Connectivity Maximization for Narrowband IoT Systems with NOMA

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Abstract—Narrowband Internet of Things (NB-IoT) is a recently standardized technology to support machine-type communications (MTC) in Long Term Evolution-Advanced (LTE-A) Pro networks. NB-IoT can enable energy-efficient communication with extended coverage on a narrow bandwidth of 180 kHz for low-cost MTC devices (MTCDs). The main challenge of supporting MTC in LTE-A Pro networks is to provide connectivity to a massive number of MTCDs. To overcome this challenge, in this paper, we propose a power-domain uplink non-orthogonal multiple access (NOMA) scheme for NB-IoT systems. By allowing multiple MTCDs to share the same sub-carrier, NOMA can provide connectivity to more MTCDs than orthogonal multiple access (OMA). We formulate a joint sub-carrier and transmission power allocation problem to maximize the number of MTCDs satisfying the quality of service (QoS) and transmission power requirements. We decompose the problem into two sub-problems and propose algorithms to solve them. Simulation results show that our proposed NOMA scheme can significantly increase the number of successfully connected MTCDs in NB-IoT systems compared to OMA.

I. INTRODUCTION

Machine-type communications (MTC) is an important enabler of the Internet of Things (IoT) [1]. MTC devices (MTCDs) are involved in a wide range of applications, such as environmental sensing, remote health monitoring, and intelligent transportation systems. It is predicted that the number of MTCDs connected to the wireless cellular networks will increase from 800 million in 2016 to 3.3 billion by 2021 [2]. The International Telecommunications Union (ITU) classifies MTC into two categories: massive MTC (mMTC) and ultra-reliable and low latency communications (URLLC) [3]. mMTC is characterized by a high connection density, i.e., a massive number of active low-cost and low-power MTCDs co-exist per cell. Sensor networks and wearables are examples of mMTC. These devices transmit small data packets with relaxed end-to-end latency requirements in the order of seconds or hours [4]. mMTC devices need to communicate with high energy efficiency to prolong their battery lifetime to reach 10 – 15 years [5] [6]. On the other hand, URLLC requires reliable data transmissions with strict latency constraints in the order of 10 milliseconds or less, as it is used for mission critical applications, such as e-health and autonomous driving.

In order to support MTC in the Long Term Evolution-Advanced (LTE-A) Pro networks, the Third Generation Partnership Project (3GPP) standardized narrowband IoT (NB-IoT) to enable low-rate energy-efficient connectivity with extended-coverage for low-cost MTCDs [7], [8]. NB-IoT can support uplink throughput of 20 – 50 kbps and enhance the network coverage by up to 20 dB [9]. This coverage enhancement is achieved by reducing the noise power through communication over a narrow bandwidth. The coverage enhancement margin makes NB-IoT communications immune against propagation and indoor penetration losses. In addition, MTCDs can reduce their uplink transmission power to prolong the battery lifetime without sacrificing connectivity.

In NB-IoT, frequency division multiple access (FDMA) [10], an orthogonal multiple access (OMA) scheme, is adopted for medium access over a narrow bandwidth of one physical resource block (PRB), i.e., 180 kHz. The system bandwidth can be equally divided into 48 or 12 sub-carriers. In the first case, each MTCD can be allocated a single sub-carrier (single-tone mode) as shown in Fig. 1(a). In the second case, each MTCD can be allocated either a single sub-carrier or a bond of 3, 6, or 12 contiguous sub-carriers (multi-tone mode) as shown in Fig. 1(b). Since FDMA only allows a single sub-carrier to be used by one MTCD, it may not be able to cope with the expected increase in the number of MTCDs connected to LTE-A Pro networks. Furthermore, the limited number of sub-carriers may result in the increase of medium access delay. Hence, the latency requirements of the MTCDs performing time-critical tasks, e.g., URLLC devices, may be violated.

