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OPTIMIZATION-BASED TELEOPERATION CONTROLLER DESIGN

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Abstract: This paper addresses issues of performance and stability robustness specifications and trade-offs, and computational techniques in optimization-based teleoperation controller design. With Youla's Q-parametrization of stabilizing controllers, a transparency measure, defined as the H_∞ distance to the ideal teleoperator model, and a robust stability constraint, defined as positive realness of the transmitted admittance to the environment, are convex in the free design parameters. Therefore, the controller design problem can be formulated as a convex optimization problem. The limit of performance achievable with the designed controller, and thus the exact form of the trade-offs between performance and robust stability, can be computed numerically. The solution procedure is illustrated by a design example for a motion-scaling teleoperation system.

Key words: Robust control, Multiobjective optimizations, Numerical methods, Robotic manipulators, Teleoperation.

1. INTRODUCTION

Teleoperation has found wide applications in space exploration, waste management and undersea exploration, by extending an operator's sensing and manipulation capabilities to remote and hazardous locations. Recently, teleoperation has started to encompass the extension of such capabilities through barriers of scale, allowing human involvement at scales much smaller or much larger than possible directly. One of the major issues in teleoperation control is to design a controller to achieve transparency while maintaining stability of the teleoperation system. Several controller design methods have been proposed for this problem. The scaling concepts of impedance and power as a function of position and force scaling based on specifying two-port hybrid parameters are discussed in (Hannaford, 1989); a four-channel control structure has been suggested to achieve transparency in (Lawrence, 1993) and the ideal response of teleoperation in (Yokokohji et al., 1994); H_∞–optimization theory has been used to best shape the closed-loop responses of interest in (Yan and Salcudean, 1996) and to shape the relationships between forces and positions at both ends of the teleoperator in (Kazerooni et al., 1993); a control algorithm was proposed based on a semi-autonomous task-oriented virtual tool (Kosuge et al., 1995); and a robust stability criterion for a power scaling teleoperation was obtained by using the structured singular values of the scattering matrix (Colgate, 1993). However, none of the above work has explicitly incorporated robust stability into the controller design.

The goal of this paper is to design robust controllers with good transparency for scaled teleoperation systems. First, the ideal teleoperator is defined by using a two-port admittance matrix that characterizes the ideal situation for scaled teleoperation. Then, by using a four-channel control structure, the controller is designed to be robustly stable for a given human operator impedance and unknown passive environments, and to match the ideal one as closely as possible. The teleoperation controller design problem is formulated as a constrained multiple objective optimization problem, which is convex and numerically solvable. In the following section, we review some basic passivity concepts and stability conditions for teleoperation systems, and present an ideal teleoperator. In section 3, the controller design problem is formulated as an optimization problem. In Section 4, a numerical solution procedure is described and illustrated by a design example of a controller for a motion–scaling system. Some concluding remarks are included in the final section.

2. AN IDEAL TELEOPERATOR

2.1. Passivity and stability

Definition 1: For a linear time-invariant (LTI) n-port network as shown in Fig. 1, the impedance matrix Z is defined as the map from v to f by $f = Zv$; the admittance matrix Y as the map from f to v by $v = Yf$; and the scattering matrix S as the map from the input wave $a \stackrel{\triangle}{=} (f+v)/2$ to the output wave $b \stackrel{\triangle}{=} (f-v)/2$, i.e., satisfying the equation $b = Sa$. These matrices are interrelated by

$$S = (I - Y)(I + Y)^{-1} = (Z - I)(Z + I)^{-1}. \quad (1)$$

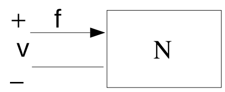

Figure 1: An n-port network

Theorem 1 (Desoer and Vidyasagar, 1975): (a) An LTI n-port as shown in Fig. 1 is *strictly passive* if and only if the matrix criterion below holds for some $\delta > 0$:

$$Y(\mathrm{j}\omega) + Y^*(\mathrm{j}\omega) \geq \delta I, \forall \omega \in \mathbf{R}. \quad (2)$$

Notice that if $n = 1$, condition (2) reduces to $\mathrm{Re}[\mathrm{Y}(j\omega)] \geq \delta/2$. (b) An LTI n-port is *passive* if and only if criterion (2) holds for $\delta = 0$.

