Robust Controller Design for Teleoperation Systems

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ABSTRACT

The controller design for a bilateral teleoperation system involves trade-offs between performance and robust stability. Beyond simple intuition, little is known how performance and robust stability trade off. This paper shows that it is possible to achieve robust stability and nominal performance of a bilateral teleoperation system by using a four-channel control architecture [1, 2]. The controller design problem is formulated as a multiple objective optimization problem, which is shown to be convex if parametrizing all stabilizing controllers via the Youla parametrization. Performance specifications, such as kinematic correspondence error, force tracking error, etc., are defined; and robust stability is also incorporated into the controller design. The controller design problem is formulated as a multiple objective optimization problem, which is shown to be convex if parametrizing all stabilizing controllers via the Youla parametrization. The limit of performance achievable with the designed controller, thus the exact form of the trade-offs between performance and robust stability, can be computed numerically. To demonstrate those, this paper treats the design of a controller for a simple one degree-of-freedom (DOF) system model of a motion-scaling teleoperation system.

I INTRODUCTION

A bilateral teleoperation system consists of five interacting subsystems: human operator, master manipulator, controller, slave manipulator and environment as shown in Fig. 1. In any bilateral teleoperation system design, the essential desire is to provide a faithful transmission of signals (positions, velocities, forces) between master and slave to couple the operator as closely as possible to the remote task. The goal of designing a bilateral teleoperation controller is to make the system stable and achieve optimal performance in the possible presence of time delays, plant disturbances, measurement noise and modelling errors.

The conventional teleoperation control schemes such as position-position and position-force schemes provide poor transparency, even at low frequencies, and poor stability properties. Recent work in teleoperation controller design has focused more on stability or/and performance, and could be categorized as stability-optimized scheme, tansparency-optimized scheme or some combinations of the two.

Passivity theory is the basis for the modifications of basic position-position and position-force schemes presented in [3, 4] to deal with time delays. Robust control ideas based on small gain theory motivate the force-force scheme proposed in [5]. Transparency performance of these teleoperation systems are not presented. Some experimental testing of the passivity concept in [3] reveals that the stability guarantee comes at the expense of the reduced stiffness, resulting in poor transparency [6].

The problem of achieving performance is also difficult, in large part because performance specifications are also likely to change with the environment. However, various control schemes have been concentrated on performance. Objectives based on specifying network theory hybrid parameters are discussed in [7]; the network hybrid parameter design problem in [8] is formulated in terms of a transparency objective, and suggests a position-position approach; a four-channel control structure has been suggested to achieve transparency in [2] and ideal response of teleoperation in [9]; H_∞–optimization theory has been used to best shape the interested closed-loop responses in [10, 1] and to shape the relationships between forces and positions at both ends of the teleoperator in [11]. However none of the above work has explicitly incorporated robust stability into the controller design.

Both robust stability and performance are treated in [12] and [13] respectively. In [12], a combined H_∞–optimization and μ–synthesis framework are used to design a teleoperator which is stable for a pre-specified time delay and fixed operator and environment impedances while optimizing performance specifications. Concepts of passivity, impedance control and H_∞–optimization theory are used in [13] to formulate the controller design as a semi-infinite optimization problem. Two criteria, "transparency distance", and "passivity distance" [14] are employed. Unfortunately, both resulting designs are not convex in design parameters, and therefore the limit of the achievable performance and the exact trade-offs between stability and performance can not be obtained.

The goal of this paper is to design a teleoperation controller that can achieve the optimal performance while maintaining the stability against any passive environment. By using the four-channel H_∞–control framework proposed by Yan and Salcudean in [1], the performance specifications, such as kinematic correspondence error, force tracking error, etc., are defined as closed-loop frequency responses. Based on passivity theory [15] and the necessary and sufficient condition for ensuring the stability of a linear system to a passive environment [16, 14], the robust stability as a design constraint is also incorporated into the controller design. After parametrizing all stabilizing controllers via the Youla parametrization, the controller design problem is formulated as a constrained multiple objective optimization problem, which is convex and numerically solvable.

In the following section, we review some basic passivity concepts and stability conditions of a linear time invariant (LTI) system coupled to passive but otherwise arbitrary environments, and apply the results to teleoperation systems. In Section III, the controller design problem is formulated and a numerical solution procedure is described. In Section IV, we give a design example of a controller for a 1-DOF teleoperation system and demonstrate the trade-offs between stability and performance. Some concluding remarks are included in the final section.