To tackle this problem, a promising approach is to use non-orthogonal multiple access (NOMA) [11] in NB-IoT systems so that multiple MTCDs can share the same sub-carrier. Power-domain NOMA has been introduced to enhance the downlink spectral efficiency of human-to-human communication systems [11] [12]. A single transmitter sends multiple
messages, differentiated in power, to multiple receivers over the same sub-carrier. The power level difference is exploited to decode these messages sequentially at the receivers. In [13], different power allocation strategies are analyzed in multiple-input multiple-output (MIMO) downlink NOMA systems to enable IoT devices to share bandwidth opportunistic with human-controlled devices. NOMA has also been considered for uplink data transmissions in [14]. In power-domain uplink NOMA, the receiver receives messages from multiple transmitters on the same sub-carrier and decodes them sequentially. In [15], a power and resource block allocation problem of uplink NOMA transmission for frequency rate maximization with power constraints is formulated.

In this paper, our goal is to maximize the connection density of low data rate MTCDs supported in an NB-IoT system. We consider quality of service (QoS) and power-aware NOMA with a focus on uplink transmission since it is the dominant traffic flow of MTC. The main contributions of this paper are:

- We propose power-domain NOMA to allow multiple MTCDs to share the same sub-carrier in an NB-IoT system thus supporting a higher connection density.
- We formulate a joint sub-carrier and transmission power allocation problem to maximize the number of successfully connected MTCDs that can satisfy their QoS requirements.
- We further decompose the problem into two sub-problems. The first sub-problem allocates sub-carriers to URLLC devices and the second sub-problem allocates sub-carriers and transmission power to mMTC devices.
- We propose two algorithms to solve these two sub-problems. Simulation results show that the number of successfully connected MTCDs per sub-carrier is greater than OMA.

The rest of this paper is organized as follows. In Section II, we present the system model and problem formulation. We propose an algorithm to solve the problem in Section III. The performance evaluation is conducted in Section IV. Finally, Section V concludes the paper.

II. SYSTEM MODEL

Consider a cellular network covered by a single base station as shown in Fig. 2. MTC is supported in the network based on NB-IoT standard [7]. Active MTCDs share a system bandwidth of one PRB for uplink data transmissions. Our work focuses on one transmission time interval (TTI), however, the findings are valid for a longer period of time. Let \( \mathcal{M} = \{m_1, \ldots, m_M\} \) and \( \mathcal{U} = \{u_1, \ldots, u_U\} \) denote the sets of \( M \) mMTC devices and \( U \) URLLC devices, respectively, and \( \mathcal{M} \cap \mathcal{U} = \emptyset \). In our proposed NOMA scheme, each sub-carrier (radio channel) can be shared by up to two MTCDs, i.e., one URLLC device and one mMTC device.

A. NOMA Scheme

We propose a power-domain NOMA scheme to enable the co-existence of mMTC and URLLC devices. A URLLC device \( u \in \mathcal{U} \) and an mMTC device \( m \in \mathcal{M} \) transmit their messages over the same sub-carrier with transmission powers \( p_u \) and \( p_m \), respectively. A combined message \( y \) with additive thermal noise \( \sigma \) is received at the base station,

\[
y = \sqrt{p_u} h_u x_u + \sqrt{p_m} h_m x_m + \sigma,
\]

where \( x_u \) and \( x_m \) are the messages transmitted by URLLC device \( u \) and mMTC device \( m \), respectively. \( h_u \) denotes the gain of the channel between URLLC device \( u \) and the base station due to propagation loss and fading. Similarly, \( h_m \) is the channel gain of the mMTC device \( m \). The NOMA scheme is designed such that the messages can be reliably extracted from the combined message \( y \) and the requirements of both MTC categories are satisfied. We require that the received power from the URLLC device \( u \) is higher than that from the mMTC device \( m \), i.e., \( |h_u|^2 p_u > |h_m|^2 p_m \). The base station begins with decoding \( x_u \). Consequently, the received signal-to-interference-plus-noise ratio (SINR) of URLLC device \( u \), \( \gamma_u \), over a sub-carrier of bandwidth \( B \) can be expressed as

\[
\gamma_u = \frac{|h_u|^2 p_u}{N_o B + |h_m|^2 p_m},
\]

where \( N_o \) is the noise power spectral density and \( |h_m|^2 p_m \) represents the interference power of mMTC device \( m \) on URLLC device \( u \). The base station employs successive interference cancellation (SIC) to deduct the successfully decoded message \( x_u \) from the received message \( y \) before decoding \( x_m \). Hence, the received SINR of mMTC device \( m \), \( \gamma_m \), is given by

\[
\gamma_m = \frac{|h_m|^2 p_m}{N_o B}.
\]

