During Short Circuit Studies, power systems are solved to obtain current magnitudes during faults at different points in the network.

Fault: “Failure in a circuit which interferes with the normal flow of current”

Purposes of a Short Circuit Study

To design a PROTECTION scheme to prevent damage to the electric equipment in case of the occurrence of a fault.

- Location of breakers
- Selection of breakers
  - Ratings of breakers
- Proper adjustment of breakers
- Coordination of the Protection
  - Interruption of the current
  - Isolation of the fault
  - Sequence of operation
  - Protection backup

When is a Short Circuit Study performed?

- When designing the electrical installation
- When changing operating conditions
- When installing or removing equipment
- When planning expansion
→ Symmetrical Faults
  ⇒ Faults involving the three-phases
     □ about only 5% of the cases
  ⇒ Easiest to evaluate
  ⇒ Required in a Short Circuit Study because they are commonly the worst case

→ Unsymmetrical Faults
  ⇒ Faults involving some unbalance
     □ Line to ground faults (one phase to ground)
       ⇒ about 70% of the faults
     □ Line-to-line faults (between two phases)
       ⇒ about 25% of the faults are line-to-line faults
  ⇒ To solve for these faults, we require the use of symmetrical components and sequence networks
Faults in a Three-Phase Line

### Solid three-phase fault
- a
- b
- c

### Three-phase to ground fault
- a
- b
- c
  - Fault impedance

### Line to ground fault
- a
- b
- c

### Line to ground fault through impedance
- a
- b
- c
  - Fault impedance

### Line-to-line fault
- a
- b
- c

### Line-to-line to ground fault
- a
- b
- c
  - Fault impedance
The solution for the current contains a forced response (steady state), and a transient response (natural):
\[ i(t) = i_{\text{steady-state}} + i_{\text{transient}} \]

The steady-state or forced response can be obtained using phasors:
\[ I_{\text{steady-state}} = \frac{V_s}{Z} = \frac{V_{\text{max}} \angle \alpha}{Z \angle \theta} = \left( \frac{V_{\text{max}}}{Z} \right) \angle (\alpha - \theta) = I_{\text{max}} \angle \theta \]
\[ i_{\text{steady-state}} = I_{\text{max}} \sin(wt + \alpha - \theta) \]

where: \[ Z = \sqrt{R^2 + w^2L^2} \quad \text{and} \quad \theta = \tan^{-1}\left( \frac{wL}{R} \right) \]

The transient response is the natural response of the circuit, which is the solution to the homogeneous differential equation: \[ Ri + L \frac{di}{dt} = 0 \]
\[ i_{\text{transient}} = Ke^{-\frac{R}{L}t} \]

Therefore, the total response is:
\[ i(t) = Ke^{-\frac{R}{L}t} + I_{\text{max}} \sin(wt + \alpha - \theta) \]

if \( i(0) = 0 \), then \[ 0 = I_{\text{max}} \sin(\alpha - \theta) + Ke^0 \]
and \[ K = -I_{\text{max}} \sin(\alpha - \theta) = I_{\text{max}} \sin(\theta - \alpha) \]
\[ i(t) = I_{\text{max}} \left[ e^{-\frac{R}{L}t} \sin(\theta - \alpha) + \sin(wt + \alpha - \theta) \right] \]
Plot for the current

\[ i(t) = I_{\text{max}} e^{-\frac{R}{L}t} \sin(\theta - \alpha) + I_{\text{max}} \sin(wt + \alpha - \theta) \]

natural

forced

total current
Short Circuit Current in a Synchronous Generator

Line current during a three-phase short circuit

Subtransient generator equivalent

\[ + \quad jX_d'' \quad r_a \quad + \quad V_t \quad - \]

\[ + \quad jX_d'' \quad r_a \quad + \quad V_t \quad - \]

Ed’’ : Internal subtransient voltage
Vt : Terminal voltage
Xd ’’ : Subtransient reactance

Transient generator equivalent

\[ + \quad jX_d' \quad r_a \quad + \quad V_t \quad - \]

\[ + \quad jX_d' \quad r_a \quad + \quad V_t \quad - \]

