Interference Diversity Gain and its Application in Multi-Channel Systems: Capacity Maximization and QoS Guarantee Strategies

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Abstract—As the spectral efficiency of wireless communications systems increases, and the frequency reuse pattern shifts towards universal frequency reuse, the capacity of future wireless and mobile systems will becomes interference limited. We explore the interference pattern of a shared channel in order to establish optimal distributed resource allocation techniques in that channel considering the mutual interference of coexisting links on each other. It is shown that the optimal usage of resources in such a shared channel can only be achieved via a multi-dimensional resource allocation strategy, taking into account not only the link’s own channel quality, but also its channel states towards coexisting links. The improvement of capacity as a result of this approach can be attributed to the interference diversity of the channel. The interference diversity gain can be harnessed in time, frequency and space domains. To this end we study two approaches, namely maximizing the capacity of the primary and secondary links under received interference constraint and minimizing the transmitted power of primary and secondary links under minimum QoS guarantee constraint. We study the Ergodic capacity to verify the significant performance improvement which can be achieved by exploiting interference diversity in multi-channel systems such as UMTS Long Term Evolution (LTE). Further we will show that utilizing this diversity gain in QoS-guaranteed scenarios results in a considerable transmission power saving. The case of Outage capacity, which is an instantaneous measure of channel throughput, is shown to be different whereby using an instantaneous received interference threshold outperforms the usage of average received interference limit.

Index Terms—Interference diversity, spectrum sharing, ergodic capacity, outage capacity, UMTS LTE.

I. INTRODUCTION

The continuous innovations in wireless communications has pushed the spectral efficiency of modern communications systems towards their theoretical boundaries. Furthermore, the required level of SINR for successful operation of such advanced systems has been reduced considerably, thereby paving the way for more efficient re-use of spectrum in cellular and broadband wireless systems. In fact many Beyond 3G (B3G) technologies such as IEEE 802.16 family (a.k.a. WiMAX) and 3GPP’s UMTS Long Term Evolution (LTE) are reaching universal frequency reuse patterns [1]. These advancements, in turn, mean that the capacity of next generation wireless systems will mainly be interference limited, which mandates intelligent interference management techniques.

The significance of interference management becomes ever more pronounced as various spectrum sharing techniques are envisioned to further increase spectrum utilization. Horizontal spectrum sharing is achieved when the coexisting systems have the same access right to the shared band, for instance as manifested in unlicensed bands [2]. However, horizontal spectrum sharing strategies are also gaining attention for licensed band operation, an example of which is the coexistence of femto-cell (called Home NodeB in 3GPP standards) and macro-cell Base Stations (BS) in the UMTS LTE context [3]. Vertical spectrum sharing, on the other hand, requires a hierarchy of access rights whereby secondary users can only access the shared band in compliance with the primary users’ QoS requirements. Such Secondary Spectrum Access (SSA) solutions require exploiting Cognitive Radios (CR) in order to reliably identify and intelligently access the secondary spectrum opportunities [5].

As demonstrated through the above examples, the issue of interference mitigation is of utmost importance in future communications systems. Hence, the goal of this paper is to study the interference pattern of a multiple-channel shared spectrum scenario, aiming to develop efficient coexistence strategies by exploiting the so-called interference diversity of the shared channel. While for the sake of consistency of the terminology we will refer to our channel set up in this paper using primary and secondary link titles (as if a vertical spectrum sharing regime is in place), note that horizontal spectrum sharing scenarios are readily accommodated in our analysis.

As a general framework for our analysis, let us use the Interference Channel (IC) set up of Fig. 1. There is, however, a major difference in the scenarios we will investigate in this paper and the classic IC scenarios, such as in [6] and [7] or with the existing (horizontal) unlicensed spectrum sharing approaches such as in [8]. In all the aforementioned references coexisting links pursue a selfish, and hence greedy, resource allocation strategy. Inspired by the spectrum sharing examples presented in the previous paragraphs, we study cases of “limited selfishness”, whereby each link behaves selfishly to

1The coexistence of femto-cell and macro-cell base-stations in a UMTS LTE network can also be achieved via vertical spectrum sharing, cf. [4].
maximize its utility of interest, bounded by constraints not only as dictated by its own limits but also constraints as regards its effect on the other coexisting links. In other words, the factor that limits the selfish behavior of a link in this case is not only its physical limitations (e.g., a link’s maximum transmit power capability), but also the state of coexisting links. This approach requires a multi-dimensional resource allocation strategy as opposed to traditional IC scenarios. Transmitter-receiver pair $Tx_i - Rx_i$, interchangeably referred to as link $i$ in the rest of paper, needs to take into account the quality of two sets of channels, i.e., its direct sub-channels denoted by the channel gain vector $g_i = (g_{i,1}, g_{i,2}, \ldots, g_{i,N})$ and the set of cross-channels with channel gain vector $h_i = (h_{i,1}, h_{i,2}, \ldots, h_{i,N})$. We will limit the scope of this study to two-dimensional analysis, as in Fig. 1. The coexisting links $i, j$, where $i \neq j$, are assumed to operate independently and hence we will not discuss cooperative communications strategies. We aim to develop optimal resource allocation techniques to achieve Ergodic and Outage capacity of each link in a distributed manner\(^2\). We also address the case of guaranteeing the QoS of coexisting links in the sequel.

The rest of the paper is organized as follows. In Section II a brief background of research efforts related to our problem is presented. Then, Section III elaborates on the case of achieving the Ergodic capacity of coexisting links with average and instantaneous received power constraint. Similarly, the case of Outage capacity is discussed in Section IV. We will study the QoS guarantee strategies in Section V, followed by the numerical results in Section VI. Finally, Section VII concludes the paper.