B. QoS and Power Constraints

The achievable data rate of an mMTC device \( m \), \( r_m \), can be expressed in terms of the aggregate rate over the number of sub-carriers \( S_m \) allocated to it. We have

\[
r_m = S_m B_M \log_2 \left(1 + \frac{|h_m|^2 p_m}{N_o B_M}\right), \quad \forall m \in \mathcal{M},
\]

where \( B_M \) is the sub-carrier bandwidth of the mMTC devices. Each mMTC device requires a minimal data rate that should not be less than a certain threshold \( R_m \), i.e.,

\[
r_m \geq R_m, \quad \forall m \in \mathcal{M}.
\]

The transmission power cannot exceed a certain maximum transmission power \( P_m \). Hence, we have

\[
0 \leq p_m \leq P_m, \quad \forall m \in \mathcal{M}.
\]
On the other hand, the achievable data rate of a URLLC device $u$, $r_u$, when it is allocated $S_u$ sub-carriers is given by
\[
r_u = \sum_{u=1}^{S_u} B_u \log_2 \left( 1 + \frac{|h_u|^2 p_u}{I_u + N_u B_u} \right), \quad \forall \ u \in \mathcal{U},
\]
where $B_u$ is the sub-carrier bandwidth of the URLLC devices and $I_u,s$ is the interference caused by an mMTC device that shares sub-carrier $s$ with URLLC device $u$.

Since URLLC devices perform critical tasks, power consumption requirements may not be crucial [4]. We set the transmission power to the maximum possible value $P_u$, i.e., $p_u = P_u$. In addition, the data rate of URLLC devices $r_u$ should exceed a minimal threshold $R_u$,
\[
r_u \geq R_u, \quad \forall \ u \in \mathcal{U}.
\]

C. Sub-carrier Allocation Constraints

The system bandwidth is equally divided into $S_M$ sub-carriers with sub-carrier bandwidth $B_M$ to serve mMTC devices. The same bandwidth is equally divided into $S_U$ sub-carriers with sub-carrier bandwidth $B_U$ to support URLLC devices. $B_M$ and $B_U$ are selected from a set of possible values $B$. The sum of the sub-carriers allocated to mMTC or URLLC devices cannot exceed the total number of sub-carriers, i.e.,
\[
\sum_{m \in \mathcal{M}} S_m \leq S_M, \quad (9)
\]
\[
\sum_{u \in \mathcal{U}} S_u \leq S_U. \quad (10)
\]

We introduce an $M \times S_M$ mMTC scheduling matrix $J$, where $j_{m,s} = 1$ indicates that mMTC device $m$ is allocated sub-carrier $s$ at a given TTI, and $j_{m,s} = 0$ otherwise. A similar $U \times S_U$ matrix $K$ is used to denote the sub-carrier allocation of URLLC devices. The sub-carrier scheduling is subject to two constraints. First, each sub-carrier is allocated to at most one URLLC device. In addition, each sub-carrier can be shared by at most one mMTC device,
\[
\sum_{m \in \mathcal{M}} j_{m,s} \leq 1, \quad s = 1, \ldots, S_M, \quad (11)
\]
\[
\sum_{u \in \mathcal{U}} k_{u,s} \leq 1, \quad s = 1, \ldots, S_U. \quad (12)
\]

Second, each device is allocated either a single sub-carrier or a contiguous set of sub-carriers. The number of sub-carriers allocated to mMTC and URLLC devices in (4) and (7) can be rewritten in terms of the sub-carrier allocation as follows:
\[
S_m = \sum_{s=1}^{S_M} j_{m,s}, \quad \forall \ m \in \mathcal{M}, \quad (13)
\]
\[
S_u = \sum_{s=1}^{S_U} k_{u,s}, \quad \forall \ u \in \mathcal{U}. \quad (14)
\]

