A Stackelberg Game for Cooperative Transmission and Random Access in Cognitive Radio Networks

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***Abstract*— In cognitive radio networks, the secondary users (SUs) can be selected as the cooperative relays to assist the transmission of the primary user (PU). In order to increase the utility, the PU needs to consider whether it is beneficial to use cooperative transmission and which SU should be chosen as the cooperative relay. In addition, if the PU selects a secondary relay, it needs to allocate time resources for cooperative transmission. Then, the SUs need to determine their strategies of random access when the licensed spectrum of the PU is available. In this paper, we first establish a model for cooperative cognitive radio networks with one PU and multiple SUs. We then propose a cooperative transmission and random access (CTRA) scheme. Based on the sequential structure of the decision-making, we study the cooperative cognitive radio network and determine the equilibrium strategies for both the PU and the SUs using the Stackelberg game. Simulation results show that both the PU and the SUs obtain higher utilities when compared with the noncooperative transmission and random access (NTRA) scheme.**

I. Introduction

Spectrum resources are scarce and the demand for the radio spectrum has been increasing in recent years. According to the report released by the Federal Communications Commission [1], fixed spectrum allocation may not always be efficient and the licensed spectrum may remain unoccupied for a long period of time. This motivates the concept of *cognitive radio* [2], which allows the secondary users (SUs) to dynamically access the licensed spectrum allocated to the primary users (PUs) when the licensed spectrum is not being utilized temporarily. Therefore, the efficiency of spectrum usage can be increased.

Motivated by the physical layer cooperative communication technique [3], *cooperative cognitive radio networks* are new cognitive radio paradigm. In [4], Jia *et al.* exploited a new research direction for cognitive radio networks by utilizing cooperative relay to assist the transmission and improve spectrum efficiency. In [5], the system enables cooperation between a primary cluster and a cognitive cluster in order to maintain the target primary throughput and provide more transmission opportunities to the SUs.

In [6], Simeone *et al.* proposed and analyzed a framework, where a PU can lease its spectrum to an ad hoc network of secondary nodes in exchange for cooperation in the form of distributed space-time coding. The framework is modeled as a Stackelberg game [7]. In [8], Zhang *et al.* proposed a cooperative cognitive radio framework. Both the PU and the SUs target at maximizing their utilities in terms of their transmission rate and revenue. This model is formulated as a Stackelberg game and a unique Nash equilibrium is characterized. In [9]. Yi *et al.* considered multiple PUs and SUs in the system model, and analyzed the optimal strategies on the relay selection and the price for spectrum leasing by a Stackelberg game. In [10], Kasbekar *et al.* formulated the price competition in a cognitive radio network as a game by taking into account both bandwidth uncertainty and spatial reuse. Niyato *et al.* studied the problem of spectrum trading with multiple PUs selling spectrum opportunities to multiple SUs in [11]. They modeled the dynamic behavior of the SUs using the theory of evolutionary game [12]. An algorithm of the evolution process implementation for the SUs was proposed.

In general, when the licensed spectrum is idle, all the SUs should have the opportunities to access the spectrum. The system models in [6], [8], [9] assumed the SUs access the licensed spectrum in a time division multiple access (TDMA) mode. Moreover, the system model in [10] assumed the PU who has unused bandwidth in a time slot can lease it to a SU for the duration of the whole slot. Therefore, the results of [6], [8]–[10] cannot be extended to the situation where the SUs can access the licensed spectrum in a random access manner.

In this paper, we consider both cooperative transmission and random access in cognitive radio networks. The problem is to find the optimal strategies for both the PU and the SUs in order to maximize their own utilities. The PU needs to determine whether it is beneficial to use cooperative transmission and which SU should be chosen as the cooperative relay. If the PU selects a secondary relay, it also needs to determine its strategy on time resource allocation for cooperative transmission. The SUs need to decide the transmission probability in random access. In this paper, we establish a model for cooperative cognitive radio networks and analyze the behaviour of the PU and the SUs using game theory.