Theorem 2 (Colgate and Hogan, 1988): Consider the network in Fig. 2, in which an LTI one-port with admittance Y is coupled to an impedance Z. This network will be stable for every strictly passive Z if and only if the one-port is itself passive, *i.e.*,

$$\mathrm{Re}[Y(\mathrm{j}\omega)] \geq 0, \forall \omega \in \mathbf{R}. \quad (3)$$

Definition 2 (Wen, 1988): The ν–index, also referred to as *passivity distance*, is defined as the distance of a stable LTI system to strict passivity. Let the system transfer function be $T(s)$, then

$$\nu \stackrel{\triangle}{=} - \inf_{\omega \in \mathrm{R}} \{\mathrm{Re}[T(\mathrm{j}\omega)]\}. \quad (4)$$

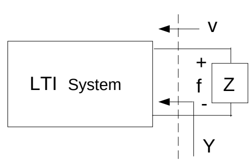

Figure 2: A representation of an LTI 1–port network, Y, coupled to Z.

A typical teleoperation system consists of five interacting subsystems: human operator, master manipulator, controller, slave manipulator and environment as shown in Fig. 3. To a first approximation, this can be modeled as an LTI 2n-port network illustrated in Fig. 4, in which the master, controller and slave are grouped into one block called the teleoperator MCS. Here, v_m is the master velocity, v_s the slave velocity, f_h the force that the operator applies to the master, f_e the force that the environment applies to the slave, and the operator hand and environment impedances are represented, respectively, by Z_h and Z_e.

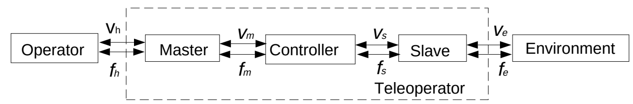

Figure 3: General teleoperation system

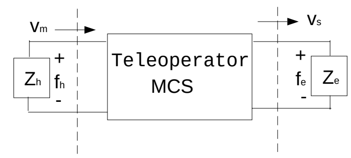

Figure 4: 2n-port representation of a teleoperation system

The following robustness criterion for the 2n-port teleoperation system is based on the finding that the system can be transformed into a structured uncertainty problem by using the scattering matrix (Colgate, 1993), which can be addressed by considering the structured singular value (Doyle, 1981).

Theorem 3 (Colgate, 1993): Consider the bilateral system shown in Fig. 4. This teleoperation system will be stable for every pair of strictly passive Z_h and Z_e if and only if the scattering matrix $S_t \stackrel{\triangle}{=} \begin{bmatrix} s^{11} & s^{12} \\ s^{21} & s^{22} \end{bmatrix}$ of the 2n-port teleoperator MCS has no poles in the closed right-half-plane, and moreover satisfies

$$\sup_{\omega} \{\mu_{\triangle}(S_t(j\omega))\} = \sup_{\omega} \inf_{\beta > 0} \overline{\sigma} \left(\begin{bmatrix} s^{11} & \beta s^{12} \\ s^{21}/\beta & s^{22} \end{bmatrix} \right) \leq 1. \quad (5)$$

Here, $\overline{\sigma}$ denotes the maximum singular value, and $\mu_{\triangle}$ denotes the structured singular value against the block structure $\triangle = \begin{bmatrix} S_h & 0 \\ 0 & S_e \end{bmatrix}$, where S_h and S_e are the scattering matrices of strictly passive Z_h and Z_e, respectively.

When the hand impedance Z_h is fixed, Theorem 3 reduces to Theorem 2 and therefore the stability criterion in (5) is less conservative than the stability criterion in (3). In practice, the variation of the operator hand impedance is relatively small if compared with drastic changes of the environment impedance. Therefore, to design a less conservative controller for teleoperation, the stability criterion in (3) could be used. The advantage of using criterion in (3) is that it is convex in design parameters, as we will show in Section 4.

