Optimal Energy Consumption Scheduling Using Mechanism Design for the Future Smart Grid

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Abstract—In the future smart grid, both users and power companies can benefit from real-time interactions and pricing methods which can reflect the fluctuations of the wholesale price into the demand side. In addition, smart pricing can be used to seek social benefits and to achieve *social objectives*. However, the utility company may need to collect various information about users and their energy consumption behavior, which can be challenging. That is, users may not be willing to reveal their local information unless there is an incentive for them to do so. In this paper, we propose an efficient pricing algorithm to tackle this problem. The benefit that each user obtains from each appliance can be modeled in form of a *utility function*, a concept from microeconomics. We propose a Vickrey-Clarke-Groves (VCG) based mechanism for our problem formulation aiming to maximize the *social welfare*, i.e., the aggregate utility functions of all users minus the total energy cost. Our design requires that each user provides some information about its energy demand. In return, the energy provider will determine each user's payment for electricity. The payment of each user is structured in such a way that it is in each user's *self interest* to reveal its local information *truthfully*. Finally, we present simulation results to show that both the energy provider and the individual users can benefit from the proposed pricing algorithm.

I. INTRODUCTION

Demand side management (DSM) is one of the key components of the future smart grid to enable more efficient and more reliable grid operation. To achieve a high level of reliability in power systems, utility companies have to design the grid for the peak demand rather than the average demand. This usually results in an under-utilized system. To remedy this problem, different programs have been proposed to shape the energy consumption pattern of the users in order to use the available generating capacity more efficiently without installing new generation and transmission infrastructure. These programs typically aim at one or both of the following design objectives: *reducing consumption* and *shifting consumption*. Adopting energy-aware consumption patterns and constructing energy efficient buildings can reduce the energy consumption of users [1]. However, there is also a need to develop practical methods to encourage users to shift their usage of high-power household appliances to off-peak hours. Considering the increasing expectations of users both in quantity and quality [2], and the emergence of new types of demand such as plug-in hybrid electric vehicles (PHEVs), which can potentially double the average household energy consumption [3], appropriate load-shifting becomes even more crucial.

Among different approaches considered for residential load management programs, smart pricing is known as one of the most effective tools that can encourage users to consume more wisely and efficiently [4]–[7]. Smart pricing can also be used to seek public benefits and to achieve *social objectives*. However, the utility company may need to collect various information about users and their energy consumption behavior, which can be challenging.

In this paper, we aim to maximize the *social welfare* (the aggregate utility of all users minus the cost of providing energy). Each appliance provides a certain amount of benefit depending on the pattern and the volume of power it consumes, and this benefit can be different from one user to another. To model the benefit obtained from each appliance, we adopt the concept of utility functions from microeconomics [8]. Knowledge of the utility functions of the users at utility company is crucial to achieve the maximum social welfare. However, in general, users are not willing to reveal such information, unless there is an incentive for them to do so. Therefore, elaborate design rules (*mechanisms*) are needed such that it is in each user's self interest to reveal its local information. This problem has already been considered for smart grid [9] and in other contexts such as in telecommunication networks [10]. However, the work in [9] and [10] assumed that users are *price takers* who *accept* the prices as fixed parameters, and do not consider the possibility that their actions may affect price values. For systems with built-in automated control units, this assumption may not be valid. Collecting local information from *price anticipating* users requires more advanced methods.

Vickrey-Clarke-Groves (VCG) mechanism is one of the well known solutions for eliciting local information from rational users. In a VCG mechanism, to determine the price charged to each user, users are asked to declare their energy demand information. The payments of the users are structured in such a way that the users have incentive to declare their local information truthfully. We note that VCG-based mechanism has already been proposed for DSM [11]. However, the problem formulation in [11] aims to change the total energy consumption, while our proposed method does not aim to change the amount of energy consumption, but instead to shift it to off-peak hours systematically. The contributions of this paper are summarized as follows:

- We propose a VCG-based mechanism for DSM programs to encourage efficient energy consumption among users.

Fig. 1. An illustration of a regional energy provider, several users, and multiple power generators as parts of the general wholesale energy market.

In our system model, each user reveals its demand information to the energy provider. By running a centralized algorithm, the energy provider computes the optimal energy consumption schedule for each user, and advertises a specific payment value for each user.