The contributions of our work are summarized as follows:

- We propose a cooperative transmission and random access (CTRA) scheme for cognitive radio networks. The PU is responsible for relay selection and resource allocation. The SUs serve as relays and transmit their own data using random access.
- We propose a game-theoretic model to study the proposed cooperative cognitive radio system. We describe the strategies available for both the PU and the SUs. In order to maximize their own utilities, we analyze the

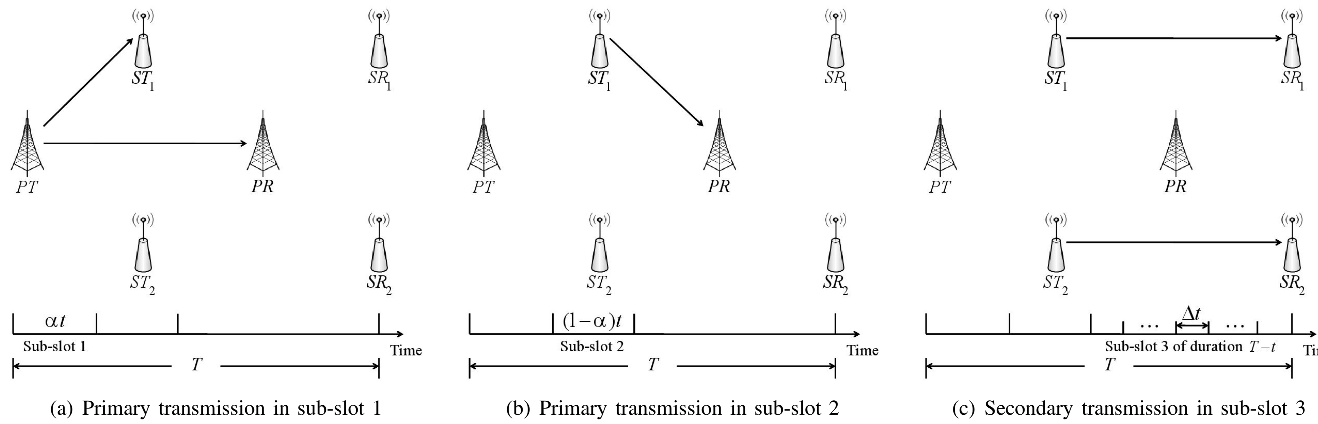

Fig. 1. An example of the proposed CTRA scheme with one primary and two secondary transmitter-receiver pairs. (a) The primary transmitter PT chooses secondary transmitter ST_1 as its cooperative relay and transmits the primary data to both the secondary relay ST_1 and the destination PR. (b) The chosen secondary relay ST_1 forwards the primary data to PR. (c) The SUs attempt to access the licensed frequency band of PT in a random access manner.

equilibrium strategies of both the PU and the SUs through a Stackelberg game.

- Simulation results show that both the PU and the SUs achieve better performance in our proposed CTRA scheme than in the noncooperative transmission and random access (NTRA) scheme. It is beneficial to exploit cooperative diversity in cognitive radio networks.

The rest of the paper is organized as follows. Section II describes the system model and the CTRA scheme. The Stackelberg game model and equilibrium analysis are presented in Section III. Section IV presents the performance evaluation results. Conclusions are given in Section V.

II. System Model

In our system model, there are one primary transmitter-receiver pair and N secondary transmitter-receiver pairs. The primary transmitter PT communicates with the intended receiver PR, and this primary transmitter-receiver pair is assigned a licensed frequency band with bandwidth B. Each secondary transmitter ST_j, where $j = 1, 2, \ldots, N$, seeks to exploit possible transmission opportunities in the licensed frequency band of the primary transmitter-receiver pair. We assume that each secondary transmitter ST_j always has data to send and it transmits data in a best-effort manner. In this paper, since the primary transmitter and secondary transmitters are responsible for data transmission, we consider the primary transmitter as the PU, and consider the secondary transmitters as the SUs.

The data transmission of PT is divided into time slots of duration T, and the CTRA scheme is performed in each time slot. As for the CTRA scheme, the primary transmitter PT either selects one secondary transmitter as its cooperative relay, or chooses not to have any cooperative relay. After that, all N SUs access the licensed spectrum in a random access manner. As an example, consider the network shown in Fig. 1, where there are one primary transmitter-receiver pair and two secondary transmitter-receiver pairs ($N = 2$) in the system. Primary transmitter PT selects secondary transmitter ST_1 as its cooperative relay. For the primary transmitter PT, one time slot can be divided into two portions, t and $T-t$ $(0 < t \leq T)$, where t is for the primary transmission and $T-t$ is for the secondary transmission. No matter PT selects secondary relay or not during the first portion, it will provide its spectrum to all the SUs during the second portion. As primary transmitter PT owns the spectrum and provides its licensed spectrum to all SUs during the second portion, PT loses the opportunity of transmitting its own primary data during the second portion $T-t$. Therefore, it is reasonable for the PU to *charge* the SUs for accessing its licensed spectrum.