II. BACKGROUND REVIEW

The concept of Z-Interference Channel (ZIC) is proposed and studied in several papers, including [9]. The Z-Interference channel models the scenarios where out of the two coexisting links only one will impose interference to the other. Hence, a “unilateral” interference from only one link to the other is investigated in ZIC. Another interesting information theoretic setup is that of the MIMO X channel, which is a generalized form containing both IC and ZIC structures [10]. The degrees of freedom and capacity bounds for this channel model is studied in the literature recently, however, optimal resource allocation strategies, which is the target of our analysis, is not vigorously studied yet.

Recently, the FCC proposed usage of a receiver-centric metric called the Interference Temperature (IT) [11]. Though this metric were later abandoned by the FCC, due to lack of “specific technical rules” [12], the concept is still very much alive and has inspired numerous studies in the literature. The first rigorous analysis of the capacity limits under received power constraint at a third party receiver was presented in [13]. In this analysis the (Ergodic) capacity of a link under the constraint of limited interference at a coexisting link was studied for the case of AWGN channels. Next, [14] extended this analysis to the case of (flat) fading channels, arguing that without fading the capacity of a link with the constraint of received interference at other link(s) is essentially a deterministically scaled version of the problem of finding the capacity of a link under maximum transmit power constraint. The (Ergodic) capacity of the secondary link in several fading channel settings, such as Rayleigh and Nakagami-m fading have been investigated in [14]. Comparing the results with the case of AWGN channels, it was clearly demonstrated that exploiting the fading states of the channel results in a considerable capacity gain for the secondary link.

Both [13] and [14] assume that an average received interference limit is in effect towards the non-target links. The authors in [15] have analyzed a scenario where both average and instantaneous received interference limits at a third party receiver are present. Furthermore, this paper has studied both the Ergodic and Outage capacity of the secondary link under such received power constraints at a third party receiver. The presented results in [15] shows that imposing the instantaneous received power limit does not considerably affect the Ergodic capacity of the secondary link as long as the average received power constraint is present. However, the Outage capacity will degrade as a result of limiting the instantaneous received power at the primary receiver. Optimal power allocation technique to achieve delay-limited and Outage capacity of a secondary link under interference limit at the primary link is also studied in [16]. For the case of delay-limited capacity only average received interference limit is considered whereas for the Outage capacity both average and instantaneous interference limits are investigated. Also the Ergodic capacity of multiple access and broadcast CR channels is investigated in [17].

The focus of studies in [13]–[15] has been the capacity of

\[ \text{Fig. 1. The interference channel set up.} \]
the secondary link. Recently, [18] has addressed the Ergodic and Outage capacity of the primary link under average or instantaneous received interference constraint (but not both, simultaneously). Intuitively, one expects that instantaneous received interference limit will better protect the primary receiver, and hence, results in a higher capacity at the primary link. This intuition is based on the fact that instantaneous receiver, and hence, results in a higher capacity at the primary link. However, as [18] notes, the reverse is indeed true. In other words, imposing a more relaxed average received interference limit results in a higher achieved capacity for the primary (and also as was known before for the secondary) link. This phenomenon is attributed to the so-called “interference diversity” of the channel [18]. The achieved gain associated with the interference diversity, as studied in [18], is only harnessed in time domain. Considering the Ergodic capacity of the links, it is well known that Ergodic capacity is the maximum long-term average rate that can be sustained in a fading channel. On the other hand, the Outage capacity is the maximum constant rate sustainable over a certain percentage of time. Though this metric is more suitable for delay-sensitive applications, nevertheless the Outage probability embodies the percentage of time that the constant rate in the channel can be maintained. In this paper we propose a generalized capacity-maximizing and QoS-guarantee resource allocation strategies that utilize the interference diversity in any given domain with an orthogonal MAC design. A comparison of assumptions pertinent to the proposed problem setting and those of [13]–[18] are summarized in Table I.

III. STRATEGY 1: MAXIMIZING THE ERGODIC CAPACITY

We will introduce two approaches to capture the interference diversity and discuss its application in multi-channel scenarios, such as OFDM-based systems, in the following. We assume each transmitter paired with its respective receiver constitutes a link, and each link is comprised of $N$ sub-channels. We will use subscript $n$ to indicate the sub-channel index; for instance we use $g_{i,n}$ to represent the direct channel gain from transmitter $i$ to receiver $i$ in sub-channel $n$ (cf. Fig. 1).

A. Instantaneous Received Interference Limit

Considering $N$ sub-channels of a shared band, each having a bandwidth of $w$ Hz, the instantaneous throughput of link $i$ in Fig. 1 is defined as

$$R_i = \sum_{n=1}^{N} R_{i,n} = \sum_{n=1}^{N} w \log \left( 1 + \frac{p_{i,n} (g_{i,n}, h_{i,n})}{\sigma_i^2 + p_{j,n} (g_{j,n}, h_{j,n})} \right),$$

where $g_{i,n}$ and $h_{i,n}$ are the i.i.d random values of the direct and cross channel gains for link $i$ in sub-channel $n$, respectively, $\forall i, j \in \{1, 2\}$ and $i \neq j$. In this paper we assume sub-channels experience i.i.d Rayleigh fading with unit mean which means $g_{i,n}$ and $h_{i,n}$ will have independent Exponential distributions with unit mean. Note that the choice of any other fading statistics (for instance Rician or Nakagami-$m$) is also supported in the proposed framework. We assume AWGN noise has a power spectral density (psd) of $\mathcal{N}_0, i$ at receiver $i$, and hence $\sigma_i^2 = wN_0, i$. Then, the Ergodic capacity of this link can be defined by (where $\mathbb{E}_x(.)$ denotes statistical expectation with respect to random variable $x$)

$$C_{E_{R,i}} = \max_{g_{i,n}, h_{i,n}} \mathbb{E}_{g_{i,n}, h_{i,n}} \left\{ \sum_{n=1}^{N} R_{i,n} \right\},$$