- We formulate an optimization problem to maximize the aggregate utility of all users while minimizing the total cost imposed to the energy provider.
- We investigate some of the desired properties of our problem formulation such as *truthfulness* and *efficiency* of the proposed mechanism.
- We compare our efficient VCG-based method with the case where users are price anticipators. We study the differences of these two systems with respect to the aggregate load, the peak-to-average ratio (PAR), and the social welfare of the system.
- Simulation results confirm that both the users and the energy provider can benefit from the proposed algorithm.

The remainder of this paper is organized as follows. In Section II, we present the system model, formulate our design problem as a convex optimization problem, and review the marginal cost pricing algorithm for the two cases of price taking users and price anticipating users. The VCG-based mechanism and its properties are presented in Section III. In Section IV, we evaluate the performance of the proposed pricing scheme. Conclusions are drawn in Section V.

II. SYSTEM MODEL

In this section, we provide a mathematical model for the residential load control problem in *real time pricing* (RTP) environments. Consider a smart power system with multiple load subscribers or users and one energy provider as part of the general wholesale electricity market shown in Fig. 1. We assume that each user is equipped with a smart meter which has an *energy consumption scheduling* (ECS) unit capable of scheduling the household energy consumption. All ECS units are connected to the energy provider through a communication infrastructure such as a local area network (LAN).

Throughout the paper, let $\mathcal{N}$ denote the set of users, where $N \triangleq |\mathcal{N}|$. We also assume that the intended time cycle for the operation of the system is divided into T time slots, where $T \triangleq |\mathcal{T}|$, and $\mathcal{T}$ is the set of all time slots.

A. Residential Load Control

For each user $n \in \mathcal{N}$, let $\mathcal{A}_n$ denote the set of the user's appliances such as dishwasher, PHEVs, etc. For each appliance

$a \in \mathcal{A}_n$, we define an energy consumption scheduling vector $\mathbf{x}_{n,a}$ as follows:

$$\mathbf{x}_{n,a} \triangleq (x_{n,a}^1, \dots, x_{n,a}^T), \quad (1)$$

where $x_{n,a}^t$ denotes the energy consumption that is scheduled for appliance $a \in \mathcal{A}_n$ in time slot $t \in \mathcal{T}$. Thus, the total load of the n th user at time slot $t \in \mathcal{T}$ is

$$l_n^t = \sum_{a \in \mathcal{A}_n} x_{n,a}^t. \quad (2)$$

For each appliance $a \in \mathcal{A}_n$, we denote the total energy needed for the operation of the appliance as $E_{n,a}$. As in [12], our designed energy consumption scheduler does not aim to change the amount of energy consumption, but instead to systematically shift it in order to maximize the *social welfare*. Using the notation in [12], we assume that for each appliance $a \in \mathcal{A}_n$, the user specifies the valid scheduling interval $\mathcal{T}_{n,a} \triangleq \{\alpha_{n,a}, \dots, \beta_{n,a}\}$, where $\alpha_{n,a}, \beta_{n,a} \in \mathcal{T}$ denote the beginning and the end of the scheduling interval, respectively. Clearly, for each appliance a , we have $\alpha_{n,a} < \beta_{n,a}$. For example, a user may select $\alpha_{n,a} = 10$ PM and $\beta_{n,a} = 7$ AM of the next day for its PHEV to have it ready in the early morning. In order to provide the required energy for each appliance $a \in \mathcal{A}_n$ within the interval $[\alpha_{n,a}, \beta_{n,a}]$, it is required that

$$\sum_{t \in \mathcal{T}_{n,a}} x_{n,a}^t = E_{n,a} \quad (3)$$

and

$$x_{n,a}^t = 0, \quad \forall t \in \mathcal{T} \setminus \mathcal{T}_{n,a}. \quad (4)$$

Clearly, for each appliance, the time interval indicated by the user needs to be larger than or equal to the time interval needed to finish the operation. We define the minimum standby power level $\gamma_{n,a}^{\min}$ and the maximum power level $\gamma_{n,a}^{\max}$ for each appliance $a \in \mathcal{A}_n$. Standby power refers to the electric power consumed by each appliance while it is either switch off or is in standby mode. Thus, we have

$$\gamma_{n,a}^{\min} \leq x_{n,a}^t \leq \gamma_{n,a}^{\max}, \quad \forall t \in \mathcal{T}_{n,a}. \quad (5)$$