Sub-carrier Bandwidth (3.75 kHz): Only single-tone mode is possible. Each MTCD $d \in \mathcal{M} \cup \mathcal{U}$ can be allocated only one sub-carrier, i.e., $S_d \in \{1\}$,
\[
\sum_{s=1}^{S_M} j_{m,s} \leq 1, \quad \forall \ m \in \mathcal{M}, \quad (15)
\]
\[
\sum_{s=1}^{S_U} k_{u,s} \leq 1, \quad \forall \ u \in \mathcal{U}. \quad (16)
\]

Sub-carrier Bandwidth (15 kHz): An MTCD $d \in \mathcal{M} \cup \mathcal{U}$ can be allocated a bond of 1, 3, 6, or 12 contiguous sub-carriers, i.e., $S_d \in \{1, 3, 6, 12\}$, based on one of $C = 19$ combinations:
- 1 sub-carrier ($S_d = 1$) (12 combinations).
- 3 sub-carriers ($S_d = 3$) starting from sub-carrier $s$ to sub-carrier $s + 2$ and $s \in \{1, 4, 7, 10\}$ (4 combinations). For example, the bond of sub-carriers $\{7, 8, 9\}$ in Fig. 1(b).
- 6 sub-carriers ($S_d = 6$) starting from sub-carrier $s$ to sub-carrier $s + 5$ and $s \in \{1, 7\}$ (2 combinations). For example, the bond of sub-carriers $\{1, \ldots, 6\}$ in Fig. 1(b).
- The whole 12 sub-carriers ($S_d = 12$) (1 combination).

We denote $v_{d,c}$ as a binary variable which is equal to 1 if MTCD $d$ is allocated sub-carriers according to combination $c$, and is equal to 0 otherwise. We have
\[
\sum_{c=1}^{C} v_{d,c} \leq 1, \quad \forall \ d \in \mathcal{M} \cup \mathcal{U}. \quad (17)
\]

For $m \in \mathcal{M}$ and $c = 1, \ldots, C$, we can define $v_{m,c}$ in terms of $j_{m,s}$ as follows:
\[
v_{m,1} = j_{m,1}, \ldots, v_{m,12} = j_{m,12}, \quad v_{m,13} = \prod_{s=1}^{3} j_{m,s},
\]
\[
v_{m,14} = \prod_{s=4}^{6} j_{m,s}, v_{m,15} = \prod_{s=7}^{9} j_{m,s}, v_{m,16} = \prod_{s=10}^{12} j_{m,s}, \quad (18)
\]
\[
v_{m,17} = \prod_{s=1}^{6} j_{m,s}, v_{m,18} = \prod_{s=7}^{12} j_{m,s}, v_{m,19} = \prod_{s=1}^{12} j_{m,s}. \quad (18)
\]

Similarly, for $u \in \mathcal{U}$, we can write $v_{u,c}$ in terms of $k_{u,s}$,
\[
v_{u,1} = k_{u,1}, v_{u,2} = k_{u,2}, \ldots, v_{u,19} = \prod_{s=1}^{12} k_{u,s}. \quad (19)
\]

D. Problem Formulation

The NB-IoT NOMA problem is formulated as a connectivity maximization problem. The goal is to maximize the number of MTCDs that guarantee their required QoS requirements and satisfy their transmission power constraints. Let vector $z_M = (z_{m_1}, \ldots, z_{m_M})$, where $z_m = 1$ if $r_m \geq R_m$ and $p_m \leq P_m$ and $z_m = 0$ otherwise. Similarly, let vector $z_U = (z_{u_1}, \ldots, z_{u_U})$, where $z_u = 1$ if $r_u \geq R_u$ and $z_u = 0$ otherwise. The problem can be formulated as follows:

maximize
\[
\text{subject to } \quad r_d \geq z_d R_d, \quad \forall \ d \in \mathcal{M} \cup \mathcal{U}, \quad (20b)
\]
\[
0 \leq z_m P_m \leq P_m, \quad \forall \ m \in \mathcal{M}, \quad (20c)
\]
\[
j_{m,s} \in \{0,1\}, \quad \forall \ m \in \mathcal{M}, s = 1, \ldots, S_M. \quad (20e)
\]

maximize
\[
\sum_{d \in \mathcal{M} \cup \mathcal{U}} w_d |z_d| + \sum_{m \in \mathcal{M}} w_M |z_M| \quad (20a)
\]