subject to

$$\sum_{n=1}^{N} p_{i,n} (g_{i,n}, h_{i,n}) h_{i,n} \leq \Gamma_{\text{inst},i},$$

and

$$\mathbb{E}_{g_{i,n}, h_{i,n}} \left\{ \sum_{n=1}^{N} p_{i,n} (g_{i,n}, h_{i,n}) \right\} \leq P_{\text{max},i},$$

where $P_{\text{max},i}$ is the average transmit power limit of transmitter $i \in \{1, 2\}$ and $\Gamma_{\text{inst},i}$ is the instantaneous total interference limits for link $i$. The interference limit constraint defined in (3) is imposed on transmitter $i$ in order to limit the instantaneous received interference at receiver $j$, where $i \neq j$. In the next section we address the case of average received interference limit, too. An important feature of the proposed framework in (2)–(4) is its flexibility to accommodate various spectrum sharing problems. By assuming a fixed, bounded value for the interference threshold at both links, scenarios of horizontal spectrum sharing nature (such as coexistence of Macro- and femto-cells in UMTS LTE, discussed in Section I) are elegantly addressed. For the problems with vertical spectrum sharing nature, where without loss of generality we assume link 1 is the primary link and link 2 is the secondary link, by defining $\Gamma_{\text{inst},1} = +\infty$, the effect of constraint (3) for the primary transmitter can be theoretically relaxed. However, in practice the imposed interference from the primary link to the secondary will always be bounded due to the limited maximum transmission power of the primary link, i.e., constraint (4). Hence, such vertical spectrum sharing scenarios can also be captured in the proposed framework by defining a much higher interference threshold for the primary link as compared with the secondary link, i.e., assuming $\Gamma_{\text{inst},1} \gg \Gamma_{\text{inst},2}$. Furthermore, the existence of interference limits at both links will result in a degree of symmetry in our analysis, whereby we only need to analyze the capacity of, say, link $i$, and the results are readily extendable to link $j$, where $i \neq j$. Since we assume independent primary and secondary links, cooperative transmission or interference cancellation techniques are not exploited in this scenario. To solve (2) subject to (3) and (4), one can decompose the problem into $N$ subproblems as follows. First, constraint (3) is re-written as

$$\sum_{n=1}^{N} \{ p_{i,n} (g_{i,n}, h_{i,n}) h_{i,n} - \alpha_{i,n} \Gamma_{\text{inst},i} \} \leq 0,$$

where $\alpha_{i,n} \in [0, 1]$ and $\sum_{n=1}^{N} \alpha_{i,n} = 1$, determines the distribution of total interference limit over the $N$ sub-channels. Corresponding to this distribution of interference over sub-channels, the total available power in (4) should also be distributed such that in sub-channel $n$ the transmit power limit of link $i$ will be $\beta_{i,n} P_{\text{max},i}$, where $\beta_{i,n} \in [0, 1]$.
and \( \sum_{n=1}^{N} \beta_{i,n} = 1 \). We will shortly discuss how to select proper values for \( \alpha_{n} \) and \( \beta_{n} \). In this sub-channel based spectrum sharing scenario it is straightforward to observe that the optimum power allocation for link \( i \) is obtained via a greedy algorithm, i.e.,

\[
p_{i,n}^{\text{inst}} = \min \left( \beta_{i,n} P_{\max,n}, \frac{\alpha_{i,n} \Gamma_{\text{inst},i}}{h_{i,n}} \right), \quad \forall \ i \in \{1, 2\}.
\]

Note that the greedy power allocation (6) only depends on the fading states of the cross channel from transmitter \( i \) to receiver \( j \). The Ergodic capacity of each link in this case can be calculated by substituting (6) into (2). This yields

\[
C_{i,j}^{\text{inst}} = \mathbb{E}_{f_{\text{h},i,n}} \left[ \sum_{n=1}^{N} R_{i,n} \left( \frac{p_{i,n}}{P_{\max,n}} \right) f_{h}(h_{i,n}) \right]
\]

\[
\times \frac{dh_{i,n}}{f_{h}(h_{i,n})} \times f_{h}(h_{i,n}) dh_{i,n},
\]

where \( f_{h}(\cdot) \) denote the probability density functions (pdf) of the channel gain \( h_{i,n} \).

As clear from (6), the optimal power allocation of link \( i \) in sub-channel \( n \) is inversely proportional to \( h_{i,n} \). Hence, intuitively, one expects that the optimal distribution of interference over the \( N \) sub-channels, determined by \( \alpha_{n} \) in (5), follows a similar pattern. In other words, if link \( i \) determines a channel that is severely faded towards the non-target link \( j \), i.e., \( h_{i,n} \to 0 \), it will allocate more power to that sub-channel as this transmitted power will be significantly attenuated before reaching receiver \( j \). Hence, we can select \( \alpha_{n} \) as

\[
\alpha_{i,n} = \frac{c_{1}}{h_{i,n}}, \quad \forall \ n \in \{1, 2, \ldots, N\},
\]

where \( c_{1} \) is chosen such that \( \alpha_{i,n} \in [0, 1] \) and \( \sum_{n=1}^{N} \alpha_{i,n} = 1 \), i.e., \( c_{1} = \frac{1}{\sum_{n=1}^{N} 1/h_{i,n}} \). Furthermore, we can use \( \beta_{i,n} = \alpha_{i,n} \) as regards the total transmit power constraint at each sub-channel.