We define $\mathbf{x}_n$ as the vector of energy consumption of user n , which is formed by stacking up the energy consumption scheduling vectors $\mathbf{x}_{n,a}$ for all appliances $a \in \mathcal{A}_n$. The *feasible* energy consumption scheduling set of user n is defined as

$$\mathcal{X}_n = \left\{ \mathbf{x}_n \mid \sum_{t \in \mathcal{T}_{n,a}} x_{n,a}^t = E_{n,a}, x_{n,a}^t = 0, \quad \forall t \in \mathcal{T} \setminus \mathcal{T}_{n,a}, \right. \\ \left. \gamma_{n,a}^{\min} \leq x_{n,a}^t \leq \gamma_{n,a}^{\max}, \quad \forall t \in \mathcal{T}_{n,a} \right\}. \quad (6)$$

B. Energy Cost Model

The *cost function* $C_t(L_t)$ represents the cost to the energy provider to supply L_t units of energy in each time slot $t \in \mathcal{T}$. We make the following assumptions:

Assumption 1: The cost functions are *increasing* with respect to the total offered energy.

Assumption 2: The cost functions are *strictly convex*.

Assumption 3: There exists a differentiable, convex, non-decreasing function $p_t(q)$ over $q \geq 0$ for each $t \in \mathcal{T}$, with $p_t(0) \geq 0$ and $p_t(q) \rightarrow \infty$ as $q \rightarrow \infty$, such that for $q \geq 0$

$$C_t(q) = \int_0^q p_t(z) dz. \quad (7)$$

Quadratic functions are among practical examples satisfying Assumptions 1-3, and are considered in this paper as well as in [12] and [13]

$$C_t(L_t) = c_1^t L_t^2 + c_2^t L_t + c_3^t, \quad (8)$$

where $c_1^t > 0$, $c_2^t \geq 0$, and $c_3^t \geq 0$ are pre-determined parameters.

C. User Preference and Utility Function

The degree to which each user n values the scheduled consumption vector $\mathbf{x}_{n,a}$ for appliance $a \in \mathcal{A}_n$ can be characterized by adopting the concept of utility functions from microeconomics [8]. However, the users also have different preferences for different appliances that should be scheduled in each time slot. In this regard, for each appliance $a \in \mathcal{A}_n$, we represent the corresponding utility function as $U_{n,a}(\mathbf{x}_{n,a}) \triangleq \sum_{t \in \mathcal{T}_{n,a}} U(x_{n,a}^t, \omega_{n,a}^t)$, where $U(x_{n,a}^t, \omega_{n,a}^t)$ denotes the utility that user n draws by running appliance a as a function of its scheduled power consumption $x_{n,a}^t$ in time slot $t \in \mathcal{T}_{n,a}$, and $\omega_{n,a}^t$ is a parameter which may vary among users and also among appliances at different times of the day, characterizing the value of electricity scheduled for appliance a at that particular time of the day. For each appliance $a \in \mathcal{A}_n$, we define $\omega_{n,a} \triangleq [\omega_{n,a}^{\alpha_{n,a}}, \dots, \omega_{n,a}^{\beta_{n,a}}]$. For notational simplicity, for each user n , we introduce vector ω_n , which is formed by stacking up vectors $\omega_{n,a}$ for all appliances $a \in \mathcal{A}_n$. We assume that the utility functions $U(x_{n,a}^t, \omega_{n,a}^t)$ fulfill the following properties:

Property 1: Utility functions are *non-decreasing* with respect to the consumed power.

Property 2: Utility functions are concave.

Property 3: The appliances can be ranked based on their utility functions. For a fixed consumption level $x_{n,a}^t$, a larger $\omega_{n,a}^t$ gives a larger $U(x_{n,a}^t, \omega_{n,a}^t)$.

Property 4: We assume that zero power consumption results in a zero utility value.

While the class of utility functions that fulfill Properties 1 and 2 is very large, it is convenient to have a linear marginal benefit [14], [15].