subject to $z_d \in \{0,1\}, \quad \forall \ d \in \mathcal{M} \cup \mathcal{U}, \quad (20b)$
\[
r_d \geq z_d R_d, \quad \forall \ d \in \mathcal{M} \cup \mathcal{U}, \quad (20c)
\]
\[
0 \leq z_m P_m \leq P_m, \quad \forall \ m \in \mathcal{M}, \quad (20d)
\]
\[
j_{m,s} \in \{0,1\}, \quad \forall \ m \in \mathcal{M}, s = 1, \ldots, S_M. \quad (20e)
\]
where $p_M = \{p_{m_1}, \ldots, p_{m_M}\}$ is a vector of the transmission power of mMTC devices. \( w_M \) and \( w_M \) are weighting factors that are selected to prioritize URLLC devices due to the criticality of their missions compared to mMTC devices. For example, when \( w_M = M + 1 \) and \( w_M = 1 \), serving one URLLC device becomes more important than serving all mMTC devices since this makes the term \( w_M^4 \|z_M\|_0 \) always greater than \( w_M^4 \|z_M\|_0 \). The formulated problem is combinatorial and exhaustive search is too costly in terms of computational complexity. Hence, we propose a two-step hierarchical algorithm to tackle this problem.

### III. Proposed Algorithm

We propose a hierarchical sub-optimal algorithm based on decomposing problem (20) into two sub-problems. First, a master problem allocates sub-carriers to URLLC devices to maximize the number of successfully connected URLLC devices that satisfy their QoS constraints. Second, a sub-problem maximizes the number of successfully connected mMTC devices that satisfy their QoS and transmission power constraints. The combined solution of the two problems provides a feasible solution for problem (20).

In the first step, since there are two possible values of URLLC sub-carrier bandwidth \( B_{ul} \) in NB-IoT, which are 3.75 kHz and 15 kHz, the master URLLC problem is solved twice. The sub-carrier bandwidth that allows serving a larger number of URLLC devices is selected and its sub-carrier allocation solution is adopted. We also solve for \( I \), which is the vector that represents the maximum tolerable interference over each sub-carrier. The second step is to solve the mMTC sub-problem twice as well to obtain the sub-carrier and transmission power allocation. Finally, we select the sub-carrier bandwidth of mMTC devices that maximizes the number of mMTC devices satisfying their QoS and power requirements.

#### A. URLLC Master Problem

For each sub-carrier bandwidth value \( B_{ul} \in B \), we solve the following problem

\[
\text{maximize}_{K, u, I} \|z_u\|_0 + \|I\|_1 \tag{21a}
\]

subject to

\[
r_u = S_u B_{ul} \log_2 \left( 1 + \frac{\|h_u\|^2 P_u}{I_{u,S_u} + N_u B_{ul}} \right), \quad \forall u \in U \tag{21b}
\]

\[
I \succeq 0, \tag{21c}
\]

constraints (10), (12), (14), (20b), (20c), (20f), (20g),

\[
B_M, B_{ul} \in B, \tag{20h}
\]

\[
\text{constraints (4) (7) (9) (14),} \tag{20i}
\]

\[
\text{constraint (15), if } B_M = 3.75 \text{ kHz,} \tag{20j}
\]

\[
\text{constraint (16), if } B_{ul} = 3.75 \text{ kHz,} \tag{20k}
\]

\[
\text{constraints (17) (18), } \forall d \in M \cup U, c = 1, \ldots, C, \tag{20l}
\]

\[
\text{constraint (17) (19), } \forall d \in U \text{ if } B_{ul} = 15 \text{ kHz,} \tag{20m}
\]

Two problems are solved at the master and sub-levels. At the master-level, we solve problem (20) for two different sub-carrier bandwidths \( B_{ul} \) and \( B_{ul} \). In Line 4, we sort the URLLC devices of \( B_{ul} \) in ascending order of \( S_u \), and in Line 5, we sort the devices from the top of the list of \( S_u \) such that \( \sum_{u \in U'} S_u \) is maximized and constraint (10) is satisfied.