Ed’ : Internal transient voltage
Vt : Terminal voltage
Xd ’ : Transient reactance

Steady-state generator equivalent

\[ + \quad jX_d \quad r_a \quad + \quad V_t \quad - \]

\[ + \quad jX_d \quad r_a \quad + \quad V_t \quad - \]

Ed : Internal steady-state voltage
Vt : Terminal voltage
Xd : Steady-state reactance

Xd > Xd’ > Xd’’
Find the subtransient, transient, and steady-state generator current when a three-phase short circuit occurs at the high-voltage transformer terminals. Before the fault, there is no load connected and the open circuit voltage at the line terminals is 69KV. Neglect all resistances.
The equivalent circuits for subtransient, transient, and steady-state periods are shown below. Solving each one will give the magnitude of the fault current during its corresponding stage (subtransient, transient, and steady-state).

Subtransient solution:

```
+ 1 p.u. j0.15 j0.10  
-                  If ’’
```

Transient solution:

```
+ 1 p.u. j0.25 j0.10  
-                  If ’
```

Steady-state solution:

```
+ 1 p.u. j0.80 j0.10  
-                  If
```
Short-Circuit Currents for the Example

Since the generator is not supplying any current, we assume $E_d=E_d'=E_d''=100\%$

For each circuit, we find the short-circuit current as:

Subtransient: If $''= \frac{1}{j0.25} = -j4$ p.u.

Transient: If $'= \frac{1}{j0.35} = -j2.857$ p.u.

Steady-state: If $= \frac{1}{j0.90} = -j1.111$ p.u.

Generator currents are found using the base current:

I_{base1} = \frac{50000}{\sqrt{3} \cdot 13.8} = 2091.85$ amperes

Generator current magnitudes are:

Subtransient: If $''= 4.000 \times I_{base1} = 8367$ amperes

Transient: If $'= 2.857 \times I_{base1} = 5976$ amperes

Steady-state: If $= 1.111 \times I_{base1} = 2324$ amperes
A three-phase short circuit occurs at point ‘P’. To evaluate a fault during the subtransient or transient period, we need to know the pre-fault current value $I_L$.

Impedance diagram for the circuit before fault:

Switch ‘S’ is normally open. Close it to simulate a fault at point ‘P’.
Pre-Fault Conditions (subtransient)

The load current (pre-fault current) will help us determine the internal voltage for the subtransient, before the fault:

\[
Ed'' = V_t + (ra + jXd'') I_L = V_f + [(ra + rt) + j(X_t + Xd'')] I_L
\]

Neglecting transformer and generator resistances:

\[
Ed''' \equiv V_t + jXd''' I_L = V_f + j(X_t + Xd''') I_L
\]

Before the fault, If $''=0.$
Pre-Fault Conditions (transient)

The corresponding internal voltage $Ed'$ for the transient is:

$$Ed' \approx Vt + jXd'I_L = Vf + j(Xt + Xd')I_L$$

Before the fault, $I_f' = 0$.

Fault Current

To simulate the fault, switch ‘S’ is now closed.

Subtransient short-circuit current: $I_f'' = \frac{Ed''}{j(Xd'' + Xt)}$

Transient short-circuit current: $I_f' = \frac{Ed'}{j(Xd' + Xt)}$

Steady-state short-circuit current: $I_f = \frac{Ed}{j(Xd + Xt)}$
Multi-Machine System

Equivalent circuit before the fault:

Steady-state:

Transient:

Subtransient:
Subtransient Short-Circuit Solution

1. Evaluate subtransient internal voltages for generator and motor under the operating conditions (switch ‘S’ open).

\[ E_g'' = V_f + j(X_t + X_g'') IL \]
\[ E_m'' = V_f - jX_m'' IL \]

2. Close switch ‘S’, and find the current contribution from generator and motor to the fault.

\[ I_g'' = \frac{E_g''}{j(X_t + X_g'')} \]
\[ I_m'' = \frac{E_m''}{jX_m''} \]

\[ I_f'' = I_g'' + I_m'' \]
Thévenin Equivalent Method

Same multi-machine example (2 machines).
Combining steps 1 and 2.

