EECE251
Circuit Analysis I
Lecture Integrated Program
Set 1: Basic Circuit Concepts and Elements

Shahriar Mirabbasi
Department of Electrical and Computer Engineering
University of British Columbia
shahriar@ece.ubc.ca
Course Material

• Lecture notes

• Required textbook: None

• A good reference:
    by Charles Alexander and Matthew Sadiku

• End of chapter problems for this book are available at bookstore (many assignment questions will be out of these problems).
Evaluation

• Assignments (8 to 10 assignments) 10%

• Quizzes (4) 30%

• Final Exam 60%
Calculator!

• HP 50g
Overview

In this slide set we will review basic concepts, electrical quantities and their units, circuit elements, basic circuit laws, and methods of circuit analysis.

What is an Electric Circuit?

• According to the book by Alexander and Sadiku:
  “In electrical engineering, we are often interested in communicating or transferring energy from one point to another.”

  “To do this requires an interconnection of electrical devices.”

  “An electric circuit is an interconnection of electrical elements.”

• Typical circuit elements that we will see in this year:
batteries or voltage sources, current sources, resistors, capacitors, inductors, diodes, transistors, operational amplifiers, …
What is a Circuit?

• According to Merriam-Webster Dictionary:
  “The complete path of an electric current including usually the source of electric energy.”

• According to Encyclopedia Britannica:
  “Path that transmits electric current.”

“A circuit includes a battery or a generator that gives energy to the charged particles; devices that use current, such as lamps, motors, or electronic computers; and connecting wires or transmission lines. Circuits can be classified according to the type of current they carry (see alternating current, direct current) or according to whether the current remains whole (series) or divides to flow through several branches simultaneously (parallel). Two basic laws that describe the performance of electric circuits are Ohm's law and Kirchhoff's circuit rules."
A Simple Circuit
A More Complicated Circuit

A Radio Receiver
Review of Basic Circuit Concepts

- Electric Charge is the basis for describing all electrical phenomena.
- **Charge** is an electrical property of the atomic particles of which matter consists and is measured in coulombs (Charles Augustin de Coulomb (1736-1806) a French Scientist)

- Inside an atom, there is negative charge on electrons, positive charge on protons and no charge on neutrons.

- The charge of an electron is equal to that of an proton and is: 
  \[ e = 1.602 \times 10^{-19} \text{ C} \]
• Note that in 1C of charge there are:
  \[ 1/ 1.602 \times 10^{-19} = 6.24 \times 10^{18} \text{ electrons} \]

• Laboratory values of charges are more likely to be a fraction of a Coulumb (e.g., pC, nC, \(\mu\)C, or mC). What are these prefixes?!

• \textit{Law of conservation of charge}: charge can neither be created nor destroyed, only transferred. (This is a law in classical physics and may not be true in some odd cases!. We are not dealing with those cases anyway.)

• Electrical effects are attributed to both separation of charges and/or charges in motion!
A Material Classification

• **Conductor:** a material in which charges can move to neighboring atoms with relative ease.
  – One measure of this relative ease of charge movement is the electric resistance of the material
  – Example conductor material: metals and carbon
  – In metals the only charged particles that can move are electrons

• **Insulator:** a material that opposes the charge movement (ideally infinite opposition, i.e., no charge movement)
  – Example insulators: Dry air and glass

• **Semi-conductor:** a material whose conductive properties are somewhat in between those of conductor and insulator
  – Example semi-conductor material: Silicon with some added impurities
Electric Current (Charges in Motion!)

- **Current**: net flow of charge across any cross section of a conductor, measured in Amperes (Andre-Marie Ampere (1775-1836), a French mathematician and physicist)

- Current can be thought of as the rate of change of charge:

  \[ i = \frac{dq}{dt} \]
Electric Current

- Originally scientists (in particular Benjamin Franklin (1706-1790) an American scientist and inventor) thought that current is only due to the movement of positive charges.

- Thus the direction of the current was considered the direction of movement of positive charges.
Electric Current

• In reality in metallic conductors current is due to the movement of **electrons**, however, we follow the universally accepted convention that current is in the direction of positive charge movement.

• Two ways of showing the same current:
Two Important Types of Current

- Direct current (DC) is a current that remains constant with time.
- Alternating current (AC) is a current that varies sinusoidally with time.
Voltage (Separation of Charge)

- **Voltage** (potential difference) is the energy required to move a unit charge through a circuit element, and is measured in Volts (Alessandro Antonio Volta (1745-1827) an Italian Physicist).

\[ v = \frac{dW}{dq} \]

- Similar to electric current, there are two important types of voltage: DC and AC
Voltage Polarity

• The plus (+) and minus (-) sign are used to define voltage polarity.