#### Algorithm 1: Algorithm to solve URLLC master problem

1. input: \( B_{ul}, S_u, h_u, P_u, R_u, \forall u \in U \)
2. \( S_u \in \{1\} \) sub-carrier if \( B_{ul} = 3.75 \text{ kHz} \), \( S_u \in \{1, 3, 6, 12\} \) sub-carrier(s) if \( B_{ul} = 15 \text{ kHz} \)
3. Initialize \( z_{ul} := 0, \forall u \in U \)
4. Initialize \( k_u, s := 0, \forall u \in U, s = 1, \ldots, S_u \)
5. Calculate \( I_{u,S_u} \) according to (22) \( \forall u \in U \)
6. Select minimum value of \( S_u \) so that \( I_{u,S_u} \geq 0 \) if feasible \( \forall u \in U \)
7. \( U' := \{u \mid u \in U, I_{u,S_u} \geq 0\} \)
8. \( S_u := 0, \forall u \in U \setminus U' \)
9. \( U' \leftarrow \) Sort URLLC devices of \( U' \) in an ascending order of \( S_u \)
10. \( U'' \leftarrow \) Sort URLLC devices of \( U'' \) in a descending order of \( I_{u,S_u} \)
11. \( z_u := 1, \forall u \in U'' \)
12. index := 1
13. \forall u \in U'' do
14. \( s := \text{index}, \ldots, \text{index} + S_u - 1, k_u := 1 \)
15. \( I_s := I_{u,S_u}, \text{index := index} + S_u \)
16. end
17. end
18. output: \( K, z_{ul}, I \).

where \( I = (I_1, \ldots, I_s, \ldots, I_{S_u}) \) and \( I_s \) is the maximum tolerable interference at sub-carrier \( s \). In this problem, we maximize \( \|z_{ul}\|_0 \) to provide connectivity to more URLLC devices. If there are multiple solutions with the same value of \( \|z_{ul}\|_0 \), we pick the solution that maximizes the term \( \|I\|_1 \) which allows for accommodating more mMTC devices by tolerating higher interference. As the values of the elements of \( I \) increase, it is more probable that mMTC devices can share sub-carriers with URLLC devices while transmitting with sufficient power to satisfy their QoS requirements. Note that \( \|I\|_1 \) is much smaller in magnitude than \( \|z_{ul}\|_0 \), which gives an inherent higher priority to URLLC devices than mMTC devices and it is consistent with problem (20) formulation.

The achievable data rate of URLLC devices is expressed in (21b), where \( I_{u,S_u} \) is the maximum tolerable interference for device \( u \) when it is allocated \( S_u \) sub-carriers. If \( r_u \) is set to be \( R_u \) in (21b) and the allocated bandwidth to URLLC device \( u \) is expressed by \( S_u B_{ul}, I_{u,S_u} \) can be calculated as follows:

\[
I_{u,S_u} = \frac{|h_u|^2 P_u}{2 \pi c_n B_{ul}} - N_u B_{ul}. \tag{22}
\]

The sub-carrier allocation algorithm for every value of \( B_{ul} \) is presented in Algorithm 1. In Lines 5 to 8, in order to maximize the connectivity, we allocate the minimum number of sub-carriers \( S_u \) such that the value of \( I_{u,S_u} \) is greater than or equal to \( 0 \), i.e., a minimum data rate \( R_u \) can be achieved if the interference power is upper bounded by \( I_{u,S_u} \). In Line 9, sorting URLLC devices according to \( I_{u,S_u} \) in an ascending order enables serving more URLLC devices by serving devices requiring less sub-carriers. On the other hand, sorting devices with equal values of \( S_u \) based on the tolerable interference in Line 10 results
in the solution that maximizes the tolerable interference over each sub-carrier. This allows mMTC devices to use a higher transmission power to achieve their data rates and enhances system capability of serving more mMTC devices. Before sub-carrier allocation, URLLC devices are sorted according to $S_u$ in a descending order to inherently allocate sub-carriers based on one of the combinations defined in (19) in case of multi-tone transmission as shown in Lines 11 to 13. Note that $I_s$ is assigned the value of $I_{u,S_u}$ of device $u$ that is allocated sub-carrier $s$, i.e., $k_{u,s} = 1$, for the minimum value of $S_u$ that makes $I_{u,S_u} \geq 0$ as shown in Lines 15 to 18.