**B. Average Received Interference Limit**

Now, let us investigate the optimal power allocation strategies under average received interference limit, i.e.,

\[
C_{E,i}^{\text{avg}} = \max_{g_{i,n}, h_{i,n}} \mathbb{E}_{g_{i,n}, h_{i,n}} \left[ \sum_{n=1}^{N} R_{i,n} \right], \quad \text{subject to (9)}
\]

\[
\mathbb{E}_{g_{i,n}, h_{i,n}} \left[ \sum_{n=1}^{N} p_{i,n} \left( g_{i,n}, h_{i,n} \right) h_{i,n} \right] \leq \Gamma_{\text{avg},i}, \quad \text{and (10)}
\]

\[
\mathbb{E}_{g_{i,n}, h_{i,n}} \left[ \sum_{n=1}^{N} p_{i,n} \left( g_{i,n}, h_{i,n} \right) \right] \leq P_{\max,i}, \quad \text{and (11)}
\]

The Lagrangian function of this convex optimization problem can be written as

\[
L(p_{i,n}, \lambda, \eta) = \mathbb{E}_{g_{i,n}, h_{i,n}} \left[ \sum_{n=1}^{N} \log \left( 1 + \frac{p_{i,n} \left( g_{i,n}, h_{i,n} \right) g_{j,n}}{\sigma_{n}^{2} + p_{i,n} \left( g_{i,n}, h_{i,n} \right) g_{i,n}} \right) \right] - \lambda \left( \mathbb{E}_{g_{i,n}, h_{i,n}} \left[ \sum_{n=1}^{N} p_{i,n} \left( g_{i,n}, h_{i,n} \right) h_{i,n} \right] - \Gamma_{\text{avg},i} \right) - \eta \left( \mathbb{E}_{g_{i,n}, h_{i,n}} \left[ \sum_{n=1}^{N} p_{i,n} \left( g_{i,n}, h_{i,n} \right) \right] - P_{\max,i} \right).
\]

Here \( \lambda \) and \( \eta \) denote the Lagrange multipliers for the constraints (10) and (11) respectively. The optimal power allocation satisfies the necessary Karush-Kuhn-Tucker (KKT) condition \( \partial L(p_{i,n}, g_{i,n}, h_{i,n}, \lambda, \eta) / \partial p_{i,n} = 0 \), which yields

\[
p_{i,n}^{\text{avg}}(g_{i,n}, h_{i,n}) = \left( \frac{w}{\lambda h_{i,n} + \eta} - \frac{\sigma_{i}^{2} + p_{j,n} h_{j,n}}{g_{i,n}} \right),
\]

\[
\forall i, j \in \{1, 2\} \quad \text{and } i \neq j \quad \text{and } (x)^{+} = \max(x, 0). \quad \text{This is a water-filling-like solution where the variables } \lambda \quad \text{and } \eta \text{ determine the water level, i.e., } \alpha_{i,n} = \frac{\sigma_{i}^{2} + p_{j,n} h_{j,n}}{g_{i,n}}. \quad \text{However, this water-filling solution is performed simultaneously over two time-varying channels, i.e., } h_{i,n} \text{ and } g_{i,n}. \quad \text{If average instantaneous received interference constraint was not present the water level was limited by (11) alone. Hence, constraint (11) defines the water level in the dimension}^{4} \text{ of direct channel gain } g_{i,n}. \quad \text{Similarly, constraint (10) determines the water level in the dimension defined by } h_{i,n}.
\]

For link \( i \), denote the received noise plus the interference from user \( j \) by

\[
I_{j,n} = \sigma_{i}^{2} + p_{j,n} h_{j,n}.
\]

Then, over the direct channel dimension of this two-dimensional water-filling solution, by replacing (12) into (11)

\[
\text{TABLE I - \text{COMPARISON OF OUR PROPOSED APPROACH TO EXISTING METHODS IN LITERATURE}}
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we get

$$P_{\text{max},i} = \int_0^\infty \int_{h_{i_n}}^{\infty} \left( \sum_{n=1}^{N} \frac{w}{\lambda h_{i_n} + \eta_n} - I_{j,n} \right) \times e^{-g_{i,n}} e^{-h_{i,n}} \left( dg_{i,n} dh_{i,n} \right)$$

$$e^{-g_{i,n}} e^{-h_{i,n}} \left( dg_{i,n} dh_{i,n} \right) = \sum_{n=1}^{N} \left[ \int_0^\infty \frac{w}{\lambda h_{i_n} + \eta_n} \times \left( e^{-g_{i,n}} e^{-h_{i,n}} \right) \right] \times \left( e^{-g_{i,n}} e^{-h_{i,n}} \right)$$

$$= \sum_{n=1}^{N} \left[ \frac{1}{\lambda_0} e^{-\frac{w}{\lambda_0}} \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) \right] \times \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right)$$

$$\left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right) - e^{-h_{i,n}} \left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right) - e^{-h_{i,n}} \left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right)$$

$$= \sum_{n=1}^{N} \left[ \frac{1}{\lambda_0} e^{-\frac{w}{\lambda_0}} \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) \right] \times \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) - e^{-h_{i,n}} \left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right)$$

$$\left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) - \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) + \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right)$$

$$= \sum_{n=1}^{N} \left[ \frac{1}{\lambda_0} e^{-\frac{w}{\lambda_0}} \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) \right] \times \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) - e^{-h_{i,n}} \left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right)$$

$$\left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) - \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) + \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right)$$

$$= \sum_{n=1}^{N} \left[ \frac{1}{\lambda_0} e^{-\frac{w}{\lambda_0}} \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) \right] \times \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) - e^{-h_{i,n}} \left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right)$$

$$\left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) - \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) + \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right)$$

In (14), $\mathcal{E}_I(x)$ is the Exponential Integral function which is defined as

$$\mathcal{E}_I(x) = \int_x^\infty \frac{e^{-t}}{t} dt.$$ (15)