• The assumption is that the potential of the terminal with (+) polarity is higher than the potential of the terminal with (-) polarity by the amount of voltage drop.

• The polarity assignment is somewhat arbitrary! Is this a scientific statement?!! What do you mean by arbitrary?!!
Voltage Polarity

• Figures (a) and (b) are two equivalent representation of the same voltage:

![Diagrams](image)

(a)  
(b)  

• Both show that the potential of terminal a is 9V higher than the potential of terminal b.
Power

- The rate of change of (expending or absorbing) energy per unit time, measured in Watts (James Watt (1736-1819) a Scottish inventor and mechanical engineer)

\[ p = \frac{dW}{dt} = \frac{dW}{dq} \times \frac{dq}{dt} = vi \]
A Classification of Circuit Components

- One common classification for circuit components is to group them in two major groups:

  1) Passive components or passive elements
     Components or elements that absorb power.

  2) Active components or active elements
     Components that are not passive! that is, components that deliver power.
Passive Sign Convention

- For calculating absorbed power: The power absorbed by any circuit element with terminals A and B is equal to the voltage drop from A to B multiplied by the current through the element from A to B, i.e., $P = V_{ab} \times I_{ab}$

- With this convention if $P \geq 0$, then the element is absorbing (consuming) power. Otherwise (i.e., $P < 0$) is absorbing negative power or actually generating (delivering) power.

- **Principle of Conservation of the Power (Energy):** The algebraic sum of the powers absorbed by all elements in a circuit is zero at any instance of time ($\Sigma P = 0$). That is, the sum of absorbed powers is equal to the sum of generated powers at each instance of time.
Passive Sign Convention

- Calculate the power absorbed or supplied by each of the following elements:
Energy Calculation

- Instantaneous power: \( p(t) = v(t)i(t) \)

- Energy absorbed or supplied by an element from time \( t_0 \) to time \( t > t_0 \)

\[
W = W(t_0, t) = \int_{t_0}^{t} p(\tau)d\tau = \int_{t_0}^{t} v(\tau)i(\tau)d\tau
\]
Ideal Voltage and Current Sources

• Independent sources: An ideal independent source is an active element that provides a specified voltage or current that is independent of other circuit elements and/or how the source is used in the circuit.

• Symbol for independent voltage source
  (a) Used for constant or time-varying voltage
  (b) Used for constant voltage (dc)

Question: Plot the v-i characteristic of the above dc source.
Ideal Voltage and Current Sources

- Equivalent representation of ideal independent current sources whose current $i(t)$ is maintained under all voltage requirements of the attached circuit:

- What is the equivalent of the ideal voltage source shown on the previous slide (Figure (a))?
Common Voltage and Current Source Labeling

- Is this different from passive sign convention?
- Can we use the passive convention for sources
Ideal Dependent (Controlled) Source

- An ideal dependent (controlled) source is an active element whose quantity is controlled by a voltage or current of another circuit element.

- Dependent sources are usually presented by diamond-shaped symbols:

![Diamond symbols for voltage (a) and current (b)]
Dependent (Controlled) Source

- There are four types of dependent sources:

  - Voltage-controlled voltage source (VCVS)
    
    \[
    V_s(t) = \alpha V(t)
    \]

  - Current-controlled voltage source (CCVS)
    
    \[
    V_s(t) = \beta I(t)
    \]
Dependent (Controlled) Source

- Voltage-controlled current source (VCCS)

\[ I(t) = V(t) + I(t) \]

- Current-controlled current source (CCCS)

\[ I_s(t) = \gamma V(t) \]

\[ I_s(t) = \delta I(t) \]
Example: Dependent Source

- In the following circuits, identify the type of dependent sources:

\[ +5 \text{ V} \]

\[ \begin{array}{c}
\text{A} \\
\text{B}
\end{array} \]

\[ +10i \]

\[ \begin{array}{c}
\text{C}
\end{array} \]

\[ +v_s \]

\[ \begin{array}{c}
\text{vs} \\
\text{i_o}
\end{array} \]

\[ +6i_o \]
Example: Power Calculation

• Compute the power absorbed or supplied by each component in the following circuit.
Resistance

• Different material allow charges to move within them with different levels of ease. This physical property or ability to resist current is known as resistance.