If the primary transmitter PT chooses the secondary transmitter ST_j as its cooperative relay, the first portion t can further be divided into two sub-slots according to a parameter α, where $\alpha \in \{0.5, 1\}$. As shown in Fig. 1(a), the first sub-slot is of duration αt and is dedicated to the transmission from the primary transmitter PT to the secondary relay ST_1 and the destination PR. The second sub-slot is of duration $(1-\alpha)t$ and is dedicated to the transmission from the chosen secondary relay ST_1 to PR, which is shown in Fig. 1(b). If the primary transmitter PT does not select any SU as its cooperative relay, then $\alpha = 1$ and the duration of the sub-slot 1 is t, which is dedicated to the transmission from PT to PR.

We denote the second portion with duration $T-t$ as the third sub-slot of PT, which is dedicated to the secondary transmission. As shown in Fig. 1(c), we assume that all the SUs access the licensed spectrum during the third sub-slot by using slotted Aloha, and each time slot for the slotted Aloha is of duration Δt. For simplicity, we assume that T is a multiple of Δt.

The channels between different users are modeled as independent complex Gaussian random variables, invariant within each time slot, but varying over time slots (block-fading channels). For convenience, we define the ratio of the transmitting power to the noise power as the signal-to-noise ratio (SNR) of the transmitter. Given the transmit SNR Γ and the complex channel gain h of the link, the transmission rate R over that link can be modeled as [6]:

$$R = B \log_2 \left(1 + |h|^2 \Gamma\right). \tag{1}$$

We consider both path loss and channel fading in the system model. Given the distance d between any two nodes in our model, the channel gain between these two nodes can be given as $|h|^2 = \delta^2/d^\eta$, where η is the path loss exponent and δ is a Rayleigh distributed random variable. The probability density function of δ ($\delta \geq 0$) for parameter $\sigma > 0$ is

$$p(\delta) = \frac{\delta}{\sigma^2} \exp\left(-\frac{\delta^2}{2\sigma^2}\right). \quad (2)$$

The following notations are used to denote the complex channel gain of different links in each block: h_P denotes the channel gain between the primary transmitter PT and the primary receiver PR; h_{PS_j} denotes the channel gain between the primary transmitter PT and the secondary transmitter ST_j; h_{S_jP} denotes the channel gain between the secondary transmitter ST_j and the primary receiver PR; h_{S_j} denotes the channel gain between the secondary transmitter ST_j and the secondary receiver SR_j. Moreover, we assume that there is no power control. That is, both the primary transmitter and the secondary transmitters are transmitting data at a fixed power level. We use Γ_P to denote the transmit SNR at the primary transmitter PT and use Γ_S to denote the transmit SNR at the secondary transmitter ST_j, where $j = 1, 2, \ldots, N$.

Given the above notations, we can model the transmission rate of any link similar to (1). For example, for the transmission rate of the primary link between PT and PR, denoted as R_P, it is given by

$$R_P = B \log_2\left(1 + |h_P|^2 \Gamma_P\right). \quad (3)$$

We use the following notations to denote the transmission rate of other links: R_{PS_j} denotes the transmission rate of the link between PT and ST_j; R_{S_jP} denotes the transmission rate of the link between ST_j and PR; R_{S_j} denotes the transmission rate of secondary link between ST_j and SR_j.

We assume that the secondary relay node uses decode-and-forward as its cooperation strategy [13], [14]. According to the analysis in [15], the achievable cooperative data rate of the primary transmitter PT under cooperative transmission with secondary relay ST_j, where $j = 1, 2, \ldots, N$, is given by

$$\begin{aligned} R_{coop,j} = \frac{1}{2} \min\{ & B \log_2\left(1 + |h_{PS_j}|^2 \Gamma_P\right), \\ & B \log_2\left(1 + |h_P|^2 \Gamma_P + |h_{S_jP}|^2 \Gamma_S\right)\}. \end{aligned} \quad (4)$$

In (4), the first term represents the maximum rate at which the secondary relay ST_j can reliably decode the source message from the primary transmitter PT. The second term represents the maximum rate at which the destination PR can reliably decode the source message given the repeated transmission from the source PT and the secondary relay ST_j. Requiring both the secondary relay ST_j and the destination PR to decode the entire codeword without error results in the minimum of the two data rates in (4).