\[ Ig" = \frac{Eg"}{j(Xt + Xg")} = \frac{Vf + j(Xt + Xg")IL}{j(Xt + Xg")} \]

\[ Ig" = \frac{Vf}{j(Xt + Xg")} + IL \]

\[ Igf'' \text{ (lets name this term: } Igf'') \]

\[ Im" = \frac{Em"}{jXm"} = \frac{Vf - jXm"IL}{jXm"} \]

\[ Im" = \frac{Vf}{jXm"} - IL \]

\[ Imf'' \text{ (lets name this term: } Imf'') \]

\[ If" = Ig" + Im" = \frac{Vf}{j(Xt + Xg")} + IL + \frac{Vf}{jXm"} - IL \]

\[ If" = \frac{Vf}{j(Xt + Xg")} + \frac{Vf}{jXm"} \]

\[ Igf'' \text{ } Imf'' \]
Thévenin Equivalent (continued)

\[ If' = \frac{V_f}{j(X_t + X_g')} + \frac{V_f}{jX_m'} \]

This expression can be represented in the following circuit:

\[ V_f \]

\[ Z_{th} \]

\[ j(X_t + X_g') \]

\[ jX_m' \]

\[ I_{gf} \]

\[ I_{mf} \]

\[ V_f \]

\[ jX_t \]

\[ jX_g' \]

\[ jX_m' \]

\[ I' \]

\[ V_f \]

\[ + \]

\[ \text{where: } X_{tg} = X_t + X_g' \]

\[ I'' = V_f \left[ \frac{1}{jX_{tg}} + \frac{1}{jX_m'} \right] \]

\[ I''' = \frac{V_f}{Z_{th}} \text{ where: } Z_{th} = j \left( \frac{(X_{tg})(X_m')}{X_{tg} + X_m'} \right) \]

\[ V_f \] is the pre-fault voltage at the fault point (Thévenin voltage)

\[ Z_{th} \] is the Thévenin Impedance seen from the fault point.

\[ I''' \] is the subtransient fault current.
Remember that the total subtransient generator current is

\[ I_{g''} = I_{gf''} + I_L \]

and the subtransient motor current is

\[ I_{m''} = I_{mf''} - I_L \]

The same problem can be solved applying superposition. The Pre-fault Solution plus the Thévenin Equivalent Solution

\[ V_f \]

is the pre-fault voltage at ‘P’ and the short-circuit is represented by two opposed Vf sources connected in series.
To obtain the total solution, we apply superposition:

1. The pre-fault solution is obtained with $E_{g''}$, $E_{m''}$ and $V_f$. This will make
   $$I_{g''1}=I_L, \quad I_{m''1}=-I_L, \quad I_f''=0.$$ 

2. The contribution to the fault is obtained with $-V_f$ only. This will make
   $$I_{g''2}=I_{gf''}, \quad I_{m''2}=I_{mf''}, \quad I_f''=I_{gf''}+I_{mf''}.$$ (Here is where we use the Thévenin equivalent)

3. Add steps 1 and 2. This will give the total fault currents
   $$I_{g''}=I_{gf''}+I_L, \quad I_{m''}=I_{mf''}-I_L, \quad I_f''=I_{gf''}+I_{mf''}.$$
Summary of Fault Analysis Using Thévenin Method

- Locate the fault point ‘P’.
- Represent the system in admittance form.
  - Convert synchronous machines to their Norton equivalents.
  - Build admittance matrix for nodal analysis $[Y_{BUS}]$.