• The resistance of any material with a uniform cross-sectional area $A$ and length $l$ is inversely proportional to $A$ and directly proportional to $l$. 
Resistance

- The constant of the proportionality is the resistivity of the material, i.e., $\rho$

\[ R \propto \frac{l}{A} \]

\[ R = \rho \frac{l}{A} \]
Resistance

- In honor of George Simon Ohm (1787-1854), a German physicist, the unit of resistance is named Ohm (Ω).

- A conductor designed to have a specific resistance is called a resistor.
Ohm’s Law

- The voltage $v$ across a resistor is directly proportional to the current $i$ flowing through the resistor. The proportionality constant is the resistance of the resistor, i.e., $v(t) = Ri(t)$.

- One can also write:

$$i(t) = \frac{1}{R} v(t)$$

- Instantaneous power dissipated in a resistor

$$p(t) = v(t)i(t) = \frac{v^2(t)}{R} = Ri^2(t)$$
Linear and Nonlinear Resistors

- Linear resistor

- Nonlinear resistor

- In this course, we assume that all the elements that are designated as resistors are linear (unless mentioned otherwise)
Resistors (Fixed and Variable)

- Fixed resistors have a resistance that remains constants.
- Two common type of fixed resistors are:
  (a) wirewound
  (b) composition (carbon film type)
Fixed Resistors

• Inside the resistor

• A common type of resistor that you will work with in your labs:
  • It has 4 color-coded bands (3 for value and one for tolerance)
    – How to read the value of the resistor?
Variable Resistors

- Variable resistors have adjustable resistance and are typically called potentiometer (or pot for short).

- Potentiometers have three terminals one of which is a sliding contact or wiper.
Conductance

• $G = 1/R$ is called the conductance of the element and is measured in siemens (S) or mho ($\Omega$).

German inventor
Ernst Werner von Siemens
(1816-1892)

• Conductance is the ability of an element to conduct current.

• A device with zero (no) resistance has infinite conductance and a device with infinite resistance has zero conductance.
Short and Open Circuits

- A device with zero resistance is called short circuit and a device with zero conductance (i.e., infinite resistance) is called open-circuit.
Capacitors

- A capacitor consists of two conductive plates separated by an insulator (or dielectric).

- Capacitors store charge and the amount of charge stored on the capacitor is directly proportional to the voltage across the capacitor. That is:

\[ q_C(t) = C v_C(t) \]

and the constant of proportionality is the capacitance of the capacitor.

- Capacitor stores energy in its electric field.
Capacitors

Dielectric with permittivity $\varepsilon$

Metal plates, each with area $A$

$$C = \frac{\varepsilon A}{d}$$

Model for a non-ideal capacitor
Capacitors

- In honor of Michael Faraday (1791-1867), an English chemist and physicist, the unit of capacitance is named Farad (F).

- The voltage-current relationship of the capacitor is:

\[ i_C(t) = C \cdot \frac{dv_C(t)}{dt} \quad \text{why?} \]

\[ v_C(t) = \frac{q(t)}{C} = \frac{1}{C} \int_{-\infty}^{t} i_C(\tau) \cdot d\tau \quad \text{or} \quad v_C(t) = \frac{1}{C} \int_{t_0}^{t} i_C(\tau) \cdot d\tau + v_C(t_0) \]
Capacitors

• Note that;
  – A capacitor acts as an open circuit when connected to a DC voltage source
  – A capacitor impede the abrupt change of its voltage

• The instantaneous power absorbed by the capacitor is:

\[ p_c(t) = i_c(t) v_c(t) = C \frac{dv_c(t)}{dt} v_c(t) \]

and the total stored energy in the capacitor is:

\[ W_c(t) = \int_{-\infty}^{t} p_c(\tau) \cdot d\tau = \int_{-\infty}^{t} C v_c(\tau) dv_c(\tau) = \frac{1}{2} C v_c^2(t) \]
Inductors

• An inductor is typically a coil of conducting wire.
• Inductor stores energy in its magnetic field.

\[ i_L(t) \]
\[ \begin{array}{c}
\text{L} \\
\end{array} \]
\[ + \]
\[ v_L(t) \]

• If current passes through an inductor the voltage across the inductor is directly proportional to the time rate of change of the current:

\[ v_L(t) = L \cdot \frac{di_L(t)}{dt} \]

The constant of proportionality is the inductance of the inductor.

\[ i_L(t) = \frac{1}{L} \int_{-\infty}^{t} v_L(\tau) \cdot d\tau \quad \text{or} \quad i_L(t) = \frac{1}{L} \int_{t_0}^{t} v_L(\tau) \cdot d\tau + i_L(t_0) \]
Inductors

\[ L = \frac{N^2 \mu A}{l} \]

Model for a non-ideal inductor
Inductors

• In honor of Joseph Henry (1797-1878), an American physicist, the unit of inductance is named Henry (H).