Similar to calculating (14), we can substitute (12) in (10) to obtain the result of (10) and present the final result here, i.e.,

$$\Gamma_{\text{avg},1} = \sum_{n=1}^{N} \left[ \frac{1}{\lambda_0} e^{-\frac{w}{\lambda_0}} \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) \right] \times \left( 1 + \frac{I_{j,n} \lambda_0}{w} \right) - e^{-h_{i,n}} \left( h_{i,n} + \frac{\eta_n}{\lambda_0} \right)$$

$$\left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) - \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right) + \eta_n \left( 1 + \frac{w}{I_{j,n} \lambda_0} \right)$$

As can be observed from (14) and (16) the inter-relation between the Lagrangian multipliers $\eta$ and $\lambda$ are nonlinear and can not be solved in closed form. Instead we use numerical approaches such as Gauss-Newton, Levenberg-Marquardt or trust-region methods (which is used in this paper) to find the value of these variables [20]. The Ergodic capacity in this case can be calculated by substituting the optimal power allocation (12) into (1) and using the resulting maximum achievable rate in (9), i.e.,

$$\Gamma_{\text{avg},1} = \sum_{n=1}^{N} \left[ \int_0^\infty \int_{h_{i_n}}^{\infty} \left( p_{i,n} | p_{i,n} = p_{i,n} \left( g_{i,n}, h_{i,n} \right) \right) \times f_{h} \left( h_{i,n} \right) dh_{i,n} dh_{i,n} \right].$$

Note that the capacity of each link is achieved by an iterative application of optimal power in (12) similar to classical iterative water-filling solution [21]. As the numerical results in Section VI shows, exploiting the interference diversity, using power allocation (12), results in a considerable performance improvement compared to the greedy approach in (6). Furthermore, this improvement does not require any coordination between primary and secondary links. However, the price to be paid for this improvement is a more complex resource allocation technique requiring numerical solution to a set of non-linear equations.

IV. STRATEGY 2: MAXIMIZING THE OUTAGE CAPACITY

The Ergodic capacity, discussed in Section III, is a valuable performance metric in circumstances of delay tolerant traffic. However, for delay sensitive applications, such as real-time voice packets, Outage capacity is a more relevant performance metric, which indicates the constant achievable throughput, irrespective of fading state of the channel. To sustain the required constant SINR at receiver $i$, transmitter $i$ should have the knowledge of the channel, effectively inverting the fading condition at the receiver, which is referred to as Channel Inversion (CI) policy [22]. In severely faded channels, such as Rayleigh fading channels, in time instants of deep fades, i.e., $g_{i,n} \to 0$, the CI policy requires a huge amount of transmission power to sustain the received SINR. Hence, a more practical transmission policy, namely Truncated Channel Inversion (TCI) [22], is to invert the channel only when the fading state of the channel is over a certain threshold.

A. Instantaneous Received Interference Limit

Similar to the Ergodic capacity analysis in Section III, the optimal power allocation of link $i$ to maximize the Outage capacity, given an instantaneous received power limit at link $j$, where $i \neq j$, should have a two-fold effect. In the direct channel dimension, TCI policy insures the transmitted power sustains a fixed SINR at the respective receiver. However, this constant SINR is, in part, a function of cross-channel dimension, taking into account constraint (3). This approach is different from [15], [18], and [16]. In all the aforementioned references, the transmission cut-off threshold (determining in which fading states TCI policy should be used) is defined as a function of the ratio of direct and cross channel gains for link $i$, i.e., $g_{i,n} / h_{i,n}$ in Fig. 1. This choice, as detailed in [16] for instance, is optimal when the interference of the primary transmitter on the secondary receiver is ignored, which is not the case in our scenario. Furthermore, the Outage only happens when the direct channel is experiencing deep fade states such that the transmitter can not sustain the target received SINR. Keeping this argument in mind, we select the channel cut-off threshold only as a function of direct channel (similar to [22]), but select the constant SINR as a function of both direct and cross channel gains. The received SINR at receiver side of link $i$ is

$$\gamma_{i,n} = \frac{p_{i,n}^{\text{inst}} g_{i,n}}{I_{j,n}}, \text{ if } g_{i,n} \geq \mu_i,$$ (17)

where $I_{j,n}$ is defined in (13), $p_{i,n}^{\text{inst}}$ is the transmitted power and $\mu_i$ is the cut-off threshold of link $i$, determining the fading states of the channel above which the TCI policy is used. Hence, to sustain $\gamma_{i,n} = \gamma_{i,n}^{\text{inst}}$, the optimal transmitted power is given by

$$p_{i,n}^{\text{inst}*} = \left\{ \begin{array}{cl} \frac{I_{j,n} \gamma_{i,n}^{\text{inst}}}{g_{i,n}}, & g_{i,n} \geq \mu_i, \\ 0, & \text{Otherwise} \end{array} \right\},$$ (18)

Throughout this section when the term "channel" is used alone, it refers to the direct channel $g_{i,n}$. 


where the cut-off value \( \mu_i \) is assumed equal for all sub-channels. Given \( \varepsilon_i \) as the target Outage probability of link \( i \) per sub-channel, the cut-off threshold can be determined from

\[
P_{\text{out}} = \mathbb{P}(g_{i,n} < \mu_i) = \varepsilon_i,
\]

(19)

where \( \mathbb{P}(x) \) indicates the probability of event \( x \). As we assumed i.i.d Rayleigh fading channels with unit mean, the transmission cut-off threshold can be calculated as a function of target Outage probability from (19), i.e., \( \mu_i = \log \frac{1}{1 - \varepsilon_i} \).