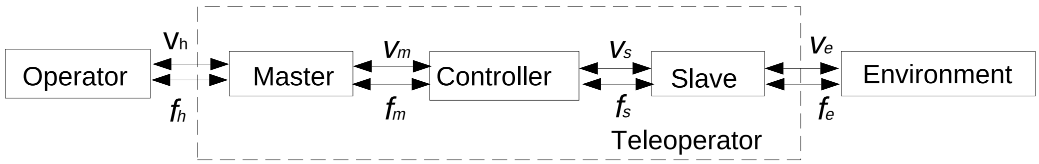

Figure 1 General Teleoperation System

II. ROBUST STABILITY OF TELEOPERATION SYSTEMS

To address the robust stability problem of teleoperation systems, we first review some basic concepts of passivity, then we provide stability conditions for an LTI system coupled to a strictly passive (but otherwise arbitrary) environment. Those conditions will provide useful design constraints for the development of robust controllers for teleoperation. The notation and conventions follow closely those of Desoer and Vidyasagar [15].

For $f: R \to R^n$, define the truncation function

$$f_T(t) = \begin{cases} f(t), & t \le T, \\ 0, & t > T. \end{cases} \quad (1)$$

Define

$$L_{2e} = \{f | f_T \in L_2[0, \infty), \forall T \ge 0\}, \quad (2)$$

and for all $x, y \in L_{2e}, \forall T \ge 0$,

$$\langle x, y \rangle_T = \int_{-\infty}^{T} x^{\tau}(t) y(t) dt. \quad (3)$$

Definition 1: [15] Let $H: L_{2e} \to L_{2e}$. H is *passive* if and only if $\exists\ \beta \in R$ such that

$$\langle Hx, x \rangle_T \ge \beta,\ \forall x \in L_{2e}. \quad (4)$$

H is *strictly passive* if and only if $\exists \delta > 0$ and $\exists \beta \in R$ such that

$$\langle Hx, x \rangle_T \ge \delta \langle x, x \rangle_T + \beta,\ \forall x \in L_{2e}. \quad (5)$$

Theorem 1: [15] Let $n = 1$, $H: L_{2e} \to L_{2e}$ and be defined by the stable and causal convention operator h as

$$Hu = h * u = \int_{-\infty}^{t} h(t - \tau) u(\tau) d\tau, \quad (6)$$

where, $u \in L_{2e}$. Then we have

(1) H is *passive* if and only if $\mathrm{Re}\left[\widehat{h}(\mathrm{j}\omega)\right] \ge 0, \forall \omega \in R$;

(2) H is *strictly passive* if, for some $\delta > 0$, $\mathrm{Re}\left[\widehat{h}(\mathrm{j}\omega)\right] \ge \delta, \forall \omega \in R$.

Theorem 2: [15] Let $H: L_{2e} \to L_{2e}$ be an $n \times n$ matrix whose elements are stable and causal, and be defined by

$$Hu = H * u, \forall u \in L_{2e}. \quad (7)$$

Then we have

(1) H is *passive* if the Hermitian matrix $\widehat{H}(\mathrm{j}\omega) + \widehat{H}^*(\mathrm{j}\omega) \ge 0, \forall \omega \in R$;

(2) H is *strictly passive* if, for some $\delta > 0$, $\widehat{H}(\mathrm{j}\omega) + \widehat{H}^*(\mathrm{j}\omega) \ge \delta > 0, \forall \omega \in R$.

Definition 2: For an LTI n-port network, the impedance matrix Z is defined as the map from v to f by $f = Zv$; the admittance matrix Y is the map from f to v by $v = Yf$; and the scattering matrix S is the map from the input wave $a := (f + v)/2$ to output wave $b := (f - v)/2$, i.e., satisfying the equation $b = S(s)a$.

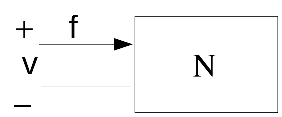

Figure 2 n-port network

Theorem 3: [16] The necessary and sufficient condition for the stability of an LTI 1–port network (Fig. 3), represented by an admittance Y, coupled to an arbitrary strictly passive environment, represented by an impedance Z, is that the LTI 1–port to be passive, or equivalently,

$$\mathrm{Re}[Y(\mathrm{j}\omega)] \ge 0, \forall \omega \in R. \quad (8)$$

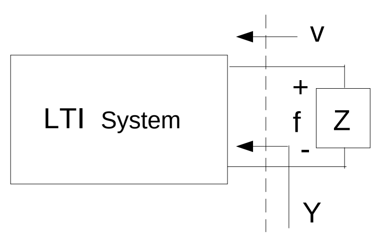

Figure 3 A representation of an LTI 1–port network, Y, coupled to environment, Z.