III. Stackelberg Game Analysis

According to the system model presented in Section II, there is a sequential structure of decision-making. First, PT determines whether it is beneficial to use cooperative transmission and which secondary transmitter should be chosen as the cooperative relay. PT also determines its optimal value t^* and the corresponding value of α. Then, all the SUs determine how to access the available licensed spectrum for the secondary transmission given the decisions made by the PU. Therefore, this cooperative cognitive radio network can be analyzed by a Stackelberg game. Suppose all the SUs are fully informed about the decisions made by the PU, our Stackelberg game has perfect information, and the decisions of the PU can be transmitted to the SUs through the control channel.

A. Stackelberg Game with Perfect Information

The set of players in this Stackelberg game are the PU and N SUs. The primary transmitter PT, which owns the licensed spectrum, plays the role of the leader. Each ST_j, where $j = 1, 2, \ldots, N$, which seeks to exploit possible transmission opportunities in the licensed frequency band of PT, plays the role of the follower.

There are two kinds of strategies available to the PU. First, PT may have $N+1$ pure strategies for the relay selection and we use $r \in \{0, 1, 2, \ldots, N\}$ to denote which SU is chosen by PT as cooperative relay. If $r = 0$, it means that PT chooses no cooperative relay and therefore $\alpha = 1$. Otherwise, PT chooses ST_r as its cooperative relay and $\alpha = 0.5$. For the second strategy, PT determines how to allocate time resources and chooses the value of t in the interval $(0, T]$.

After PT chooses its strategies r, t and α, all the SUs choose their strategies simultaneously. The strategies available to these SUs are denoted by a vector $\boldsymbol{p} = (p_1, p_2, \ldots, p_N)$. The value of element p_i, where $i = 1, 2, \ldots, N$, is chosen in the interval $[0, 1]$ and it corresponds to the transmission probability of ST_i during the third sub-slot.

In order to find the Nash equilibrium of this Stackelberg game, *backward induction* is used. We start by finding the equilibrium strategies of N SUs given the PU's decisions on its strategies r, t and α. Then, taking these actions as given, we find the optimal strategies of the PU who makes the decisions before all the SUs.

B. Strategies for Secondary Users

Given the time allocation t determined by the PU, we can calculate the number of slots for the slotted Aloha as

$$n = \left\lfloor \frac{T - t}{\Delta t} \right\rfloor. \quad (5)$$

In each slot, each secondary transmitter ST_i, where $i = 1, 2, \ldots, N$, determines its own transmission probability p_i. ST_i may transmit data (i.e., $0 < p_i \leq 1$) or not (i.e., $p_i = 0$). If ST_i transmits data in one slot, the transmission may either be a success or a failure due to packet collisions with other secondary transmissions. Besides, ST_i should pay for accessing the licensed spectrum with probability p_i. ST_i wants to achieve better performance in terms of transmitting more data and paying less money. Therefore, the utility function of secondary transmitter ST_i is defined as

$$U_{S_i}(\boldsymbol{p}, r) = n\big(P_{suc,i}(\boldsymbol{p}) U_{d_i} - p_i U_{c_i}(p_i, r)\big), \quad (6)$$

where U_{d_i} denotes the successful data transmission utility of ST_i in each slot, $U_{c_i}(p_i, r)$ denotes the licensed spectrum accessing payment from ST_i to the primary transmitter PT in each slot, and $P_{suc,i}(\boldsymbol{p})$ is the successful transmission probability of ST_i. The utility function in (6) means that, in each slot of slotted Aloha, ST_i can get the successful data transmission utility U_{d_i} with probability $P_{suc,i}(\boldsymbol{p})$ and has to make the payment $U_{c_i}(p_i, r)$ with probability p_i. The successful data transmission utility U_{d_i} of ST_i is defined as

$$U_{d_i} = \omega_S R_{S_i} \Delta t, \tag{7}$$

where ω_S is the equivalent utility per unit secondary data transmission, and R_{S_i} is the secondary data rate of ST_i. The successful transmission probability $P_{suc,i}(\boldsymbol{p})$ of ST_i is given by

$$P_{suc,i}(\boldsymbol{p}) = p_i \prod_{j \neq i} (1 - p_j). \tag{8}$$

If the secondary transmitter ST_i attempts to access the licensed channel of PT with probability p_i, it should make a payment to the primary transmitter PT with probability p_i. We define the accessing payment $U_{c_i}(p_i, r)$ as