As mentioned before, while the channel cut-off threshold only depends on the direct channel gain of link \( i \), the value of the sustained SINR is determined, in part, from the constraints limiting the interference of this link on link \( j \), where \( i \neq j \). From (5) we have \( p_{i,n} \leq \frac{\alpha_{i,n} \Gamma_{i,n}}{h_{i,n} I_{j,n}} \), resulting in a received SINR given by

\[
g_{i,n} \leq \gamma_{i,n} = \frac{\alpha_{i,n} \Gamma_{i,n} g_{i,n}}{h_{i,n} I_{j,n}}, \quad \text{if } g_{i,n} \geq \mu_i.
\]

On the other hand, constraint (4) dictates that \( p_{i,n} \leq \beta_{i,n} P_{\text{max},i} \), which in turn means

\[
g_{i,n} \leq \gamma_{i,n} = \frac{\beta_{i,n} P_{\text{max},i} g_{i,n}}{I_{j,n}}, \quad \text{if } g_{i,n} \geq \mu_i.
\]

Hence, the sustainable SINR in (18) is the minimum of the above calculated SINRs, i.e.,

\[
\gamma_{i,n}^{\text{inst}} = \min (\gamma_{i,n}, \gamma_{i,n}).
\]

(20)

Hence, the optimal power allocation maximizing the Outage capacity with instantaneous received interference threshold is indeed similar to the power policy maximizing the Ergodic capacity with instantaneous received interference threshold, i.e.,

\[
p_{i,n}^{\text{inst}} = \min \left( \frac{\beta_{i,n} P_{\text{max},i} g_{i,n}}{I_{j,n}}, \quad \frac{\alpha_{i,n} \Gamma_{i,n}}{h_{i,n}} \right), \quad \text{if } g_{i,n} \geq \mu_i,
\]

\[
\text{Otherwise}.
\]

(21)

The Outage capacity can then be derived as

\[
\mathcal{C}_{\text{Out},i}^{\text{inst}} = \sum_{n=1}^{N} \log \left( 1 + \gamma_{i,n}^{\text{inst}} \right).
\]

(22)

B. Average Received Interference Limit

The case of maximizing the Outage capacity, given an average received interference at a third party receiver can be solved by the optimal power allocation given by

\[
p_{i,n}^{\text{avg}} = \min \left( \frac{I_{j,n} g_{i,n}}{\gamma_{i,n}^{\text{avg}}}, \quad \frac{\alpha_{i,n} \Gamma_{i,n}}{h_{i,n}} \right), \quad \text{if } g_{i,n} \geq \mu_i,
\]

\[
\text{Otherwise}.
\]

(23)

where \( I_{j,n} \) is defined in (13), \( \gamma_{i,n}^{\text{avg}} \) is the sustained SINR at receiver \( i \), and \( \mu_i \) is the cut-off threshold related to the Outage probability \( \varepsilon_i \), as in (19). In order to satisfy the average received interference limit, by substituting \( p_{i,n} \) in (10) with (23), we have

\[
\Gamma_{i,n}^{\text{avg}} = \int_0^{\infty} \int_{\mu_i}^{\infty} \sum_{n=1}^{N} \frac{I_{j,n} \gamma_{i,n}^{\text{avg}} h_{i,n}}{g_{i,n}} e^{-g_{i,n}} e^{-h_{i,n}} dg_{i,n} dh_{i,n},
\]

\[
= -\mathcal{E}_f (-\mu_i) \sum_{n=1}^{N} I_{j,n} \gamma_{i,n}^{\text{avg}},
\]

(24)

where \( \mathcal{E}_f (x) \) is defined in (15). Similarly, from the average transmit power limit (11) we have

\[
P_{\text{max},i} = -\mathcal{E}_f (-\mu_i) \sum_{n=1}^{N} I_{j,n} \gamma_{i,n}^{\text{avg}}.
\]

(25)

Hence, from (24) and (25), the sustainable SINR at receiver \( i \) should be calculated from

\[
\gamma_{i,n}^{\text{avg}} = \frac{-\mathcal{E}_f (-\mu_i) I_{j,n}}{\mathcal{C}_{\text{avg},i}/N}.
\]

(26)

The optimal power allocation policy to achieve the Outage capacity is calculated by replacing (27) in (23) and the resulting Outage capacity can be calculated as

\[
\mathcal{C}_{\text{Out},i}^{\text{avg}} = \sum_{n=1}^{N} \log \left( 1 + \gamma_{i,n}^{\text{avg}} \right).
\]

(28)

The sustained level of SINR using instantaneous versus average interference threshold in (20) and (27) can not be compared in their closed form, however, as the numerical results in Section VI verifies, the usage of interference diversity is not optimal for the Outage capacity. In other words, instantaneous interference threshold performs better than average interference threshold when dealing with an instantaneous measure of the channel, i.e., the Outage capacity.

V. STRATEGY 3: GUARANTEEING THE QoS

In this section we approach the optimal coexistence problem of two links in a shared band from a different angle, i.e., minimizing the transmission power of each links such that a minimum rate (as a measure of QoS level) for both systems can be guaranteed. The constraint here is defined on the QoS rather than the received interference which was used in Sections III and IV, i.e., the upper bound limit on the interference, for instance in (3) or (10), is translated into a lower bound limit on the achievable rates in this case. Recall from Section III that we study a generalized format of the problem by considering a level of protection for both links, thereby we propose a minimum QoS guarantee level for both primary and secondary links.