Definition 3: [14] The ν–index, also referred to as *passivity distance*, is defined as the distance of a stable LTI system to strict passivity. Let the system transfer function be $T(s)$, then

$$\nu \triangleq - \inf_{\omega \in \mathbb{R}} \{\mathrm{Re}[T(\mathrm{j}\omega)]\}. \tag{9}$$

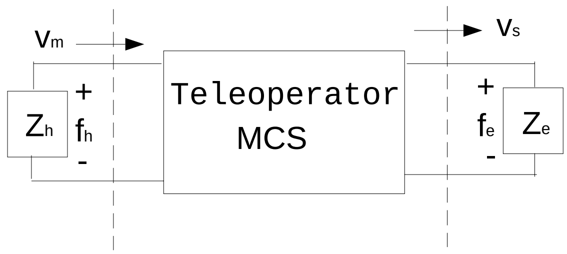

Figure 4 Two-port representation of a bilateral teleoperation system

Now let's model the teleoperator (master-controller-slave, MCS) as a controlled LTI system and assume that both operator and environment are strictly passive (but otherwise arbitrary) as shown in Fig. 4, then the robust stability criterion for the teleoperation system can be defined using the structured singular value as follows [17]:

$$\sup_{\omega} \{\mu(S_T(j\omega))\} \leq 1, \tag{10}$$

where, S_T is the scattering matrix describing the teleoperator. This criterion is less conservative than that of passivity, and is broadly applicable to robustness analysis; however, because of the non-convexity in design parameters it is hard to incorporate into the controller design.

In practice, the variation of the operator hand impedance is relatively small if compared with that of the environment impedance. To design a less conservative controller for teleoperation, we may assume that the operator hand impedance is fixed. Under this assumption, the teleoperation system can be modelled as a controlled LTI 1–port network, Y_T, coupled to any strictly passive environment impedance Z_e, as shown in Fig. 3; by Theorem 3, (8) is the necessary and sufficient condition for the coupled stability; and therefore the robustness criterion for the teleoperation system becomes:

$$\mathrm{Re}[Y_T(j\omega)] \geq 0, \forall \omega \in R. \tag{11}$$

It will be shown in the next section that this robustness criterion is convex in design parameters.

III PROBLEM FORMULATION AND SOLUTION

System modelling: Consider 1-DOF master and slave models in terms of Laplace-transforms:

$$\mathrm{Master}: x_m = P_m(f_h + f_m), \tag{12}$$

$$\mathrm{Slave}: x_s = P_s(f_e + f_s). \tag{13}$$

x_m and x_s are, respectively, the master and slave positions, P_m and P_s are, respectively, the master and slave plants after using the local stabilizing controllers, f_h denotes the force that the operator applies to the master, and f_e denotes the force that the environment applies to the slave. Actuator driving forces are represented by f_m f_s, respectively.

The dynamics of the operator interacting with the master is modelled by the following linear system:

$$f_h = f_{ha} - Z_h s x_m = f_{ha} - H x_m, \tag{14}$$

where Z_h denotes the operator hand impedance, $H \triangleq sZ_h$,and f_{ha} denotes an active exogenous force.

Similarly the dynamics of the environment interacting with the slave is modelled by

$$f_e = f_{ea} + Z_e s x_s = f_{ea} - E x_s, \tag{15}$$

where Z_e denotes the environment impedance, $E \triangleq sZ_e$, and f_{ea} denotes the active exogenous force.

It should be noted that both the hand impedance Z_h and the environment impedance Z_e may change during operation and therefore neither is constant. But the change of Z_h is relatively small when compared with that of Z_e.

