_i(f)|^2+\sigma^2_n) \) depends on the power allocation. The power allocation problem will be tackled in the next section.

C. Optimal Power Allocation

We introduce \( S(f) \), defined as \( S(f) \triangleq -1/M(f) \), with

\[
M(f) \triangleq q^T(f)(\Gamma(f)+D(f)/p(f))^{-1}q^*(f) + |F(f)|^2 + \xi.
\]

Based on these definitions, the equalizer output SNR (8), the original optimization problem (12), and the optimal frequency
response direction in (14), we can formulate the power allocation problem as

$$\max_{p(f)} \int_{-1/2}^{1/2} S(f) \, df$$

(19a)

$$\text{s.t.} \int_{-1/2}^{1/2} p(f) \, df \leq P \quad \text{or equivalently}$$

$$\max_{p(f)} S(f) - \mu p(f)$$

(19b)

$$S'(f) \triangleq \frac{\partial S(f)}{\partial p(f)} = \mu.$$ 

(20)

Since $\partial M(f)/\partial p(f) < 0$ and $\partial S(f)/\partial M(f) > 0$ for $M(f) > 0$, the power allocation problem is convex. The corresponding Lagrange dual function is

$$\mathcal{D}(\mu) = \max_{p(f)} \int_{-1/2}^{1/2} (S(f) - \mu p(f)) \, df$$

$$= \int_{-1/2}^{1/2} \max_{p(f)} (S(f) - \mu p(f)) \, df,$$ 

(21)

where $\mu \geq 0$ is the Lagrangian multiplier. The second line in (20) can be established because the total power constraint (19b) is implicitly captured by the dual variable $\mu$ and the maximization over $p(f)$ can be moved inside the integration. Therefore, for a given $\mu$, $p(f)$ can be obtained from

$$\max_{p(f)} S(f) - \mu p(f)$$

(19c)

or equivalently

Note that constraint (19c), which has been ignored in (20), can be taken into account by evaluating $S'(f) \triangleq \partial S(f)/\partial p(f)$ for $p(f) \to 0^+$. In particular, since $S'(f)$ is a monotonic decreasing function of $p(f)$ for all considered equalization schemes, for a given $\mu$, $S'(f) = \mu$ does not have a positive solution if $\lim_{p(f) \to 0^+} S'(f) < \mu$, and we set $p(f) = 0$ in this case. Otherwise, we find $p(f)$ from (22) by using e.g. the bisection search method [11]. On the other hand, the optimal value $\mu = \mu_{\text{opt}}$ that ensures the power constraint is satisfied can be found iteratively by another bisection search. However, if the corresponding total power $P_R = \int_{-1/2}^{1/2} p(f) \, df$ is less than the maximum power $P$ for a given $\mu$, the Lagrange multiplier $\mu$ has to be decreased, whereas it is increased if $P_R > P$.

We note that since the frequency axis is real valued, in practice, $f$ has to be discretized in $-1/2 \leq f \leq 1/2$ to make the problem computationally tractable. A summary of the numerical algorithm for finding the optimal power allocation, $p_{\text{opt}}(f)$, for discrete frequency points is given in Algorithm I. Applying $p_{\text{opt}}(f)$ found with Algorithm I in (17), yields the optimal FF–BF filter frequency response $A_{i,\text{opt}}(f)$ for relay $i$, $1 \leq i \leq N_R$.

IV. OPTIMAL FIR FF–BF FILTERS

In practice, it is not possible to implement the IIR FF–BF filters discussed in the previous section since they would require the feedback of an infinite number of filter coefficients. However, the performance achievable with these IIR FF–BF filters provides a useful upper bound for the FIR FF–BF filters considered in this section. In particular, the performance of the IIR solution can be used for optimizing the FIR FF–BF length to achieve a desired trade–off between the amount of feedback and performance. We note that although FIR FF–BF filters are considered in this section, the equalizer at the destination is still assumed to employ IIR filters.