### B. mMTC Sub-problem

In this sub-problem, the mMTC devices in the set $M$ are allocated sub-carriers subject to QoS and power constraints. The solution of the URLLC master problem is used to enforce an upper bound on the transmission power of mMTC devices on each sub-carrier if $S_M = S_U$ as given by

$$\sum_{m \in M} j_{m,s} |h_m|^2 p_m \leq I_s, \quad s = 1, \ldots, S_M.$$  \hspace{1cm} (23)

On the other hand, when $S_M > S_U$, the interference limit at one sub-carrier of URLLC devices is applicable to $S_M/S_U$ sub-carriers of mMTC devices. For example, if $S_M = 48$ and $S_U = 12$, $I = (I_1, \ldots, I_{12})$ is rewritten to be $I = (I_1, I_1, I_1, I_1, \ldots, I_1, I_1, I_1)$. The mMTC sub-problem can be expressed as

$$\text{maximize } ||z_M||_0$$

subject to constraints (4), (9), (11), (13), (20b) – (20e), (23),

constraint (15), if $B_M = 3.75$ kHz,

constraint (17), (18), if $B_M = 15$ kHz.

This sub-problem can be solved by using Algorithm 2. In Lines 5 to 8, we find the minimum power $p_{m,S_m}$ required to guarantee a minimum data rate $R_m$ if device $m$ is allocated $S_m$ sub-carriers, where

$$p_{m,S_m} = \left( \frac{R_m}{h_m} \right)^2 N_o B_M.$$  \hspace{1cm} (25)

Similar to the URLLC master problem, sorting is used to maximize the number of mMTC devices that can guarantee their required QoS level in Lines 9 to 12. During sub-carrier allocation process in Lines 13 to 21, the interference caused by mMTC devices cannot exceed a threshold as specified by (23) in Line 14 to maintain the obtained QoS of URLLC devices from the solution of the master problem.

### IV. PERFORMANCE EVALUATION

In this section, we compare the total number of successfully connected MTCDs, i.e., MTCDs that satisfy their QoS and power requirements per one PRB at one TTI using the proposed NOMA scheme with that of the OMA scheme. We also calculate the percentage increase in the number of successfully connected MTCDs due to using NOMA. Note that we maximize the number of MTCDs that can be successfully connected within one TTI instead of a long period of time.

However, the achievable performance gain at one TTI can reflect the gain over a longer period of time.

URLLC and mMTC devices are uniformly distributed within a 1 km² region. They are served by one base station that allocates one PRB to support MTC traffic. We consider flat Rayleigh fading channels since the total system bandwidth is as narrow as 180 kHz. The distance-dependent path loss $PL(D)$ at 900 MHz carrier frequency is calculated by [16]

$$PL(D) = 120.9 + 37.6 \log(D/1000) + L + AG,$$  \hspace{1cm} (26)

where $D$ is the distance between an MTCD and the base station in meters, $AG$ is the antenna gain of $-4$ dB, and $L$ is the indoor penetration loss that is assumed to be 20 dB for 80% of MTCDs (indoor MTCDs) and 0 dB for the remaining 20% MTCDs (outdoor MTCDs). We consider additive white Gaussian noise with power spectral density $-174$ dBm/Hz and noise figure of 5 dB. The maximum transmission power, $P_t$, $\forall d \in \mathcal{M} \cup \mathcal{U}$, is 23 dBm [16]. Both NOMA and OMA select the values of $B_M$ and $B_U$ from set $B$. The OMA sub-carrier allocation does not prioritize URLLC devices over mMTC devices in order to compare NOMA with OMA at its best connectivity performance, i.e., URLLC devices do not occupy all sub-carriers with their high QoS requirements.