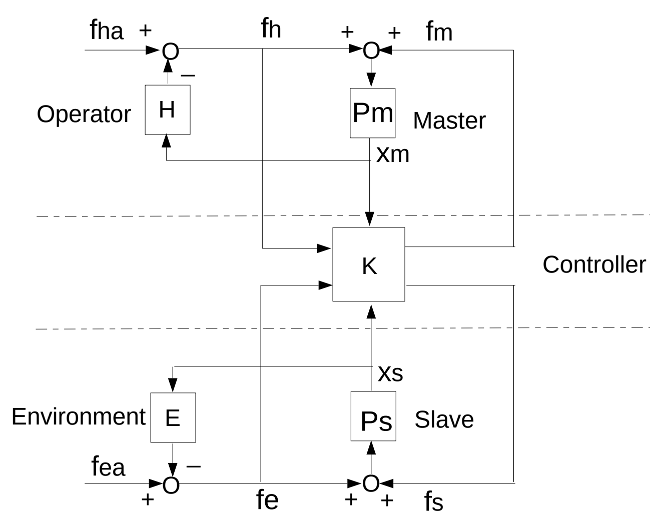

Figure 5 Four channel control structure.

Framework for controller design: We assume that positions and forces from both master and slave are available and therefore we can use the four channel control structure proposed in [1, 2] and illustrated in Figure 5. The controller takes forces and positions from both master and salve, which gives sufficient freedom to shape various closed-loop frequency responses of interest as shown in [1]. For simplicity, we assume that the hand impedance Z_h is fixed and we do not consider the master and salve modelling errors, the measurement noise, the control disturbances and the time delays.

Optimization problem formulation: The above framework can be transformed into the standard H_∞–optimization setup shown in Fig. 6, to define the performance specifications and the robustness constraint. There are four vector-valued signals of interest in the standard setup: the plant exogenous inputs w, the compensator control signals u, the measurements y used in the control law and the outputs of interest z, mostly error signals to be minimized. The generalized plant G and controller K are assumed to be proper, real-rational transfer matrices. Frequency dependent weighting functions which characterize the desired behavior are assumed to be absorbed in the plant G.

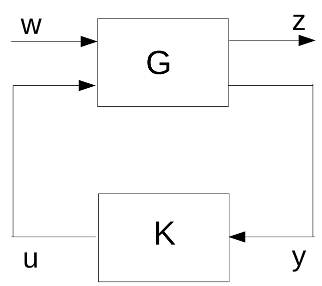

Figure 6 Standard H_∞–optimization problem

Define the signals

$w = [f_{ha}\ f_e]^T$,
$u = [f_m\ f_s]^T$,
$y = [f_h\ f_e\ x_m\ x_s]^T$, and a vector z of signals of interest. As an example, a possible set of components in z are:

- $z_1 = W_1(f_h - k_f f_e)$, *i.e.,* the *force tracking error* for some frequency range of interest. Since it is more important at lower frequencies, W_1 is chosen to be a low-pass filter. k_f is the force scaling ratio from the environment to the hand.
- $z_2 = W_2(x_s - G_c x_m)$, *i.e.,* the *kinematic correspondence error*. If $G_c = k_p$, where k_p is a constant, then the system is in *position control* mode; If $G_c = k_r/s$, where k_r is a constant, then the system is in *rate control* mode. Again, W_2 is chosen to be a low-pass filter.
- $z_3 = sx_s \triangleq v_s$: this is an unweighted output, which can be used to define the *robust stability constraint* since the map from f_e to v_s is the transmitted admittance 'felt' by the environment.

Then we have

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} W\overline{G}_{zw} & W\overline{G}_{zu} \\ G_{yw} & G_{yu} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}, \tag{16}$$

where,

$$\overline{G}_{zw} = \begin{bmatrix} 1 - HP_H & -k_f \\ -G_c P_H & P_s \\ 0 & sP_s \end{bmatrix}, \tag{17}$$

$$\overline{G}_{zu} = \begin{bmatrix} -HP_H & 0 \\ -G_c P_H & P_s \\ 0 & sP_s \end{bmatrix}, \tag{18}$$

$$G_{yw} = \begin{bmatrix} 1 - HP_H & 0 \\ 0 & 1 \\ P_H & 0 \\ 0 & P_s \end{bmatrix}, \tag{19}$$

and

$$G_{yu} = \begin{bmatrix} -HP_H & 0 \\ 0 & 0 \\ P_H & 0 \\ 0 & P_s \end{bmatrix}. \tag{20}$$

Here, for notational convenience, $W \triangleq \mathrm{diag}(W_1, W_2, 1)$, and $P_H \triangleq P_m/(1 + P_m H)$.