D. Problem Formulation and Efficient Scheduling

In this section, we consider the problem of energy consumption scheduling. From a social fairness point of view, it is desirable to utilize the available generated power provided by the energy provider such that the sum of the utility functions of *all* users is maximized and the cost imposed to the energy provider is minimized. In this regard, we assume that the energy provider is regulated in such a way that its objective

is not to maximize its profit through selling electricity, but rather to maximize the social welfare. If centralized control is feasible and we can collect all information about the users' utility functions, an efficient energy consumption schedule can be characterized as the solution of the following problem:

$$\underset{\mathbf{x}_n \in \mathcal{X}_n, n \in \mathcal{N}}{\text{maximize}} \sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}) - \sum_{t \in \mathcal{T}} C_t \left(\sum_{n \in \mathcal{N}} l_n^t \right), \quad (9)$$

where l_n^t is defined in (2). Problem (9) is a concave maximization problem and can be solved in a centralized fashion using *convex programming* techniques such as the interior point method (IPM) [16]. Since it is assumed that the vector ω_n of each user n is local information, the energy provider may not have sufficient information to solve problem (9). Each user tries to optimize its own objective. To align these individual objectives with the social objective, some elaborately designed pricing scheme is needed. Next, we investigate different approaches of users in responding to the price values.

E. Equilibrium Among Users

In general, users may have different approaches in responding to the price values set by the energy provider. This can lead to different equilibriums among users. We are especially interested in analyzing *competitive equilibrium* and *Nash equilibrium*. In competitive equilibrium, each user acts as a *price taker*. That is, it does not consider the effect of its actions on the price. However, in Nash equilibrium, it is assumed that users are price anticipators, i.e., they consider the effect of their actions on the price set by the energy provider.

1) *Price Taking Users:* Given a price vector $\lambda \triangleq \{\lambda_t | t \in \mathcal{T}\}$, a user n chooses its energy consumption schedule $\mathbf{x}_n$ such that its *payoff* function,

$$P_n(\mathbf{x}_n) = \sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}) - \sum_{t \in \mathcal{T}} \lambda_t l_n^t, \quad (10)$$

is maximized, where λ_t is the price in time slot t . We call a pair $(\mathbf{x}, \lambda)$ a *competitive equilibrium* if users maximize their own payoff function defined in (10) for a given price vector λ , i.e.,

$$P_n(\mathbf{x}_n) \geq P_n(\bar{\mathbf{x}}_n), \quad \bar{\mathbf{x}}_n \in \mathcal{X}_n, \quad n \in \mathcal{N}, \quad (11)$$

where vector $\mathbf{x}_n$ is the solution to the problem defined in (9). It has been shown that under Properties 1-4 and Assumptions 1-3, a competitive equilibrium always exists [9]. A decentralized algorithm is proposed in [9] for the smart grid in which each user receives a real-time price vector from the energy provider and chooses its energy consumption schedule to maximize its own payoff function introduced in (10). On the other hand, the energy provider uses marginal cost pricing to update the real-time price. It has been shown that the proposed algorithm in [9] converges to the social optimal solution. However, when the price taking assumption is violated, the model changes into a game, and the assumptions required to have competitive equilibrium no longer hold. We investigate this scenario in the next sub-section.

2) *Price Anticipating Users*: If users are price anticipators, i.e., they consider the effect of their actions on the price, then we need to analyze the Nash equilibrium of the game which is played among multiple users who compete for the available energy provided by the energy provider. We consider the following pricing scheme for resource allocation. Given $\mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_N)$, the energy provider sets a single price for each time slot t as $\mu_t(\mathbf{x}) = p_t(\sum_{n \in \mathcal{N}} l_n^t)$, where $p_t(\cdot)$ is defined in Assumption 3. We use the notation $\mathbf{x}_{-n}$ to denote the vector of all consumption powers chosen by users other than user n , i.e., $\mathbf{x}_{-n} = (\mathbf{x}_1, \dots, \mathbf{x}_{n-1}, \mathbf{x}_{n+1}, \dots, \mathbf{x}_N)$. Then, given $\mathbf{x}_{-n}$, the payoff of each user n is

$$Q_n(\mathbf{x}_n; \mathbf{x}_{-n}) = \sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}) - \sum_{t \in \mathcal{T}} p_t \left(\sum_{m \in \mathcal{N}} l_m^t \right) l_n^t. \quad (12)$$

From (12), the payoff of each user depends on its energy scheduling as well as the energy scheduling of other users. Hence, we will have the following game among the users:

- **Players**: Registered users in set $\mathcal{N}$.
- **Strategies**: Each user $n \in \mathcal{N}$ selects its energy consumption schedule $\mathbf{x}_n \in \mathcal{X}_n$ to maximize its payoff.
- **Payoffs**: $Q_n(\mathbf{x}_n; \mathbf{x}_{-n})$ for each user $n \in \mathcal{N}$ as in (12).