Fig. 3 shows the number of successfully connected MTCDs versus the total number of MTCDs. The QoS requirements and the ratio $M/U = 2$ are fixed. We consider two cases that represent low and high QoS requirements. In both cases, the number of successfully connected devices in OMA saturates at the maximum number of sub-carriers of NB-IoT system, i.e., 48, as each sub-carrier can be allocated to at most one MTCD (a URLLC or an mMTC device). However, NOMA increases

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**Algorithm 2: Algorithm to solve mMTC sub-problem**

1. input: $B_M$, $S_M$, $h_m$, $P_m$, $R_m$, $\forall m \in M$, and $I$
2. $S_m \in \{1\}$ sub-carrier if $B_M = 3.75$ kHz, $S_m \in \{1, 3, 6, 12\}$ sub-carrier(s) if $B_M = 15$ kHz
3. Initialize $z_m := 0$, $\forall m \in M$
4. Initialize $j_{m,s} := 0$, $\forall m \in M, s = 1, \ldots, S_M$
5. Calculate $p_{m,S_m}$, $\forall m \in M$, using (25)
6. Select minimum $S_m$ such that $p_{m,S_m} \leq P_m$ if feasible
7. $\mathcal{M'} := \{m | \forall m \in M, p_{m,S_m} \leq P_m\}$
8. $p_m := 0$, $\forall m \in \mathcal{M} \setminus \mathcal{M'}$
9. $\mathcal{M'} \leftarrow$ Sort mMTC devices of $\mathcal{M'}$ in an ascending order of $S_m$
10. $\mathcal{M'} \leftarrow$ Sort mMTC devices of $\mathcal{M'}$ with equal $S_m$ in a descending order of $p_m$
11. $S := \{1, \ldots, S_M\}$
12. $m \leftarrow$ first element of $\mathcal{M'}$
13. while $S \neq \emptyset$
14. $S' := \{s | m \in S, \{s\} = S_m$, constraints (18) and (23) are satisfied$
15. \text{if } S' \neq \emptyset$
16. $z_m := 1, j_{m,s} \in S'$
17. \text{else}
18. $z_m := 0, p_m := 0$
19. $\mathcal{S} := S \setminus S', \mathcal{M'} := \mathcal{M'} \setminus \{m\}$, $m \leftarrow$ first element of $\mathcal{M'}$
20. $\mathcal{S} := S \setminus S'$
21. $S' := S \setminus S'$

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the number of successfully connected MTCDs per one PRB in one TTI by up to 100% in the low QoS requirement case (Fig. 3(a)) and 79% in the high QoS requirement case (Fig. 3(b)). The remaining non-connected MTCDs can be served in the following TTIs or using additional PRBs. To reflect the gains in practical numbers, if NB-IoT can support 52500 MTCDs sending small payload in area of 0.86 km² with OMA [16], it can generally support up to 105000 MTCDs with NOMA in the low QoS requirement case.

Fig. 4 illustrates the impact of the URLLC QoS requirement on the number of successfully connected MTCDs. The data rate requirements of URLLC devices are uniformly distributed between 0.1 kbps and \( R_{\text{max}} \), where \( R_{\text{max}} \) takes the values \( 2, 5, 10, 20, 50, 70, 100, 150, \) and 200 \( \text{kbps} \). 100 MTCDs are deployed in the region and we consider two cases of different \( M/U \) ratios, which are 50/50 = 1 (Fig. 4(a)) and 85/15 = 5.67 (Fig. 4(b)). In the former case, the proposed NOMA can support up to 100% more MTCDs than OMA. In the latter case, NOMA provides connectivity up to 31% more MTCDs than OMA. In both cases, OMA supports a constant number of MTCDs equivalent to the maximum number of sub-carriers of an NB-IoT system, which is 48, due to reaching the maximum connectivity capacity by allocating each sub-carrier to one MTCD. In addition, the gains decrease with \( R_{\text{max}} \). It becomes less probable to share sub-carriers among URLLC and mMTC devices since the QoS requirements of mMTC devices cannot be satisfied given the maximum tolerable interference of prioritized URLLC devices.

V. CONCLUSION

In this paper, we proposed using NOMA in an NB-IoT system in order to improve the connectivity of MTCDs. Each sub-carrier can be shared by a pair of MTCDs: a URLLC device and an mMTC device. We formulated a joint sub-carrier and transmission power allocation problem to maximize the number of MTCDs satisfying their QoS and power constraints. We proposed a two-step hierarchical algorithm to solve the problem. Simulation results show that NOMA enables a significant increase in connectivity compared to OMA. For future work, we will group more than two MTCDs to share a single sub-carrier along with prioritizing the URLLC devices and include power consumption minimization into the objective function.

REFERENCES