• Note that:
  – An inductor acts like a short circuit to DC current.
  – Inductor impede instantaneous changes of its current.
• Instantaneous power delivered to the inductor is:

\[
p_L(t) = v_L(t) i_L(t) = L \frac{di_L(t)}{dt} i_L(t)
\]

The total stored energy is:

\[
W_L(t) = \int_{-\infty}^{t} p_L(\tau) \cdot d\tau = \int_{-\infty}^{t} L i_L(\tau) di_L(\tau) = \frac{1}{2} L i_L^2(t)
\]
Terminology (Nodes and Branches)

• **Note:** our definition of nodes (and branches) is slightly different from traditional definitions used in the textbooks!

• Please note that almost all components that we deal with in this course are two-terminal components (resistors, capacitors, inductors, sources, …)

• A **true node** (or node for short) is the point of connection of three or more circuit elements. (The node includes the interconnection wires.)

• A **binary node** (or b-node for short) has only two components connected to it.
Example

• In the following circuit identify the nodes (and their types).
Branch

• A branch is a collection of elements that are connected between two “true nodes” that includes only those two true nodes (and does not include any other true nodes).

• In our example:
Loop

- A “loop” is any closed path in the circuit that does not cross any true node but once.

- A “window pane loop” is a loop that does not contain any other loops inside it.

- An “independent loop” is a loop that contains at least one branch that is not part of any other independent loop.
Example

- In the following circuit, find the number of branches, nodes, and window pane loops. Are the window pane loops independent?
Series and Parallel Connections

- Two or more elements are connected “in series” when they belong to the same branch (even if they are separated by other elements).
- In general, circuit elements are in series when they are sequentially connected end-to-end and only share binary nodes among them.
- Elements that are in series carry the same current.
Series and Parallel Circuits

- Two or more circuit elements are “in parallel” if they are connected between the same two “true nodes”.
- Consequently, parallel elements have the same voltage.

![Parallel Circuit Diagram]
Kirchhoff’s Current Law (KCL)

- Gustav Robert Kirchhoff (1824-1887), a German physicist, stated two basic laws concerning the relationship between the currents and voltages in an electrical circuit.

- **KCL:** The algebraic sum of the currents entering a node (or a closed boundary) is zero.

- The current entering a node may be regarded as positive while the currents leaving the node may be taken as negative or vice versa.
KCL

• KCL is based on the law of conservation of charge.

• Example: Write the KCL for the node A inside this black box circuit:

![Black box circuit](image-url)
• **Alternative statement of KCL:** For lumped circuits, the algebraic sum of the currents leaving a node (or a closed boundary) is zero.

• Can you think of another statement for KCL?
The sum of the currents entering a node is equal to the sum of the currents leaving that node.

\[ \Sigma i_{in} = \Sigma i_{out} \]
Closed Boundary

• A closed boundary is a closed curve (or surface), such as a circle in a plane (or a sphere in three dimensional space) that has a well-defined inside and outside.
• This closed boundary is sometimes called supernode or more formally a Gauss surface.

• Johann Carl Friedrich Gauss (1777-1855) 
  German mathematician
KCL Example

• Draw an appropriate closed boundary to find $I$ in the following graphical circuit representation.
Kirchhoff’s Voltage Law (KVL)

- **KVL**: The algebraic sum of the voltage drops around any closed path (or loop) is zero at any instance of time.

- Write KVL for the above circuit.

Sum of voltage drops = Sum of voltage rises
KVL Example

- Find $V_{AC}$ and $V_{CH}$ in the following circuit.
Example

- In the following circuit, find $v_o$ and $i$. 
Some Interesting Implications of KCL and KVL

• A series connection of two different current sources is impossible. Why?