To achieve the performance specifications while ensuring the robust stability against *any strictly passive environment* Z_e, we have to shape the following closed-loop transfer functions:

- $T_1(K)$: from w to z_1. $||T_1(K)||_\infty$ should be minimized to get good force transparency at the master.
- $T_2(K)$: from w to z_2. $||T_2(K)||_\infty$ should be minimized to get good kinematic correspondence.
- $T_3(K)$: from f_e to z_3. This transfer function is the transmitted admittance 'felt' by the environment. Since we have assumed that the hand impedance Z_h is fixed, the teleoperation system will be stable when in contact with an arbitrary strictly passive environment impedance if and only if $\mathrm{Re}[T_3(K)(j\omega)] \geq 0, \forall\omega \in R$, *i.e.,* the environment at the slave "sees" a passive one-port.

Therefore the problem of designing a robustly stable controller for the teleoperation system is a constrained multiple objective optimization problem as follows:

$$(P^0): \min_{stabilizing\ K} \max\{\|T_1(K)\|_\infty, \|T_2(K)\|_\infty\} \tag{21}$$

$$\text{s.t.}: \inf_\omega \{\mathrm{Re}[T_3(\mathrm{K})(j\omega)]\} \geq -\nu. \tag{22}$$

The positive parameter ν is used to ensure a given distance to passivity defined as in (9) and determines the degree of conservatism of the design.

As shown in [18, 19, 20], the set of all controllers K that stabilize a given plant G is given parametrically as follows:

$$K(s) = (Y - MQ)(X - NQ)^{-1}, X - NQ \neq 0. \tag{23}$$

Here $Q(s)$ is the parameter, free to range over all stable real-rational transfer functions, while $N, M, X,$ and Y are fixed stable real-rational transfer functions chosen to satisfy the coprime factorization conditions

$$G = NM^{-1} \text{ and } XM - YN = I. \tag{24}$$

This is called the Youla parametrization.

By introduction of the Youla parametrization of the stabilizing controllers, $T_i\,(i \in \underline{3})$ are affine transfer functions in $Q \in RH_\infty$, i.e.,

$$T_i = T_{i,1} + T_{i,2}QT_{i,3},\ i \in \underline{3}, \tag{25}$$

where $T_{i,1}, T_{i,2}$ and $T_{i,3}$, $i \in \underline{3}$, are known proper and stable transfer functions, which can be obtained directly from the plant G. It can be shown that the scattering operator, $S_T : (f_e + v_s) \rightarrow (f_e - v_s)$ is not affine in $Q \in RH_\infty$, so $\|S_T\|_\infty \leq 1$ is not a convex constraint, and neither is $\sup_\omega\{\mu(S_T(\mathrm{j}\omega))\} \leq 1$. But the transmitted admittance Y_T :

$f_e \rightarrow v_s$ is affine in $Q \in RH_\infty$, which makes the robustness criterion (11) convex. This is why we take v_s as an output signal in the setup.

The problem (P^0) is equivalent to

$$(P^1): \min_{Q \in RH_\infty} \max \{ \|T_1(Q)\|_\infty, \|T_2(Q)\|_\infty \} \quad (26)$$

$$\text{s.t.}: \inf_\omega \{ \mathrm{Re}[T_3(Q)(j\omega)] \} \geq -\nu. \quad (27)$$

Numerical solution: Problem (P^1) in (26)-(27) is infinite-dimensional. To produce a finite-dimensional approximation, $Q \in H_\infty$ can be approximated as a linear combination of fixed stable basis functions $Q_i \in RH_\infty$, as in

$$Q(X_1, X_2, ..., X_N) = \sum_{i=1}^{N} X_i Q_i, \; X_i \in \mathrm{R}^{4 \times 2}, \quad (28)$$

where the N real-valued matrices $X_i (i = 1, 2, ..., N)$ are the design parameters, and the basis functions can be chosen as all-pass functions

$$Q_i = \left(\frac{s-p}{s+p} \right)^{N-i}, \mathrm{Re}(p) > 0, \quad (29)$$

as in [21].

The linear approximation in (28) reduces problem (P^1) to a finite-dimensional *convex* program. There are some numerical solvers specifically developed to solve this program [21, 22]. We will use a cutting-plane-based solver, which we developed in frequency domain, for a design example in the next section.

IV DESIGN EXAMPLE

The design problem considered here is that of controlling the force-scaling and motion-scaling system, a prototype telerobotic system for use in microsurgery experiments [1].