$$U_{c_i}(p_i, r) = c_i(p_i, r)\Delta t = c_{max,i}(r) p_i \Delta t, \tag{9}$$

where $c_i(p_i, r)$ is the secondary accessing price for the ST_i per unit access time. We choose a linear pricing function [16] so that the accessing price $c_i(p_i, r)$ increases with the transmission probability p_i. It means that ST_i should pay more if it attempts to access the licensed spectrum aggressively. Moreover, we define the maximum accessing price $c_{max,i}(r)$ as a function of the secondary relay selection r, which is given by

$$c_{max,i}(r) = \begin{cases} c_{coop}, & r = i, \\ c_{non}, & r \neq i, \end{cases} \tag{10}$$

where c_{coop} stands for the maximum accessing price when ST_i is selected as the cooperative relay by the PU (i.e., $r = i$), and c_{non} stands for the maximum accessing price when ST_i is not selected as the cooperative relay (i.e., $r \neq i$). Since the secondary relay consumes energy to forward the primary data during sub-slot 2, it is reasonable for the chosen secondary relay ST_r to pay less than the other SUs. Therefore, we have $c_{non} > c_{coop} > 0$ in order to differentiate the secondary accessing price.

By substituting (7)$-$(9) into (6), the utility of secondary user ST_i is given by

$$U_{S_i}(\boldsymbol{p}, r) = n\Delta t \left(\omega_S R_{S_i} p_i \prod_{j \neq i} (1 - p_j) - c_{max,i}(r) p_i^2 \right). \tag{11}$$

Theorem 1: There exists a Nash equilibrium in the secondary users' game.

Proof: According to [17], a Nash equilibrium exists in the secondary users' game if two conditions are satisfied. First, the strategy set is nonempty, convex and compact. Second, $U_{S_i}(\boldsymbol{p}, r)$ is continuous in $\boldsymbol{p}$ and concave in p_i. The first condition is satisfied because $0 \leq p_i \leq 1$, for all $i = 1, 2, \ldots, N$. For the second condition, since $U_{S_i}(\boldsymbol{p}, r)$ is continuous in $\boldsymbol{p}$, we only need to prove $U_{S_i}(\boldsymbol{p}, r)$ is concave in p_i. We take the first and second order derivative of $U_{S_i}(\boldsymbol{p}, r)$ with respect to p_i to show its concavity. The derivatives are given as follows:

$$\frac{\partial U_{S_i}(\boldsymbol{p}, r)}{\partial p_i} = n\Delta t \left(\omega_S R_{S_i} \prod_{j \neq i} (1 - p_j) - 2c_{max,i}(r) p_i \right). \tag{12}$$

$$\frac{\partial^2 U_{S_i}(\boldsymbol{p}, r)}{\partial {p_i}^2} = -2n\Delta t c_{max,i}(r). \tag{13}$$

In (13), we have $n > 0$ because there is no secondary users' game if $n = 0$. Besides, we notice that Δt and $c_{max,i}(r)$ are positive. The second order derivative in (13) is always negative, which means $U_{S_i}(\boldsymbol{p}, r)$ is concave in p_i. Thus, there exists at least one Nash equilibrium in the secondary users' game. ■

In order to find the Nash equilibrium, an evolutionary algorithm [18] can be used. By using the evolutionary algorithm as a tool to determine the Nash equilibrium, we can find the equilibrium strategy $\boldsymbol{p^*}$ for the SUs.

C. Strategies for Primary User

Based on the results of the secondary users' game, the leader of the Stackelberg game (i.e., the PU), can determine its strategies (r, t, α) in order to maximize its own utility. The utility of the PU is composed of two parts. The first part is the equivalent revenue of primary data transmission, and the second part is the payment made by the SUs for accessing the licensed spectrum. Therefore, we define the utility function of PT to be

$$U_P(\boldsymbol{p}, r, t) = \omega_P R(r) t + \left\lfloor \frac{T - t}{\Delta t} \right\rfloor \sum_{i=1}^{N} p_i U_{c_i}(p_i, r), \tag{14}$$

where ω_P is the equivalent revenue per unit primary data transmission utility. $R(r)$ is the primary transmission rate, which is defined as

$$R(r) = \begin{cases} R_P, & r = 0, \\ R_{coop,j}, & r = j \text{ where } j = 1, 2, \ldots, N. \end{cases} \tag{15}$$

In order to maximize U_P, the PU first chooses the value of r, and find the optimal value of t according to (14). For each value of r, the one that maximizes PU's utility function is chosen as the cooperative relay, and the corresponding values of α and t^* can then be determined.