2.2. An ideal scaled teleoperator

An ideal teleoperator is one that provides complete transparency of the man-machine interface such that the operator has the perception of working directly on the task environment. In order to achieve this, for non-scaled teleoperation, the effort and impedance of the operator port should be identical to the effort and impedance of the environment and vice versa (Hannaford, 1989; Lawrence, 1993), *i.e.*, $v_m = v_s,\ f_h = f_e$. This can be represented by an infinitely stiff and weightless mechanical connection between the master and the slave (Dudragne, 1989), which cannot be physically realized. For scaled teleoperation, an ideal teleoperator is proposed here and is modeled by a two-port network as illustrated in Fig. 5. Instead of eliminating the dynamics of the teleoperator and realizing the ideal response defined by $v_m = n_p v_s,\ f_h = n_f f_e$, where n_p and n_f are constant scaling ratios of position and force respectively, a passive tool represented by impedance Z_d is used by the operator, as similarly done in (Kosuge et al., 1995). The force is multiplied by n_f and directly fed forward to the operator's hand so that the operator can get a feel of the task, and the motion command from the operator is divided by n_p. For micro-manipulations, $n_f, n_p > 1$. There are instances in which $n_f, n_p = 1$ (passive tool) or $n_f, n_p < 1$, *i.e.*, manipulation is at a large scale. Here Z_{th} denotes the transmitted impedance to the hand, and Z_{te} the transmitted impedance to the environment.

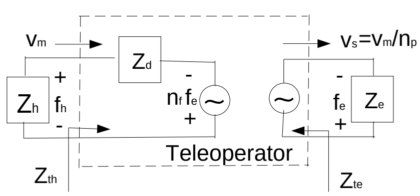

Figure 5: Two-port representation of ideal scaled teleoperation

The dynamics of this MCS teleoperator block are given by:

$$v_m = y_d(f_h + n_f f_e), \tag{6}$$

$$v_s = y_d\left(\frac{1}{n_p} f_h + \frac{n_f}{n_p} f_e\right), \tag{7}$$

where $y_d \triangleq 1/Z_d$. Therefore, the ideal MCS teleoperator block can also be represented by the following admittance matrix

$$Y_d = \begin{bmatrix} 1 & n_f \\ \frac{1}{n_p} & \frac{n_f}{n_p} \end{bmatrix} y_d. \tag{8}$$

When $n_f = n_p = 1$, this corresponds to the case in which the operator manipulates the task environment directly with the assistance of a tool with impedance Z_d. The important properties of this MCS teleoperator block are summarized in the following theorem.

Theorem 4: (1). $Z_{th} = Z_d + \frac{n_f}{n_p} Z_e,$ and $Z_{te} = \frac{n_p}{n_f} Z_d + \frac{n_p}{n_f} Z_h$. (2). If y_d is passive, then Y_d is passive *if and only if* $n_f(j\omega)n_p^*(j\omega) = 1, \forall\omega \in \mathbf{R}$. (3). (3a). If y_d is passive, then $\sup_\omega \{\mu_\triangle(S_d(j\omega))\} = 1$ for any constant, positive and real scalings n_f and n_p. (3b). If $\left(1 + \frac{n_f}{n_p}\right) y_d$ is passive, then $\sup_\omega \{\mu_\triangle(S_d(j\omega))\} = 1$ for any frequency dependent scalings $n_f(j\omega)$ and $n_p(j\omega)$ satisfying $\left|\frac{n_f}{n_p}(j\omega)\right| = \mathrm{Re}\left\{\frac{n_f}{n_p}(j\omega)\right\}, \forall\omega \in \mathbf{R}$. Here, S_d is the scattering matrix of the teleoperator MCS, and the block structure is given by $\triangle = \begin{bmatrix} S_h & 0 \\ 0 & S_e \end{bmatrix}$ with $||S_h||_\infty < 1$ and $||S_e||_\infty < 1$. (4). If $|y_d(j\omega)| \to \infty, \forall\omega \in \mathbf{R}$, then $S_d \to \frac{1}{n_f + n_p} \begin{bmatrix} n_p - n_f & 2n_f n_p \\ 2 & n_f - n_p \end{bmatrix}$.