A *Nash equilibrium* of the game defined by $(Q_1, \dots, Q_N)$ is a vector $\mathbf{x}$ such that for all $n \in \mathcal{N}$,

$$Q_n(\mathbf{x}_n; \mathbf{x}_{-n}) \geq Q_n(\bar{\mathbf{x}}_n; \mathbf{x}_{-n}) \quad \bar{\mathbf{x}}_n \in \mathcal{X}_n. \quad (13)$$

It can be shown that a Nash equilibrium exists for this game. However, in general, the Nash equilibrium of a resource allocation game may not be optimal [17], [18]. That is, the energy consumption profile obtained at the Nash equilibrium in a distributed pricing scenario may not necessarily be the same as the optimal solution of the optimization problem in (9). Next, we investigate how the price values can be set by the utility company such that the system performance becomes optimal at the aforementioned Nash equilibrium.

III. THE VICKREY-CLARKE-GROVES (VCG) APPROACH

In the previous section, we considered a mechanism which uses only a single price for all users to allocate the provided power. As mentioned before, the main obstacle in solving problem (9) centrally is the lack of information about the utility functions of the users. However, if we remove the restriction that the mechanism only chooses a single price, it is possible to elicit the utility information of the users. One of the best known approaches to convince users to declare their utility functions is the VCG mechanism [19].

In the VCG class of mechanisms, each user is asked to specify its utility function, which reduces to revealing a vector ω_n and a set of constraints $\mathcal{X}_n$, which can be represented as a single matrix. For each user n , based on the declared vector $\hat{\omega}_n$, we use $\hat{U}_{n,a}$ to denote the declared utility function and $\hat{\mathbf{U}}_n \triangleq (\hat{U}_{n,1}, \dots, \hat{U}_{n,|\mathcal{A}_n|})$ to denote the vector of declared utility functions of user n . We also define $\hat{\mathbf{U}} \triangleq (\hat{\mathbf{U}}_1, \dots, \hat{\mathbf{U}}_N)$ to denote the vector of all declared utility functions.

If user n has an energy consumption schedule $\mathbf{x}_n$, but has to pay θ_n , then the payoff function of user n is

$$\sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}) - \theta_n. \quad (14)$$

On the other hand, the social objective is in the form of

$$\sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}) + \sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} U_{m,a}(\mathbf{x}_{m,a}) - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}} l_m^t \right),$$

where $\mathcal{N}_{-n}$ is the set of all users except user n . For a given vector of declared utility functions $\hat{\mathbf{U}}$, the VCG mechanism chooses the energy consumption scheduling vector $\mathbf{x}(\hat{\mathbf{U}})$ as an optimal solution to problem (9) and calculates optimal energy consumption vectors as

$$\mathbf{x}(\hat{\mathbf{U}}) = \arg \max_{\mathbf{x}_n \in \mathcal{X}_n, n \in \mathcal{N}} \left\{ \sum_{n \in \mathcal{N}} \sum_{a \in \mathcal{A}_n} \hat{U}_{n,a}(\mathbf{x}_{n,a}) - \sum_{t \in \mathcal{T}} C_t \left(\sum_{n \in \mathcal{N}} l_n^t \right) \right\} \quad (15)$$

and the payments are structured such that

$$\theta_n(\hat{\mathbf{U}}) = - \left(\sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} \hat{U}_{m,a}(\mathbf{x}_{m,a}) - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}} l_m^t \right) \right) + h_n(\hat{\mathbf{U}}_{-n}), \quad (16)$$

where h_n is an arbitrary function of the declared utility functions other than n , denoted by $\hat{\mathbf{U}}_{-n}$. We note that the definition of the payments in (16) is a natural way to align user objectives with the objective of a social planner.

We note that the cost term $C_t(\cdot)$ in (15) couples the consumption power variables, $\mathbf{x}_n$, of all users. This term makes the whole problem not only a utility maximization but also a cost minimization problem, and thus, the *system objective* is different from the normal objective of VCG mechanisms studied in other contexts [20]–[22]. These changes in our problem formulation require the verification of some desired properties of the proposed VCG-based mechanism for the new scenario. This is shown in the following proposition.