Proposition 1: For each possible value of r, the corresponding value of t^* is a multiple of Δt, and the optimal value t^* can only be chosen from a finite set $\{\Delta t, 2\Delta t, \ldots, T\}$.

Proof: Suppose t is not a multiple of Δt. From (5), we can calculate the remaining time in each time slot T as

$$t_{rem} = T - t - n\Delta t. \tag{16}$$

Since

$$0 < t_{rem} < \Delta t, \tag{17}$$

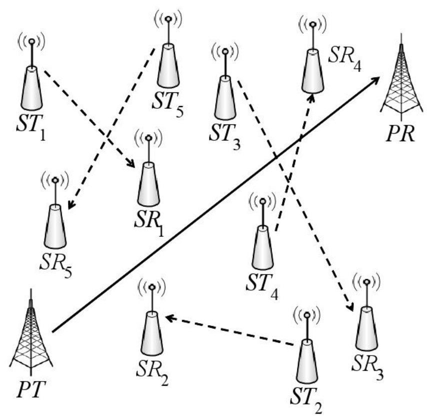

Fig. 2. Simulation model with one PU and five SUs.

it implies that the remaining time in each time slot T is greater than zero and less than the duration of slot Δt. If t is not a multiple of Δt, then the remaining time t_{rem} in each time slot T is wasted. From (14), the equivalent revenue of primary data transmission is an increasing function of t. If the remaining time t_{rem} is used for primary data transmission, the utility of PT will be increased. From (16), $t + r_{rem}$ is a multiple of Δt. From the above, the optimal value t^* must be a multiple of Δt. Since we assume T is a multiple of Δt, t^* can only be chosen from the finite set $\{\Delta t, 2\Delta t, \ldots, T\}$. ■

Based on the results of the secondary users' game, we can find the optimal strategies for the PU according to (14) and Proposition 1. The primary transmitter PT first chooses the value of r, and for each value of r, the corresponding value of t^* which maximizes U_P in (14) can only be chosen from a finite set $\{\Delta t, 2\Delta t, \ldots, T\}$. Once the optimal strategies r^* and t^* have been found, the corresponding value of α can be determined by the cooperative transmission scheme, which is

$$\alpha = \begin{cases} 1, & r^* = 0, \\ 0.5, & r^* = j \text{ where } j = 1, 2, \ldots, N. \end{cases} \tag{18}$$

From the above analysis, we can determine the equilibrium strategies for both the PU and the SUs. The equilibrium strategy for the PU is (r^*, t^*, α), and the equilibrium strategies for the N SUs are given by the vector $\boldsymbol{p^*}$.

IV. Performance Evaluation

In this section, we evaluate the performance of our proposed CTRA scheme with the NTRA scheme, where the PU will not select any SU as its cooperative relay. In the simulation model, we consider there are one primary transmitter-receiver pair and N ($N > 1$) secondary transmitter-receiver pairs. The distance between the primary transmitter PT and the primary receiver PR is equal to 10 m. The positions of the SUs are randomly placed in a 10 m×10 m square region. An example of simulation model with $N = 5$ is given in Fig. 2. For the path loss and channel fading, we choose $\eta = 2$ and $\sigma = 0.15$.

We assume that both the primary transmitter and secondary transmitters transmit at a fixed power level without power control, and we choose $P = 10$ mW for both the primary and secondary transmitters. The noise power is $N_0 = 0.1$ mW,

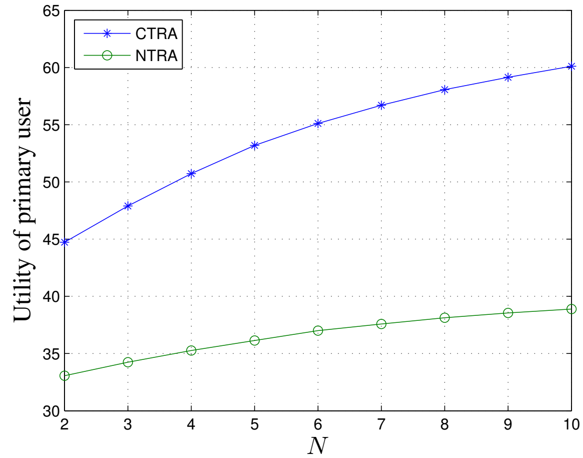

Fig. 3. The utility of the PU versus the number of SUs N ($\omega_P = 1$, $\omega_S = 0.1$, $c_{non} = 18$).