3. CONTROLLER DESIGN

A general four channel structure in "admittance" form is used for controller design as shown in Fig. 6. Laplace transforms and transfer function notation are assumed throughout. For simplicity, we consider only the one-degree-of-freedom case. In this figure, $x_m = v_m/s$ is the master position, $x_s = v_s/s$ the slave position, P_m the master plant, and P_s the slave plant. The hand force f_h is decoupled into an active component f_{ha} and a passive component $-Hx_m$, and similarly, the environment force f_e is decoupled into f_{ea} and $-Ex_s$, where $H = sZ_h$ and $E = sZ_e$.

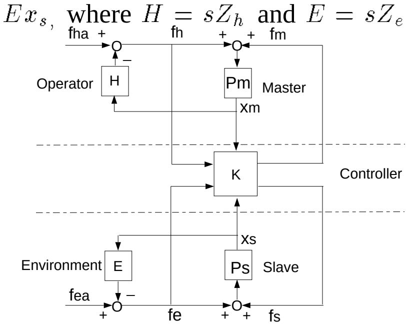

Figure 6: A four channel control structure.

The controller K, which is a 2 by 4 matrix of real-rational transfer functions, takes forces and positions from both master and slave, and generates the actuator driving forces f_m on the master and f_s on the slave. Our objective here is to design the controller K to realize scaled teleoperation with good transparency while maintaining stability. This controller design problem can be put into the standard H_∞–framework (Francis, 1988; Boyd and Barratt, 1991) shown in Fig. 7, where we define the signals $w = [f_h\ f_e]^T$, $u = [f_m\ f_s]^T$, $y = [f_h\ f_e\ x_m\ x_s]^T$, and $z = [v_m\ v_s]^T$.

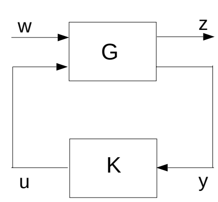

Figure 7: General control system

The generalized plant is described by

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}, \tag{9}$$

where

$$G_{zw} = G_{zu} = \begin{bmatrix} sP_m & 0 \\ 0 & sP_s \end{bmatrix}, \tag{10}$$

and

$$G_{yw} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ P_m & 0 \\ 0 & P_s \end{bmatrix},\; G_{yu} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ P_m & 0 \\ 0 & P_s \end{bmatrix}. \tag{11}$$

The master and slave transfer matrices P_m and P_s are real-rational and strictly proper, and therefore G will be proper and H_∞–optimization design techniques can be applied.

The teleoperator can be described by the admittance matrix $Y_t : w \to z$. Clearly, to get good transparency, it should be designed to match the model Y_d for the ideal teleoperator MCS proposed in the previous section as closely as possible. Therefore, a *transparency measure* can be defined as

$$\|W[Y_t(K) - Y_d]\|_\infty, \tag{12}$$

where W is a weighting matrix that reflects the frequency bands of interest. To guarantee stability against any strictly passive environment or operator impedances, the scattering matrix $S_t \triangleq (Y_t - I)(Y_t + I)^{-1}$ should satisfy the stability criterion in (5). Therefore the problem of designing a robustly stable controller for the scaled teleoperation system is a constrained optimization problem as follows:

$$\left(P^1\right) : \min_{stabilizing\ K} \|W[Y_t(K) - Y_d]\|_\infty \tag{13}$$

$$\text{s.t.} : \sup_\omega \mu_\Delta(S_t(K)(j\omega)) \le 1. \tag{14}$$

The H_∞ theory can also be applied in the simpler case, where the hand impedance is assumed to be known and fixed and the environment impedance is arbitrary but strictly passive. Here the signals u, y, and z are the same as those defined before except now $w = [f_{ha}\ f_e]^T$. Then, the generalized plant becomes