Proposition 1: If the VCG mechanism defined in (15) and (16) is used to select electricity price values, then declaring $\hat{\mathbf{U}}_n = \mathbf{U}_n$ is a dominant strategy for each user n , and following this strategy results in an efficient allocation.

The proof of Proposition 1 is given in the Appendix. For the following, we need to specify the function h_n introduced in (16). Here, we will use the most popular choice for this function, which is referred to as Clarke tax [19],

$$h_n(\hat{\mathbf{U}}_{-n}) = \sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} U_{m,a}(\mathbf{x}_{m,a}(\hat{\mathbf{U}}_{-n})) - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}_{-n}} l_m^t(\hat{\mathbf{U}}_{-n}) \right), \quad (17)$$

where $\mathbf{x}_{m,a}(\hat{\mathbf{U}}_{-n})$ is the VCG allocation choice introduced in (15) but with user n excluded from the system. The payment

of user n is

$$\begin{aligned} \theta_n(\hat{\mathbf{U}}) = & - \left(\sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} U_{m,a}(\mathbf{x}_{m,a}(\hat{\mathbf{U}})) \right. \\ & \left. - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}} l_m^t(\hat{\mathbf{U}}) \right) \right) \\ & + \sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} U_{m,a}(\mathbf{x}_{m,a}(\hat{\mathbf{U}}_{-n})) \\ & - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}_{-n}} l_m^t(\hat{\mathbf{U}}_{-n}) \right). \end{aligned} \quad (18)$$

The payment of user n is the difference in the social welfare of other users with and without the presence of user n .

IV. PERFORMANCE EVALUATION

In this section, we present simulation results and assess the performance of the proposed load control algorithm. In our simulation model, we consider a residential area comprising $N = 10$ users, and each user has around 10 major appliances including refrigerator-freezer (daily usage: 2.32 kWh), electric stove (daily usage: 4 kWh), lighting (daily usage: 2 kWh), heating (daily usage: 7.1 kWh), dishwasher (daily usage: 1.44 kWh), washing machine (daily usage: 3.4 kWh), clothes dryer (daily usage: 2.5 kWh), PHEV (daily usage: 9.9 kWh), and entertainment (daily usage: 9 kWh) [12]. Here, we aggregate different appliances such as TV and PC as one item named entertainment. We assume the energy needed for the operation of each appliance is slightly different for different users. We consider 24 time slots representing the 24 hours of a day. A day starts at 8 AM, i.e., the first time slot corresponds to the hour between 8 AM to 9 AM. For the purpose of study and for each appliance, we consider a quadratic concave utility function for each time slot defined as

$$U(x, \omega) = \begin{cases} \omega x - \frac{\delta}{2} x^2, & \text{if } 0 \leq x < \frac{\omega}{\delta}, \\ \frac{\omega^2}{2\delta}, & \text{if } x \geq \frac{\omega}{\delta}, \end{cases} \quad (19)$$

where δ is a pre-determined parameter. However, we set $\delta = 0.5$ for all time slots. For different appliances, parameter ω is selected from different sets indicating the preference of the user. The ω sets used for different appliances are as follows: $\{2, 4, 6, 8, 10\}$ for refrigerator, $\{10, 12, 14, 16, 18\}$ for electric stove, $\{2, 4, 6, 8, 10, 12\}$ for lightening, $\{2, 4, 6, 8, 10, 12\}$ for heating, $\{4, 6, 8, 10\}$ for dishwasher and washing machine, $\{2, 4, 6, 8\}$ for PHEV, and $\{6, 8, 10, 12, 14, 16, 18, 20\}$ for entertainment. We assume that the cost of energy is higher at the beginning of the day and also at the beginning hours of night, i.e., we set the parameter c_1^t of the cost function to 0.07 (8 AM-2 PM), 0.04 (2 PM-7 PM), 0.1 (7 PM-12 AM), 0.03 (12 AM-5 AM), and 0.04 (5 AM-8 AM), respectively.