With FIR FF–BF filters of length $L_a$ at the relays the relays the length of the equivalent CIR $h_{eq}[k]$ (3) is given by $L_{eq} = L_a + L_g + L_h - 2$. In this case, the Fourier transform of $h_{eq}[k]$ can be expressed as $H_{eq}(f) = d^H(f) Q a$ with $d(f) \triangleq [1, e^{2\pi f}, \ldots, e^{2\pi f(N-1)}]^T$, $\Gamma$ the FF–BF coefficient vector $a \triangleq a_0[0] \, a_1[1] \, \ldots \, a_{N_b}[N_b]$, where $Q$, is an $L_{eq} \times L_a$ column–circulant matrix with vector $H_{eq}(f) \, \Gamma$ in the first column. Here, $H_{eq}$ is an $(L_h + L_g - 1) \times L_a$ column–circulant matrix with vector $[h_0[0] \, \ldots \, h_{L_{eq}-1}[L_a-1]]^T$ in the first column and $g_i \triangleq [g_i[0] \, \ldots \, g_i[L_a-1]]^T$.

The noise whitening filter in the FIR case is given by

$$W(f) = (a^H \tilde{\Gamma}(f) a + 1)^{-1/2}$$

(23)

with $L_a N_R \times L_a N_R$ block diagonal matrix $\tilde{\Gamma}(f) \triangleq \begin{pmatrix} \Gamma(f) & \cdots & \Gamma_{N_b}(f) \end{pmatrix}$ of rank $N_R$, where $\tilde{\Gamma}(f) \triangleq H_{eq}^H d^H(f) H_{eq}$ is an $L_a \times L_a$ matrix of rank 1. Here, $H_{eq}$ is an $(L_h + L_g - 1) \times L_a$ column–circulant matrix with vector $[h_0[0] \, \ldots \, h_{L_{eq}-1}[L_a-1]]^T$ in the first column and $d(f) \triangleq [1, e^{2\pi f}, \ldots, e^{2\pi f(L_h+L_g-2)}]^T$. Therefore, after noise whitening, the frequency response of the overall channel is

$$H_{eq}(f) = d^H(f) Q a (a^H \tilde{\Gamma}(f) a + 1)^{-1/2}.$$ 

(24)

We note that for a practical implementation, the noise whitening filter does not have to be implemented. Instead, the noise correlation can be directly taken into account for equalizer filter design [9]. However, in order to be able to exploit the simple existing expressions for the SNR of the equalizer output given in [8]–[10], it is advantageous to assume the presence of a whitening filter for FIR FF–BF filter design.

A. Problem Formulation

Similar to the IIR case in (7), also for the FIR case it is convenient to introduce the definition

$$Z(a) \triangleq |H_{eq}(f)|^2 = \frac{a^H H_{eq}(f) Q a}{a^H \tilde{\Gamma}(f) a + 1}.$$ 

(25)

Note, however, that this is a slight abuse of notation since while the argument of $Z(a(f))$ in (7) is a vector containing all frequency responses of the IIR FF–BF filters, the argument of $Z(a)$ in (25) is a real scalar.
In this section, we present simulation results for the SNR and the bit error rate (BER) of a cooperative network with FF–BF. Throughout this section we assume \( \sigma_z^2 = 2 + 1 = 1 \). This allows us to decompose the CIRs as \( h_i[k] = \sqrt{\gamma_i} \tilde{h}_i[k] \), \( g_i[k] = \sqrt{\gamma_i} \tilde{g}_i[k] \), and \( f[k] = \sqrt{\gamma_f} \tilde{f}[k] \), where \( \gamma_i \), \( \gamma_f \), and \( \gamma_f \) denote the transmitter SNRs of the relay–destination, the source–relay, and the source–destination links, respectively. The normalized CIRs \( \tilde{h}_i[k] \), \( \tilde{g}_i[k] \), and \( \tilde{f}[k] \) include the effects of multipath fading and path–loss. All IIR and FIR FF–BF filters were obtained using Algorithm I and Algorithm II, respectively. For all results shown the GA in Algorithm II was initialized with a scaled all–one vector.