If switch open $I_f = 0$
If switch closed $V_f = 0$
Step 1

→ Find the pre-fault operating conditions.
  ⇐ Use steady-state values.
  ⇐ Name Vf the pre-fault voltage at point ‘P’.
  ⇐ With nodal analysis or Load Flow analysis obtain
    prefault voltages V₁, V₂, V₃, … Vₙ
  ⇐ Calculate currents I₁, I₂, … I₁, I₂, …

\[ V_f = V_{\text{pre-fault}} \]

\[ I_f = 0 \]

\[ \mathbf{V}_f = \mathbf{V}_{\text{pre-fault}} \]

Iₙ = Fault current
Step 1 (continued)

\[ V_f = V_{pre-fault \ voltage} \]

\[ I_f = 0 \]

\[ Y_{BUS} \bar{V} = \bar{J} \]

\[ \bar{V} = [Y_{BUS}]^{-1} \bar{J} = Z_{BUS} \bar{J} \]

\[
\begin{bmatrix}
V_1^o \\
V_2^o \\
\vdots \\
V_p^o \\
V_f \\
\vdots \\
V_n^o
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & Y_{12} & \ldots & Y_{1p} & \ldots & Y_{1n} \\
Y_{21} & Y_{22} & \ldots & Y_{2p} & \ldots & Y_{2n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
Y_{p1} & Y_{p2} & \ldots & Y_{pp} & \ldots & Y_{pn} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
Y_{n1} & Y_{n2} & \ldots & Y_{np} & \ldots & Y_{nn}
\end{bmatrix}^{-1}
\begin{bmatrix}
J_1 \\
J_2 \\
\vdots \\
J_p \\
0 \\
\vdots \\
J_n
\end{bmatrix}
\]

This term is zero because before the fault there is no fault current (switch is open)
→ Find the Thévenin contribution to the fault.
  ⇔ Set all sources to zero (including synchronous motors internal sources).
  ⇔ Use the subtransient, transient or steady-state impedances depending on the solution desired.
  ⇔ Apply a voltage source ‘-Vf’ at point ‘P’ and solve the network with this source only. This source injects current ‘-If’ into faulted node.

- This will give the fault current If and all changes in voltages and currents needed.

- Name voltage changes $\Delta V_1, \Delta V_2, \Delta V_3…$
- Name current changes $\Delta I_{g1}, \Delta I_{g2},…, \Delta I_{\text{line}1}, \Delta I_{\text{line}2}…$
Step 2 (continued)

Machine Impedances
(for Subtransient, Transient,
or Steady-state),
Transformers,
and Transmission
Lines

\[
\begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\vdots \\
\Delta V_p \\
\Delta V_n
\end{bmatrix}
= \begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\vdots \\
-Vf \\
\Delta V_n
\end{bmatrix}
= \begin{bmatrix}
Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1n} \\
Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2n} \\
\vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pn} \\
Z_{n1} & Z_{n2} & \cdots & Z_{np} & \cdots & Z_{nn}
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
\vdots \\
-If \\
0
\end{bmatrix}
\]

This term equals \(-Vf\) because \(-Vf\) is the voltage that we need to add to the prefault voltage \(Vf\) to have a zero voltage at point ‘P’

This matrix \(Z_{BUS}\) is formed using the appropriate impedances (subtransient, transient or steady-state) to form \(Y_{BUS}\) before inverting.
Step 2 (continued)

The last equations can be simplified to:

\[
\begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
. \\
- V_f \\
. \\
\Delta V_n
\end{bmatrix}
= \begin{bmatrix}
Z_{1p} \\
Z_{2p} \\
. \\
Z_{pp} \\
. \\
Z_{np}
\end{bmatrix}
\begin{bmatrix}
- I_f
\end{bmatrix}
\]

or

\[
\Delta V_1 = - Z_{1p} I_f ,
\]

\[
\Delta V_2 = - Z_{2p} I_f ,
\]

\[
\ldots.
\]

\[
\Delta V_p = - V_f = - Z_{pp} I_f ,
\]

\[
\ldots.
\]

\[
\Delta V_n = - Z_{np} I_f
\]

from which

\[
V_f = Z_{pp} I_f \quad I_f = \frac{V_f}{Z_{pp}}
\]

where

\( I_f \) is the fault current
\( V_f \) is the pre-fault voltage at point 'P'
\( Z_{pp} \) is the Thévenin Impedance
Step 3

→ Add solutions for steps 1 and 2.
  ⇔ This is equivalent to solving the network with the switch closed.