A. Instantaneous Minimum Rate Guarantee

The optimization problem for link \( i \) that models our scenario here is defined as follows.

\[
\begin{align*}
\text{Minimize } & E_{g_{i,n}} \left\{ \sum_{n=1}^{N} p_{i,n} (g_{i,n}) \right\}, \quad \text{subject to }
\end{align*}
\]

(29)
the achievable rate of user allocation in (29) is not explicitly a function of the cross channel gain $h_{i,n}$. The effect of interference of link $i$ on link $j$, where $i \neq j$, in this case, is implicitly captured in the achievable rate of user $j$. Further to the discussion at the beginning of Section V, by selecting $R_{QoS,2} = 0$, a theoretical analysis of vertical spectrum sharing scenarios can be achieved. In order to solve this optimization problem, we can rewrite constraint (30) as

$$
\sum_{n=1}^{N} \alpha_{i,n} R_{QoS,i} - R_{i,n} \leq 0,
$$

where $\alpha_{i,n}$ determines the allocation level of the guaranteed rate in sub-channel $n$. As the capacity of each sub-channel is directly proportional to its direct channel gain we can define $\alpha_{i,n}$ as $\alpha_{i,n} = \frac{g_{i,n}}{\sum_{n=1}^{N} g_{i,n}}$, so that $\alpha_{i,n} \in [0,1]$ and $\sum_{n=1}^{N} \alpha_{i,n} = 1$, $\forall i \in \{1, 2, \ldots, K\}$. Corresponding to this distribution of rate over $N$ sub-channels, the total available power should also be distributed such that in sub-channel $n$ the total transmit power limit will be $\beta_{i,n}P_{max,i}$, where we assume $\beta_{i,n} = \alpha_{i,n}$. This sub-channel based problem can be easily solved by re-arranging constraint (32) in terms of the power instead of rate, i.e.,

$$
p_{i,n} \geq \left( e^{\alpha_{i,n} R_{QoS,i}/w} - 1 \right) \frac{I_{j,n}}{g_{i,n}} = p_{i,n}^*,
$$

where $I_{j,n}$ is defined in (13). The lower bound for transmission power in (33) is the optimal power allocation as long as it is lower than the total transmit power limit, i.e.,

$$p_{i,n}^* = \begin{cases} p_{i,n} & \text{if } p_{i,n} < \beta_{i,n}P_{max,i} \\ 0 & \text{otherwise}. \end{cases}
$$

It is worth noting that, the optimal power (34) is directly proportional to the received interference, $I_{j,n}$, and inversely proportional to the direct channel gain $g_{i,n}$. Therefore, a desirable fading channel state (as to facilitate interference diversity gain) is to have a (relatively) high direct channel gain and a low cross channel gain simultaneously.

B. Average Minimum Rate Guarantee

In this case, the following optimization problem should be solved.

Minimize $\mathbb{E}_{g_{i,n}} \left\{ \sum_{n=1}^{N} p_{i,n} (g_{i,n}) \right\}$, subject to

$$R_{QoS,i} \leq \mathbb{E}_{g_{i,n}} \left\{ \sum_{n=1}^{N} R_{i,n} \right\}, \quad \text{and} \quad \mathbb{E}_{g_{i,n}} \left\{ \sum_{n=1}^{N} p_{i,n} (g_{i,n}) \right\} \leq P_{max,i},
$$

We can express the Lagrangian as,

$$L(p_{i,n}, \lambda_0, \eta_0) = \mathbb{E}_{g_{i,n}} \left\{ \sum_{n=1}^{N} p_{i,n} (g_{i,n}) \right\} + \lambda_0 \left( R_{QoS,i} - \mathbb{E}_{g_{i,n}} \left\{ \sum_{n=1}^{N} w \log \left( 1 + \frac{p_{i,n} g_{i,n}}{\sigma^2 + p_{j,n} h_{j,n}} \right) \right\} \right) + \eta_0 \left( \mathbb{E}_{g_{i,n}} \left\{ \sum_{n=1}^{N} p_{i,n} (g_{i,n}) \right\} - P_{max,i} \right),
$$

where $\lambda_0$ and $\eta_0$ are the lagrange multipliers for the constraints (36) and (37), respectively. The optimal power allocation in this case is given by

$$p_{i,n}^* = \left( \frac{w \lambda_0}{1 + \eta_0} - \frac{I_{j,n}}{g_{i,n}} \right)^+.
$$

For the case of Rayleigh fading channels, by substituting (38) into (37) we have

$$P_{max,i} = \sum_{n=1}^{N} \frac{w \lambda_0 e^{-(1+\eta_0)I_{j,n}}}{w \lambda_0 + I_{j,n} E_I \left( \frac{I_{j,n} (1+\eta_0)}{w \lambda_0} \right)}.
$$

Similarly, using (36) we have,

$$R_{QoS,i} = w \sum_{n=1}^{N} \left[ -E_I \left( \frac{-(1+\eta_0) I_{j,n}}{w \lambda_0} \right) \right].
$$

To calculate the optimal Lagrange multipliers this set of non-linear algebraic equations needs to be solved numerically.

VI. NUMERICAL RESULTS

Using extensive simulation results in this section, we demonstrate the performance gain achieved by exploiting the interference diversity of a shared channel. We will study the effect of involved parameters in the performance of the proposed resource allocation techniques, namely the number of sub-channels, the maximum transmit power, the received interference threshold and the minimum QoS requirements. For clarity of comparisons, we assume the same limit for average and instantaneous received interference as well as the average and instantaneous guaranteed QoS level. Furthermore, to clearly identify how the interference diversity behaves in a multiple-channel system, we study the cases with fixed rate and power distribution over all sub-channels, i.e., $\alpha_{i,n} = \beta_{i,n} = 1/N$, versus the case of adaptive rate and power distribution given by (8). In the former case, only time domain diversity can be achieved (due to averaging process and assumption of ergodicity of channel gains), whereas in the latter case time and frequency domain diversity is achieved. Finally, in order to capture the performance of the proposed methods in spectrum sharing scenarios, in some simulations we have used a different interference or QoS limit for the two links.