We consider the cooperative relay network shown in Fig. 2 with \( N_{RF} = 5 \) relays at locations (a)–(e). The normalized distance between the source and the destination is equal to 2 and the normalized horizontal distance between the source and the relays is \( d \). A path–loss exponent of 3 with reference distance \( d_{ref} = 1 \) is assumed. The CIR coefficients of all links are modeled as independent quasi–static Rayleigh fading with \( L_g = L_h = L_f = 5 \) and following an exponential power delay profile \( p[k] = \gamma_0 \sum_{l=0}^{L_g-1} e^{-k/\sigma_l} \delta[k-l] \), where \( L_g \in \{ L_{g1}, L_{g2}, L_{g3} \} \) and \( \sigma_t \) characterizes the delay spread [12]. All SNR results were averaged over 100,000 independent realizations of the fading channels.

In Fig. 3 we show the average SNR vs. distance \( d \) for various FF–BF filter and equalization designs. Here, \( \sigma_t = 2 \) and \( \gamma_0 = \gamma_{th} = \gamma_f = 10 \) dB. We compare the performance of the proposed FF–BF filter design with MMSE–LE at the destination with the FF–BF filter design in [5] which assumed a slicer at the receiver and does not exploit the source–destination link. Clearly, by adding a simple linear equalizer at the destination and by exploiting the source–destination link, performance gains of several dB can be achieved for all considered distances \( d \). For very small and very large distances, the scheme in [5] may even be outperformed by direct transmission without relay. It should be noted that for a given filter length \( L_o \) the feedback requirements and the relay complexity for the proposed FF–BF scheme with equalization and the scheme in [5] without equalization are identical. Fig. 3 also shows that as \( L_o \) increases, for the proposed scheme with FIR FF–BF, the performance of IIR FF–BF with MMSE–LE at the destination is approached, which validates the GA in Algorithm II used for FIR FF–BF.

**Algorithm II:** Gradient algorithm (GA) for calculation of near–optimal FIR FF–BF filter vector \( a \). Termination constant \( \epsilon \) has a small value (e.g. \( \epsilon = 10^{-5} \)).

1. Let \( i = 0 \) and initialize vector \( v \) with some \( v_0 \) fulfilling \( v^H v = P \).
2. Update the vector \( v \):
   \[
   v_{i+1} = v_i - \frac{1}{\epsilon} \left[ \frac{1}{\epsilon} \int_{-1/2}^{1/2} \frac{\Phi(f)}{J(f)} df \right] v_i
   \]
   (Note that normalization of vector \( v_{i+1} \) is not necessary since \( v^H_{i+1} v_i = P \).)
3. Compute SNR(\( v_{i+1} \)) based on (8).
4. If \( |\text{SNR}(v_{i+1}) - \text{SNR}(v_i)| < \epsilon \), goto Step 5, otherwise increment \( i = i + 1 \) and goto Step 2.
5. \( v_{i+1} \) is the desired vector, and the corresponding optimum FF–BF filter is \( a = D^{-1/2} v_{i+1} \).

**V. Simulation Results**

Algorithm II: Gradient algorithm (GA) for calculation of near–optimal FIR FF–BF filter vector \( a \). Termination constant \( \epsilon \) has a small value (e.g. \( \epsilon = 10^{-5} \)). \( i \) denotes the iteration index and \( \delta_i \) is the adaptation step size chosen through a backtracking line search [11].

1. Let \( i = 0 \) and initialize vector \( v \) with some \( v_0 \) fulfilling \( v^H v = P \).
2. Update the vector \( v \):
   \[
   v_{i+1} = v_i - \frac{1}{\epsilon} \left[ \frac{1}{\epsilon} \int_{-1/2}^{1/2} \frac{\Phi(f)}{J(f)} df \right] v_i
   \]
   (Note that normalization of vector \( v_{i+1} \) is not necessary since \( v^H_{i+1} v_i = P \).)
3. Compute SNR(\( v_{i+1} \)) based on (8).
4. If \( |\text{SNR}(v_{i+1}) - \text{SNR}(v_i)| < \epsilon \), goto Step 5, otherwise increment \( i = i + 1 \) and goto Step 2.
5. \( v_{i+1} \) is the desired vector, and the corresponding optimum FF–BF filter is \( a = D^{-1/2} v_{i+1} \).