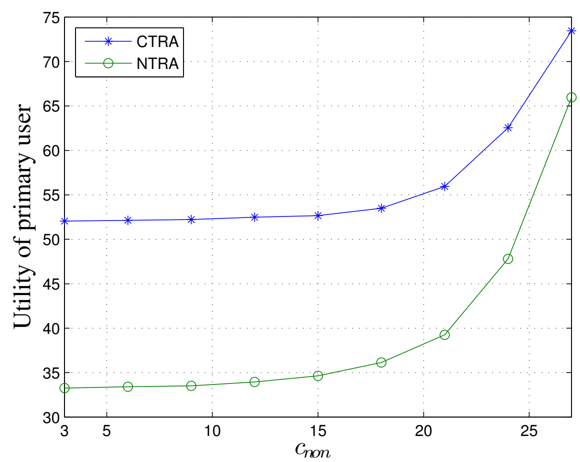

Fig. 4. The utility of the PU versus the maximum accessing price c_{non} ($\omega_P = 1$, $\omega_S = 0.1$, $N = 5$).

therefore, the transmit SNRs of both the primary transmitter and the secondary transmitters are $\Gamma_P = \Gamma_S = P/N_0 = 100$. Other parameters used in our simulation are as follows: the bandwidth of the primary licensed frequency band is $B = 500$ kHz; the length of the whole time slot $T = 1$ s; the duration of each slot for random access $\Delta t = 0.1$ s; the maximum accessing price for cooperative relay $c_{coop} = 1$. The simulation results are averaged over 10000 simulation runs.

Fig. 3 shows the utility of the PU versus the number of SUs N when the equivalent revenue per unit primary data transmission utility $\omega_P = 1$, the equivalent revenue per unit secondary data transmission utility $\omega_S = 0.1$ and the maximum accessing price for noncooperative relay $c_{non} = 18$. Since cooperative diversity is exploited in CTRA, we can see that the utility of the PU under CTRA is higher than that under NTRA. When the number of SUs N increases, the PU has more options for cooperative relay selection and there is a better chance for the PU to select a secondary relay which gives a higher cooperative transmission rate. Therefore, the utility of the PU increases with N under the CTRA scheme. As for the NTRA scheme, the utility of the PU increases with

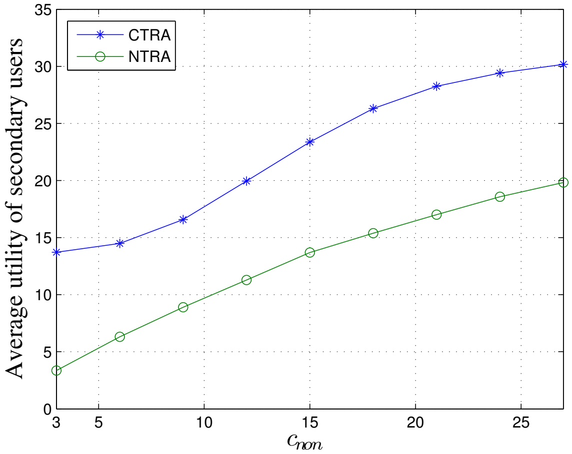

Fig. 5. The average utility of the SUs versus the maximum accessing price c_{non} ($\omega_P = 1$, $\omega_S = 0.8$, $N = 5$).

N moderately because cooperative transmission is not used in the NTRA scheme.

Fig. 4 shows the utility of the PU versus the maximum accessing price for noncooperative relay c_{non} when $\omega_P = 1$, $\omega_S = 0.1$ and $N = 5$. We can see that the utility of the PU under CTRA is higher than that under NTRA, because cooperative transmission is used and cooperative diversity is exploited in CTRA. The utility of the PU increases with c_{non} under both the CTRA and NTRA schemes. When the maximum accessing price c_{non} is low, the equivalent revenue of primary data transmission is much greater than the payment from secondary random accessing, therefore, the utility of PU does not increase too much. When the maximum accessing price c_{non} increases, the payment from the secondary random accessing increases and a majority of PU's utility comes from the payment of random accessing, therefore, t^* decreases and U_P increases with c_{non} significantly.

Fig. 5 shows the average utility of the SUs versus the maximum accessing price c_{non} when $\omega_P = 1$, $\omega_S = 0.8$ and $N = 5$. We can see that the average utility of the SUs under CTRA is larger than that under NTRA. Also, the average utility of the SUs increases with c_{non} under both the CTRA scheme and NTRA scheme. As the maximum accessing price c_{non} increases, the PU decreases the value of t^* in order to obtain more revenue from secondary random accessing. Therefore, the number of slots for the slotted Aloha will increase and the average utility of SUs will increase as well. In summary, Figs. 3$-$5 show that both the PU and the SUs can obtain higher utilities under CTRA scheme, and they have the incentive to engage in cooperative transmission.