A. Ergodic Capacity Maximization

Consider a horizontal spectrum sharing scenario where both links have a similar interference threshold. Further assume both links have a similar average transmit power limit of 5 Watts in (4) and (11). The effect of number of sub-channels, $N$, on the achievable Ergodic capacity is depicted in Fig. 2. First
note that the average interference limit significantly (by orders of magnitude) outperform the instantaneous interference limit. Comparing the graphs for the instantaneous interference limit, it is interesting to observe that the Ergodic capacity is higher when using adaptive values for the interference distribution, $\alpha_{\text{avg}}$. In Fig. 2, it is further notable that the higher the interference threshold is, the higher the achieved Ergodic capacity is. This is due to the fact that a more relaxed interference cap allows both links to transmit with a higher power. Although transmitting with a higher power means more received interference, on average the performance of links have improved. Finally, as the total transmit power of links are limited, as the number of sub-channels increase the Ergodic capacity decreases as less power per sub-channel will be allocated. The average allocated power for the scenario of Fig. 2 is shown in Fig. 3.

An important characteristics of the proposed symmetric framework of multi-dimensional resource allocation technique is its applicability to SSA scenarios. To verify this capability we have repeated the simulation presented in Fig. 2 and Fig. 3 with a differentiated interference limit for the primary and secondary links, where we assume a more relaxed $\Gamma_{\text{inst,1}} = \Gamma_{\text{avg,1}} = -5$ dB for the primary link whereas assuming a much tighter $\Gamma_{\text{inst,2}} = \Gamma_{\text{avg,2}} = -15$ dB threshold for the secondary link. The achievable Ergodic capacity of each link is depicted in Fig. 4. This figures shows that the proposed resource allocation technique can clearly differentiate the primary and secondary links, hence effectively implementing an SSA spectrum sharing policy in the target frequency band. Next, we have studied the effect of maximum average transmit power available to each link in Fig. 5. The total number of sub-channels is fixed at 500. Increasing the total available transmit power slightly improves the Ergodic capacity in Fig. 5 as the average allocated power per sub-channel increases, as shown in Fig. 5. Note that this increase in performance is not sustainable in single-dimensional resource allocation approaches such as that of IC problem due to mutual interfering effect of links on each other.

B. Outage Capacity Maximization

In Fig. 7, the achieved Outage capacity for the cases of uniform and adaptive interference distribution with instantaneous interference limit as well as the case of average interference limit are presented, assuming in all cases that $P_{\text{out}} = 0.1$. As expected, each method sustains a level of SINR at the respective receiver side of the links. However, the more adaptive the resource allocation strategy is, the less the achieved Outage capacity will be. This somewhat surprising phenomenon can be intuitively justified. As discussed in Section IV the power allocation to maximize the Outage capacity should combat two issues in the channel. First is the the fading state of the channel which is compensated using the TCI power allocation. The second issue is the received interference in the channel. The proposed interference threshold will maintain the disruptive
the received interference from the coexisting link, i.e., by (28) for the average interference case, does depend on set-up, given by (22) for instantaneous interference case and scenario. Also note that the Outage capacity of a link in our drained of transmission power in less favorable sub-channels, SINR at each instant of time. He resource allocation periods and needs to sustain the received for more desirable states of the channel in the following scheduler in this case can not afford the luxury of waiting time. Unlike the Ergodic capacity maximization strategy, the effect of the received interference below a certain threshold. However, the desirable states of the channel will only occur in a limited percentage of sub-channels at any instance of time. Unlike the Ergodic capacity maximization strategy, the scheduler in this case can not afford the luxury of waiting for more desirable states of the channel in the following resource allocation periods and needs to sustain the received SINR at each instant of time. Hence, the scheduler will be drained of transmission power in less favorable sub-channels, resulting in a much lower capacity than the Ergodic capacity scenario. Also note that the Outage capacity of a link in our set-up, given by (22) for instantaneous interference case and by (28) for the average interference case, does depend on the received interference from the coexisting link, i.e., $I_{j,n}$. This interference term can reduce the sustainable SINR of each sub-channel and our simulation results show that under these circumstances instantaneous interference limit can help increase the Outage capacity of the shared channel.

The power allocation for the scenario of Fig. 7 is shown in Fig. 8, which further verifies our intuition. Comparing the power allocation in Fig. 8 with similar graphs in Fig. 6 and Fig. 3 reveals that a lower average transmit power per sub-channel is assigned in the Outage maximization strategy.

C. QoS Guarantee: Instantaneous and Average Minimum Rate

Let us now study the QoS-guarantee scenarios where the primary and secondary links have a different QoS level. As the resource allocation strategy in this case ensures that achieved throughput converges to the requested minimum rate level, we only need to examine the power allocation policy of schedulers using average or instantaneous received interference limit. As is clear from Fig. 9, average instantaneous received interference limit outperforms all other power allocation policies, i.e., it will guarantee the requested QoS level with the minimum transmit power possible. This power saving is the dual of higher achievable Ergodic capacity in Section III and the corresponding numerical results in Section VI-A. In other word, exploiting the interference diversity gain in multiple-channel systems helps reduce the amount of transmit power required to guarantee a certain QoS level. Similarly, in the instantaneous received interference regime, average distribution of QoS level over sub-channels results in a lower transmit power compared to a uniform QoS distribution over all sub-channels.

VII. CONCLUSION

This paper introduces a generalized framework for a multi-dimensional resource allocation technique which takes the coexistence effect of each link on the other links into account. The proposed approach utilizes the interference-diversity gain of a shared channel and can be readily applied to vertical as well as horizontal spectrum sharing scenarios. To demonstrate the performance improvement harnessed by this approach we elaborated on the Ergodic and Outage capacity of a multiple-channel system, such as an OFDM-based network, and derived optimal power allocation strategies in this setting.
Further, we discussed QoS guarantee strategies based on this coexistence principle and verified the power saving capability of our method compared with existing techniques in the literature. Application of such a multi-dimensional resource allocation paradigm can pave the way towards more efficient coexistence of future wireless technologies, examples of which are Cognitive Radio Networks and UMTS LTE systems with universal frequency reuse pattern.

REFERENCES