**B. Near–Optimal FIR FF–BF Filters**

Similar to problem (26), the FIR FF–BF optimization problem in (31) is a difficult non–convex optimization problem and it does not seem possible to find the globally optimal solution in an efficient way. In order to obtain a practical and simple method for finding a locally optimal solution for the FIR FF–BF coefficient vectors, we propose a gradient algorithm (GA).

The GA improves vector \( v_i \) from iteration \( i \) in the direction of the steepest ascent [11] \( \int_{-1/2}^{1/2} \frac{\Phi(f)}{J(f)} df \) of the objective function in (31a). The GA is summarized in Algorithm II. Although, in principle, the GA may not be able to find the globally optimal solution, extensive simulations have shown that for the problem at hand the performance achievable with GA is practically independent of the initialization \( v_0 \). More importantly, for sufficiently large FIR filter lengths \( L_o \), the solution found with the GA closely approaches the performance of the optimal IIR FF–BF filter. This suggests that the solution found by the GA is at least near optimal. Exemplary simulation results confirming these claims are provided and discussed in the next section.
The BER of BPSK transmission by approximating performance degradation compared to IIR equalization filters of lengths $N_f = 3$ and exponentially decaying power delay profile with $\sigma_t = 2$ and $L_a = L_b = L_f = 5$, and $\gamma_f = \gamma_h = \gamma_t = 10$ dB. For comparison the SNRs of FF–BF without (w/o) equalization (EQ) [5] at the destination and without relaying are also shown, respectively.

BF filter optimization. The loss compared to an idealized MF receiver, which is the ultimate performance bound for any equalizer architecture [13], exceeds 1 dB only for $d < 0.7$.

Fig. 4 shows the BERs of BPSK modulation vs. transmit SNR, $\gamma = \gamma_0 = \gamma_h = \gamma_f$, for FIR and IIR FF–BF filters. The BERs for FIR FF–BF filters were simulated by implementing MMSE–LE scheme with IIR equalization filters of lengths $4 \times \max(L_{eq}, L_f)$, which caused negligible performance degradation compared to IIR equalization filters. The BERs for IIR FF–BF were obtained by approximating the BER of BPSK transmission by $BER = Q(\sqrt{2SNR})$ [10]. The BER is averaged over 100,000 channel realizations. Fig. 4 shows that equalization at the destination is very beneficial in terms of the achievable BER and large performance gains are realized compared to FF–BF without equalization [5]. Also, for sufficiently long FF–BF filters ZF–LE and MMSE–LE receivers achieve practically identical BERs and the gap to the idealized MF receiver is less than 0.6 dB. Fig. 4 also shows that for MMSE–LE at the destination, FIR FF–BF filters of length $L_a = 7$ achieve a performance close to that of IIR filters.

VI. CONCLUSIONS

In this paper, we have considered FF–BF for frequency–selective cooperative relay networks. In contrast to prior work, we have assumed that the destination is equipped with a simple linear equalizer. The FF–BF filters at the relays were optimized for maximization of the SNR at the equalizer output under a joint relay power constraint. For IIR FF–BF filters, we found a closed–form expression for the frequency response of the optimal filters, and proposed a simple algorithm with guaranteed convergence for optimization of the power allocation factor included in the optimal frequency response. For FIR FF–BF filters, a difficult non–convex optimization problem resulted and we proposed a simple and efficient gradient algorithm to find near–optimal filter coefficients, which for sufficiently large filter lengths closely approach the performance of the optimal IIR FF–BF filters. Our simulation results confirmed that the proposed FF–BF scheme achieves large performance gains compared to a previously proposed scheme without equalization at the destination.

References