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G_{zw} & G_{zu} \\ G_{yw} & G_{yu} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}, \tag{15}$$

where

$$G_{zw} = G_{zu} = \begin{bmatrix} sP_H & 0 \\ 0 & sP_s \end{bmatrix}, \tag{16}$$

$$G_{yw} = \begin{bmatrix} 1 - HP_H & 0 \\ 0 & 1 \\ P_H & 0 \\ 0 & P_s \end{bmatrix},\; G_{yu} = \begin{bmatrix} -HP_H & 0 \\ 0 & 1 \\ P_H & 0 \\ 0 & P_s \end{bmatrix}, \tag{17}$$

and $P_H \triangleq P_m/(1 + P_m H)$. To get transparency, we have to shape the closed-loop transfer matrix $Y_{t,H} : w \to z$ to match the matrix $Y_{d,H}$ as in the model in the ideal teleoperation. To maintain stability against any arbitrary but strictly passive environment, the admittance seen from the environment, $Y_{te} : f_e \to v_s$, should be passive according to the stability criterion in (3). Therefore the robust controller design problem for this simpler case is

$$\left(P^2\right) : \min_{stabilizing\ K} \|W_H[Y_{t,H}(K) - Y_{d,H}]\|_\infty \tag{18}$$

$$\text{s.t.} : \inf_\omega \{\mathrm{Re}[Y_{te}(\mathrm{K})(j\omega)]\} \ge -\nu, \tag{19}$$

where W_H is a weighting matrix penalizing the frequencies of interest, and the positive parameter ν is used to ensure a given distance to passivity defined as in (4) and determines the degree of conservatism of the controller design.

4. NUMERICAL SOLUTION

4.1. Convex optimization

The Youla parametrization of the stabilizing controllers K (Francis, 1988) makes the transfer matrices $Y_t, Y_{t,H}$ and Y_{te} affine functions of $Q \in RH_\infty^{2\times4}$. However the scattering operator S_t is not affine in Q, so the scalar quantity $\sup_\omega \{\mu_\Delta(S_t(\mathrm{j}\omega))\} \le 1$ is not a convex function of Q. Therefore, the general controller design for scaled teleoperation is a *non-convex optimization problem*:

$$\left(P^3\right) : \min_{Q \in RH_\infty^{2\times4}} \|W[Y_t(Q) - Y_d]\|_\infty \tag{20}$$

$$\text{s.t.} : \sup_\omega \mu_\Delta(S_t(Q)(j\omega)) \le 1, \tag{21}$$

and only in the simpler case, does it reduce into a *convex optimization problem*:

$$\left(P^4\right) : \gamma = \min_{Q \in RH_\infty^{2\times4}} \|W_H[Y_{t,H}(Q) - Y_{d,H}]\|_\infty \tag{22}$$

$$\text{s.t.} : \inf_\omega \{\mathrm{Re}[Y_{te}(\mathrm{Q})(j\omega)]\} \ge -\nu. \tag{23}$$

In this paper, we only solve convex problem $\left(P^4\right)$. This problem is infinite-dimensional. To produce a finite-dimensional approximation, $Q \in RH_\infty^{2\times4}$ can be approximated as a linear combination of fixed scalar stable basis functions $Q_i \in RH_\infty^1$, as in

$$Q(X_1, X_2, ..., X_N) = \sum_{i=1}^{N} X_i Q_i,\ X_i \in \mathrm{R}^{2\times4}, \tag{24}$$

where the N real-valued matrices $X_i (i = 1, 2, ..., N)$ are the design parameters. For example, the basis functions can be chosen as all-pass functions

$$Q_i = \left(\frac{s - p}{s + p}\right)^{N-i} \tag{25}$$

for some fixed p with positive real part. The linear approximation in (24) reduces problem $\left(P^4\right)$ to a finite-dimensional *convex* program.

There are several numerical solvers that can be used to solve this program (e.g., Boyd et al., 1988, Polak and Salcudean, 1989; Boyd et al., 1994). Of those, the following were tried:

- Quadratic program based solver: This solver was developed based on functions provided by the Matlab optimization toolbox. Even though it worked but was very slow.