The transfer functions mapping force to position for the master $P_m(s)$ and $P_s(s)$ are, respectively,

$$P_m(s) = \frac{1}{0.62s^2 + 3s + 150}, \quad (30)$$

and

$$P_s(s) = \frac{1}{0.035s^2 + 0.17s + 8.6}. \quad (31)$$

We assume that the operator's hand is a constant mass-spring-damper system with impedance :

$$Z_h = 0.5s + 5 + \frac{20}{s}. \quad (32)$$

In this example, we chose the force scaling ratio and the motion scaling ration as $k_f = 10 \text{ and } G_c = 1/10$, respectively. The weighting functions are selected as

$$W_1(s) = \frac{0.01(s+100)^2}{(s+25)^2}, \quad (33)$$

and

$$W_2 = \frac{(s+40)^2}{(s+10)^2}, \quad (34)$$

which reflect the frequency bandwidths of force transparency and kinematic correspondence, and are low-pass filters. For example, for the kinematic correspondence, we want the error to be low for frequencies below 10 rad/sec but allow errors above 40 rad/sec.

In order to display the trade-offs between performance and stability robustness, the convex problem (P^1) was solved with different values of ν. If we define the performance index as $\gamma = \max \{ \|T_1(Q)\|_\infty, \|T_2(Q)\|_\infty \}$, the performance *vs* robust stability trade-off curve can be plotted as shown in Figure 7. As expected, the performance gets worse as the passivity distance increases. The results for two extreme designs: one without any passivity constraint ($\nu = -\infty$ in (27)) and the other with passivity constraint ($\nu = 0$ in (27)) are shown in Figure 8. Note the drastic reduction in performance index imposed by the passivity requirement.

V CONCLUDING REMARKS

In this paper, a controller design approach for teleoperation systems that optimizes performance subject to robust stability for all passive environments is proposed. The transmitted admittance is used to define the robustness criterion, which is shown to be easily incorporated into the controller design. With the four channel control structure, the controller design problem can be formulated as a convex optimization problem with several competing objectives. By defining passivity distance as a robustness measure, the trade-off between performance and robustness can be clearly displayed. Even though the convex-optimization-based controller design method presented here is for 1–DOF systems, it is certainly applicable to multi-DOF systems as well. However, there is much work to do in the future, especially the work on simulation and experimentation.

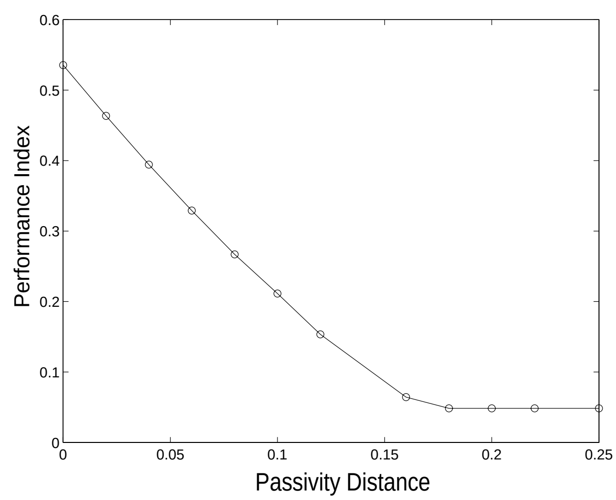

Figure 7 Trade-offs between Performance and Robustness

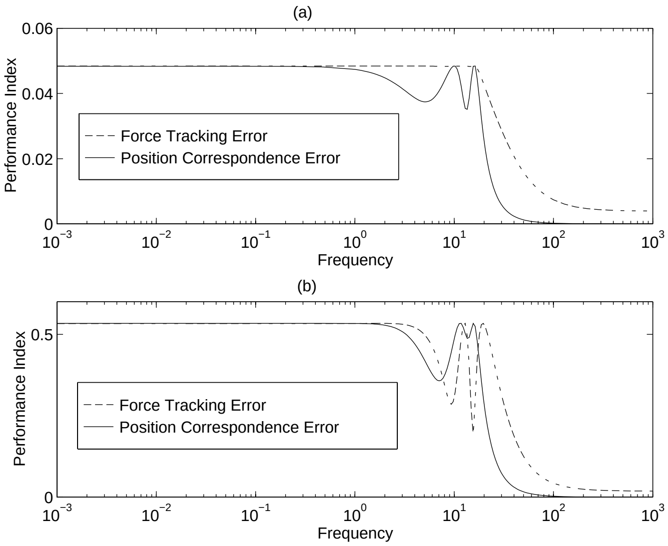

Figure 8 Controller design results: (a) without robustness, and (b) with robustness

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