• A parallel connection of two different voltage sources is impossible. Why?
Some Interesting Implications of KCL and KVL

- A current source supplying zero current is equivalent to an open circuit:

\[ i(t) = 0 \]

- A voltage source supplying 0V is equivalent to a short circuit:
Series Resistors and Voltage Division

- The equivalent resistance of any number of resistors connected in series is the sum of the resistors (Why?)

\[ R_{eq} = R_1 + R_2 + \cdots + R_n \]

or

\[ \frac{1}{G_{eq}} = \frac{1}{G_1} + \frac{1}{G_2} + \cdots + \frac{1}{G_n} \]

- The voltage drop across the resistor \( R_j \) for \( j=1,2, \ldots, n \) is:

\[ v_j(t) = \frac{R_j}{R_1 + R_2 + \cdots + R_n} v_{in}(t) \]
Parallel Resistors

- The equivalent conductance of resistors connected in parallel is the sum of their individual conductances:

\[ G_{eq} = G_1 + G_2 + \cdots + G_n \quad \text{or} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \]

- Why?
Current Division

- The current through the resistor $R_j$ for $j=1,2,\ldots,n$ is:

$$i_j(t) = \frac{G_j}{G_1 + G_2 + \cdots + G_n} i_{in}(t)$$

- Why?
Parallel Resistors and Current Division Example

- For the special case of two parallel resistors

\[ R_{eq} = \frac{R_1 R_2}{R_1 + R_2}, \quad i_1(t) = \frac{R_2}{R_1 + R_2} i(t), \quad \text{and} \quad i_2(t) = \frac{R_1}{R_1 + R_2} i(t) \]

- Why?
Example

• In the following circuit find $R_{eq}$:

![Circuit Diagram](image)
Example

- In the following circuit, find the equivalent resistance $R_{eq}$. Assume $g_m=0.5S$. 

![Circuit Diagram]
Example

• In the following circuit find the current $i$. 
Example

- Find $v_1$, $v_2$, power dissipated in the 3kΩ and 20kΩ resistors and the power supplied by the current source.
Wye-Delta Transformations

• In some circuits the resistors are neither in series nor in parallel.

• For example consider the following bridge circuit:

how can we combine the resistors $R_1$ through $R_6$?
Wye and Delta Networks

• A useful technique that can be used to simply many such circuits is transformation from wye (Y) to delta (Δ) network.

• A wye (Y) or tee (T) network is a three-terminal network with the following general form:

![Diagram of wye (Y) and delta (Δ) networks](image-url)
Wye and Delta Networks

- The delta (Δ) or pi (Π) network has the following general form:
Delta-Wye Conversion

- In some cases it is more convenient to work with a Y network in place of a Δ network.

- Let’s superimpose a wye network on the existing delta network and try to find the equivalent resistances in the wye network.
Delta-Wye Conversion

• We calculate the equivalent resistance between terminals a and c while terminal b is open in both cases:

\[ R_{ac}(Y) = R_1 + R_3 \]

\[ R_{ac}(\Delta) = R_b \parallel (R_a + R_c) \]

\[ R_{ac}(Y) = R_{ac}(\Delta) \Rightarrow R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \]

Similarly:

\[ R_1 + R_2 = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} \]

\[ R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \]
Delta-Wye Conversion

• Solving for $R_1$, $R_2$, and $R_3$ we have:

\[
R_1 = \frac{R_b R_c}{R_a + R_b + R_c}
\]

\[
R_2 = \frac{R_c R_a}{R_a + R_b + R_c}
\]

\[
R_3 = \frac{R_a R_b}{R_a + R_b + R_c}
\]

• Each resistor in the Y network is the product if the resistors in the two adjacent $\Delta$ branches, divided by the sum of the three $\Delta$ resistors.
**Wye-Delta Conversion**

- From the previous page equations, we have:

\[ R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c(R_a + R_b + R_c)}{(R_a + R_b + R_c)^2} \]

\[ = \frac{R_aR_bR_c}{R_a + R_b + R_c} \]

- Dividing this equation by each of the previous slide equations:

\[ R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}, \quad R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}, \quad \text{and} \quad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \]

- Each resistor in the $\Delta$ network is the sum of all the possible products of Y resistors taken two at a time, divided by the opposite Y resistor
Wye-Delta Transformations

- Y and Δ networks are said to be balanced when:

\[ R_1 = R_2 = R_3 = R_Y \quad \text{and} \quad R_a = R_b = R_c = R_\Delta \]

- For balanced Y and Δ networks the conversion formulas become:

\[ R_Y = \frac{R_\Delta}{3} \quad \text{and} \quad R_\Delta = 3R_Y \]
Example

- For the following bridge network find $R_{ab}$ and $i$. 

![Bridge Network Diagram](image-url)