- LMI based solver: Problem (P^4) could also be expressed in terms of state-space linear matrix inequalities (LMI) (Chen and Wen, 1994), which can be solved by interior point methods (Boyd et al., 1994). In our design examples, these methods only worked for low-order approximation of Q in (24). With an increase of the order of the approximated Q in (24), the resulting LMI's for (P^4) became ill-conditioned and therefore failed to produce the desired results.

- Cutting-plane based solver: We found that the cutting-plane algorithm with rules of dropping old constraints developed by (Elzinga and Moore, 1975) is most effective for solving problem (P^4). The cutting-plane method is an iterative algorithm. It is attractive since the subproblem to be solved at each iteration is a simple linear program that changes only sightly from one iteration to the next and no line searches are required. The original cutting-plane algorithm developed by (Kelley, 1960) suffers the size problem, i.e., the number of constraints in the linear program that must be solved at each iteration grows rapidly with the total number of elapsed iterations. The cutting-plane algorithm by (Elzinga and Moor, 1975) drops old constraints, so that the linear programs solved at each iteration does not grow rapidly in size as the algorithm proceeds. Therefore this algorithm was used in the development of the cutting-plane-based solver. To apply the cutting-plane algorithm, we simply approximate the semi-inifinite constraint and objective by discretization, *e.g.*, replacing a H_∞ norm objective by some number of single frequency objectives log-spaced in a specified frequency range. For our design examples, the algorithm by Elzinga and Moor reduced the computation time by an order of magnitude compared with the Kelley's algorithm.

4.2. Design example

The design problem considered here is that of controlling the force-scaling and motion-scaling system, a prototype telerobotic system for use in microsurgery experiments (Yan and Salcudean, 1996). Both force and position need to be scaled down from the operator's hand to the task. The transfer functions mapping force to position for the master $P_m(s)$ and $P_s(s)$ are, respectively,

$$P_m(s) = \frac{1}{0.62s^2 + 3s + 150}, \text{ and} \qquad (26)$$

$$P_s(s) = \frac{1}{0.035s^2 + 0.17s + 8.6}. \qquad (27)$$

We assumed that the operator's hand is a constant mass-spring-damper system with impedance $Z_h = 0.5s + 2.5 + \frac{120}{s}$, and specified the tool's impedance as $Z_d = 0.62s + 3 + \frac{150}{s}$. In this example, we chose the force scaling ratio and the motion scaling ratio as $n_f = 10$ and $n_p = 5$, respectively. The weighting matrix was selected as

$$W_H(s) = \frac{\omega_y}{s + \omega_y} I_{2\times 2},\ \omega_y = 5\pi \text{ rad/sec}, \qquad (28)$$

which reflects the frequency bandwidth of transparency, and is a low-pass filter.

This controller design problem was formulated in the form of (P^2), which was approximated by (P^4) with $N = 10$ and $p = 1$, and was solved by the cutting-plane-based solver. First we solve (P^4) when $\nu = 0$. The obtained controller is robustly stable for any strictly passive environments. To look at transparency of the designed teleoperator, we compute its transmitted impedances to the hand, $Z_{th} = f_h/v_m$, corresponding to three different environment impedances: (a) $Z_e = 0$ (free motion), (b) $Z_e = 5s + 10 + \frac{20}{s}$ (soft environment), and (c) $Z_e = 10s + 100 + \frac{1000}{s}$ (stiff environment) and compare with $Z_d + \frac{n_f}{n_p} Z_e$, where $\frac{n_f}{n_p} = 2$ was obtained from the proposed ideal teleoperator. The Bode plots of the transmitted impedances Z_{th} and $Z_d + 2Z_e$ are presented in Fig. 8 to Fig. 10, from which we can see that the designed teleoperator shows very good transparency in the specified frequency range $(< 5\pi \text{ rad/sec})$. In order to display the trade-offs between performance and stability robustness, the convex problem (P^4) was solved with different values of ν. The performance *vs* robust stability trade-off curve can be plotted as shown in Figure 11. As expected, the performance gets better as the passivity distance increases. Note the big increase in performance index imposed by the passivity requirement.