To have a baseline to compare with, we consider two different systems. The first system is the one *without* ECS deployment, i.e., each appliance $a \in \mathcal{A}_n$ for each user $n \in \mathcal{N}$ is assumed to start operation right at the beginning of the

TABLE I
DIFFERENT ASPECTS OF THE VCG-BASED SYSTEM, THE PRICE ANTICIPATING SYSTEM, AND THE SYSTEM WITHOUT ECS DEPLOYMENT

	VCG-based system	Price anticipating system	System without ECS deployment
Energy cost	\$469.97	\$548.43	\$967.65
PAR	1.86	1.97	2.48
Social welfare	\$3294.6	\$3162.7	\$2587.6

Fig. 2. Power consumption for the VCG-based system, the system without ECS deployment, and the system which has price anticipating users.

time interval $[\alpha_{n,a}, \beta_{n,a}]$ and at its typical power level. The second system consists of price anticipating users and employs marginal cost pricing. The simulation results on total power consumption for the proposed VCG-based system, the system without ECS deployment, and the system which has price anticipating users are depicted in Fig. 2. As illustrated in Fig. 2, since we assume the lowest cost of providing energy for the hours between midnight and 5 AM, in the VCG-based system, users prefer to shift their energy consumption to low cost hours which results in a lower energy cost in the VCG-based system compared to the two other systems. The proposed algorithm also helps to reduce the PAR of the system compared to the system without ECS deployment and the system which has price anticipating users. Simulation results for the cost of providing energy and the PAR of the system are summarized in Table I.

In addition to advantages of the proposed algorithm for the energy provider, deployment of the proposed energy consumption scheduling can also benefit individual users. Simulation results for the payoff (utility minus cost) of the individual users for the VCG-based system, the system without ECS deployment, and the system which has price anticipating users are shown in Fig. 3. The payoff of each user is much higher for the proposed VCG-based algorithm compared to the case where users do not employ ECS units. For the system with price anticipating users, since users are playing a game, some users gain and some users loose compared to the proposed method. In our proposed method, we structure the energy consumption schedule such that the maximum social welfare is guaranteed. Simulation results for the social welfare of different methods are also presented in Table I.

Fig. 3. Payoff of each user for the VCG-based system, the system without ECS deployment, and the system which has price anticipating users.

V. CONCLUSIONS

In this paper, we proposed a VCG-based mechanism for DSM in the future smart grid. The proposed mechanism aims to maximize the aggregate utility of all the users while minimizing the total energy cost. We investigated some of the main properties of the proposed mechanism such as truthfulness and efficiency. Through simulation analysis, we compared our proposed method with two other systems, a system without ECS deployment and a system which has price anticipating users and employs marginal cost pricing. Simulation results confirmed that by using our proposed method, along with maximizing the social welfare, the energy provider will also benefit in terms of energy cost and PAR. The idea developed in this paper can be extended in several directions. For example, a system with multiple energy providers can be considered, and the effect of malicious users can be explored.

APPENDIX

Considering each user's payment in (16), since user n cannot affect the term h_n through the choice of $\hat{U}_n$, it chooses $\hat{U}_n$ to maximize

$$W_n(\mathbf{x}_n(\hat{\mathbf{U}}), t_n(\hat{\mathbf{U}})) = \sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}(\hat{\mathbf{U}})) + \sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} \hat{U}_{m,a}(\mathbf{x}_{m,a}(\hat{\mathbf{U}})) - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}} l_m^t(\hat{\mathbf{U}}) \right).$$

However, the above expression is bounded above by

$$\begin{aligned} \max_{\mathbf{x}_n \in \mathcal{X}_n, n \in \mathcal{N}} \quad & \sum_{a \in \mathcal{A}_n} U_{n,a}(\mathbf{x}_{n,a}) + \sum_{m \in \mathcal{N}_{-n}} \sum_{a \in \mathcal{A}_m} \hat{U}_{m,a}(\mathbf{x}_{m,a}) \\ & - \sum_{t \in \mathcal{T}} C_t \left(\sum_{m \in \mathcal{N}} l_m^t \right). \end{aligned}$$

We notice that $\mathbf{x}(\hat{\mathbf{U}})$ satisfies (15), and user n can achieve this maximum payoff by truthfully declaring $\hat{U}_n = U_n$ for solving (15). Since this optimal strategy does not depend on the utility functions declared by other users, it confirms that truthful declaration is a dominant strategy for each user. ■

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