V. Conclusions

In this paper, we investigated the behaviour of the PU and the SUs under our proposed CTRA scheme in a cooperative cognitive radio network. According to the sequential structure of decision making, we analyzed the established cooperative cognitive radio network using a Stackelberg game with perfect information. Given the decisions made by the PU and the utility function we defined for the SUs, we showed that there exists a Nash equilibrium in the secondary users' game. We determined the optimal strategies of the PU by using backward induction. Simulation results showed that both the utility of the PU and the average utility of the SUs under our proposed CTRA scheme are higher than those under the NTRA scheme. Thus, a win-win situation can be achieved by exploiting cooperative diversity in our proposed CTRA scheme. For future work, we will apply game theory to analyze cooperative cognitive radio networks with multiple PUs and multiple SUs under the proposed CTRA scheme.

References

[1] "Spectrum policy task force," Federal Communications Commission, Tech. Rep., 2002.

[2] J. Mitola III and G. Q. Maguire Jr., "Cognitive radio: Making software radios more personal," *IEEE Personal Communications*, vol. 6, no. 4, pp. 13–18, Aug. 1999.

[3] M. O. Hasna and M.-S. Alouini, "End-to-end performance of transmission systems with relays over Rayleigh-fading channels," *IEEE Trans. on Wireless Communications*, vol. 2, no. 6, pp. 1126–1131, Nov. 2003.

[4] J. Jia, J. Zhang, and Q. Zhang, "Cooperative relay for cognitive radio networks," in *Proc. of IEEE INFOCOM*, Rio de Janeiro, Brazil, April 2009.

[5] I. Krikidis, J. N. Laneman, J. S. Thompson, and S. McLaughlin, "Protocol design and throughput analysis for multi-user cognitive cooperative systems," *IEEE Trans. on Wireless Communications*, vol. 8, no. 9, pp. 4740–4751, Sept. 2009.

[6] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary ad hoc networks," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 203–213, Jan. 2008.

[7] H. von Stackelberg, *Market Structure and Equilibrium*. Springer, 2011.

[8] J. Zhang and Q. Zhang, "Stackelberg game for utility-based cooperative cognitive radio networks," in *Proc. of ACM MobiHoc*, New Orleans, LA, May 2009.

[9] Y. Yi, J. Zhang, Q. Zhang, T. Jiang, and J. Zhang, "Cooperative communication-aware spectrum leasing in cognitive radio networks," in *Proc. of IEEE Symposium on New Frontiers in Dynamic Spectrum (DySPAN)*, Singapore, April 2010.

[10] G. S. Kasbekar and S. Sarkar, "Spectrum pricing games with bandwidth uncertainty and spatial reuse in cognitive radio networks," in *Proc. of ACM MobiHoc*, Chicago, IL, Sept. 2010.

[11] D. Niyato, E. Hossain, and Z. Han, "Dynamics of multiple-seller and multiple-buyer spectrum trading in cognitive radio networks: A game-theoretic modeling approach," *IEEE Trans. on Mobile Computing*, vol. 8, no. 8, pp. 1009–1022, Aug. 2009.

[12] M. J. Osborne, *An Introduction to Game Theory*. Oxford University Press, 2003.

[13] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. on Information Theory*, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.

[14] A. Host-Madsen and J. Zhang, "Capacity bounds and power allocation for wireless relay channels," *IEEE Trans. on Information Theory*, vol. 51, no. 6, pp. 2020–2040, June 2005.

[15] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. on Information Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[16] Y. Tan, S. Sengupta, and K. P. Subbalakshmi, "Competitive spectrum trading in dynamic spectrum access markets: A price war," in *Proc. of IEEE Globecom*, Miami, FL, Dec. 2010.

[17] G. Debreu, "A social equilibrium existence theorem," *Proc. of the National Academy of Sciences (PNAS)*, vol. 38, no. 10, pp. 886–893, Oct. 1952.

[18] R. Rajabioun, E. Atashpaz-Gargari, and C. Lucas, "Colonial competitive algorithm as a tool for Nash equilibrium point achievement," in *Proc. of Int'l Conf. on Computational Science and Its Applications (ICCSA)*, Perugia, Italy, June 2008.