5. CONCLUDING REMARKS

An optimization method has been used to design a controller for the teleoperation system that has not only good transparency but also robust stability. Formulation of the controller design problem in optimization form was described, the solution procedure was presented, and some computational issues were discussed. A design example demonstrates the effectiveness of the approach.

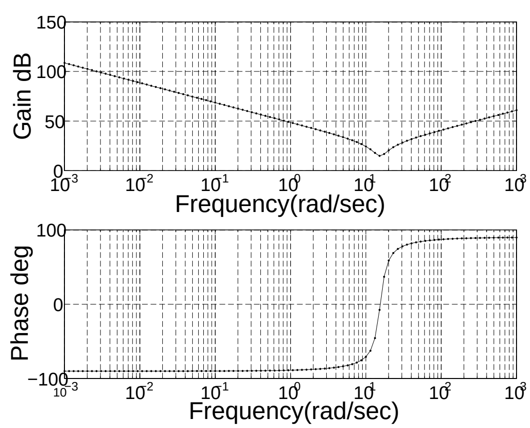

Figure 8: Transmitted impedance Z_{th} (solid line) and $Z_d + 2Z_e$ (dotted line) : with $Z_e = 0$.

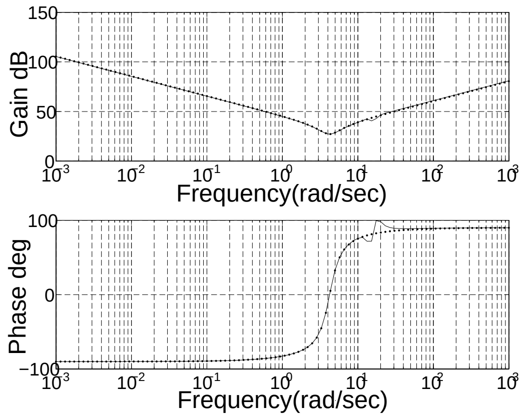

Figure 9: Transmitted impedance Z_{th} (solid line) and $Z_d + 2Z_e$ (dotted line) : with $Z_e = 5s + 10 + \frac{20}{s}$.

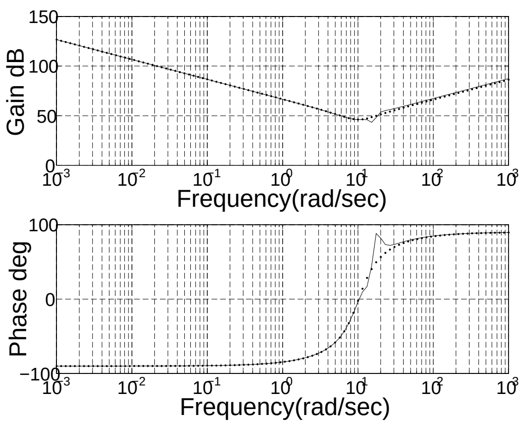

Figure 10: Transmitted impedance Z_{th} (solid line) and $Z_d + 2Z_e$ (dotted line) : with $Z_e = 10s + 100 + \frac{1000}{s}$.

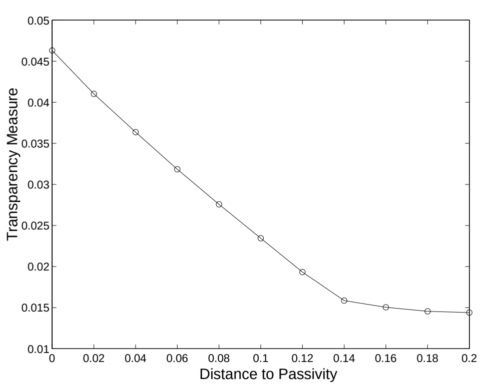

Figure 11: Trade-offs between transparency and robustness

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