→ Voltages during the fault are
  \[ V_1^f = V_1^0 + \Delta V_1, \quad V_2^f = V_2 + \Delta V_2, \quad V_3^f = V_3 + \Delta V_3, \ldots \]

→ at the fault \( V_p = V_f + (-V_f) = 0 \)

→ Currents during the fault are
  \[ I_{g1} + \Delta I_{g1}, \quad I_{g2} + \Delta I_{g2}, \ldots \]
  \[ I_{line1} + \Delta I_{line1}, \quad I_{line2} + \Delta I_{line2}, \ldots \]

![Diagram of a power system with loads and the system matrix][1]

[1]: [Image of a power system with loads and the system matrix]
Study Case: General Solution for Symmetrical Faults

The power system shown operates under steady-state conditions with $E_{g1}=1 \angle 0^\circ$ p.u. and $E_{g2}=0.9 \angle 30^\circ$ p.u. when a solid three-phase fault occurs at node 2. Obtain the transient short-circuit currents in lines, generators and transformers. Evaluate the transient node voltages $V_1^f$, $V_2^f$ and $V_3^f$ during the fault (transient period).

![Diagram of the power system](image)
Step 1. Pre-Fault Solution

Impedance diagram (all values in p.u.)

Admittance diagram (all values in p.u.)

To solve it, we use nodal analysis.

\[
\begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
Y_{31} & Y_{32} & Y_{33}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
=
\begin{bmatrix}
J_1 \\
J_2 \\
J_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.405 - j5.466 & -0.187 + j2.486 & -0.198 + j1.980 \\
-0.187 + j2.486 & 0.399 - j5.180 & -0.198 + j1.980 \\
-0.198 + j1.980 & -0.198 + j1.980 & 0.496 - j3.960
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
=
\begin{bmatrix}
0.02 - j0.9996 \\
0.333 - j0.549 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.929 \angle 7.1^\circ \\
0.916 \angle 10.2^\circ \\
0.920 \angle 7.2^\circ
\end{bmatrix}
\]
Pre-Fault Currents

**Line Currents:**

**Line 1-2**
\[ I_{12} = y_{12} (V_1 - V_2) \]
\[ I_{12} = (0.187 - j2.486)(0.929 \angle 7.1^\circ - 0.916 \angle 10.2^\circ) \]
\[ I_{12} = 0.129 \angle -151.9^\circ \text{ p.u.} \]

**Line 1-3**
\[ I_{13} = y_{13} (V_1 - V_3) \]
\[ I_{13} = (0.198 - j1.980)(0.929 \angle 7.1^\circ - 0.920 \angle 7.2^\circ) \]
\[ I_{13} = 0.019 \angle -87.4^\circ \text{ p.u.} \]

**Line 2-3**
\[ I_{23} = y_{23} (V_2 - V_3) \]
\[ I_{23} = (0.198 - j1.980)(0.916 \angle 10.2^\circ - 0.920 \angle 7.2^\circ) \]
\[ I_{23} = 0.096 \angle -18.9^\circ \text{ p.u.} \]

**Generator Currents:**

**Generator 1**
\[ I_{G1} = I_{12} + I_{13} = 0.129 \angle -151.9^\circ + 0.019 \angle -87.4^\circ \]
\[ I_{G1} = 0.138 \angle -144.6^\circ \]

**Generator 2**
\[ I_{G2} = -I_{12} + I_{23} = -0.129 \angle -151.9^\circ + 0.096 \angle -18.9^\circ \]
\[ I_{G2} = 0.224 \angle -24.2^\circ \]
Power Balance (Pre-Fault)

To verify the solution, a real Power Balance is now calculated (as an exercise):

**Generated Power**

**Generator 1 + Transformer 1:**

\[
S_{G1} = V_1 I_{G1}^* = (0.929 \angle 7.1^\circ)(0.138 \angle 144.6^\circ)
\]

\[
S_{G1} = -0.113 + j0.061
\]

\[
P_{G1} = -0.113 \quad \text{(where the minus sign means this generator absorbs P=0.113 p.u. and therefore is acting as a motor)}
\]

\[
Q_{G1} = 0.061
\]

**Generator 2 + Transformer 2:**

\[
S_{G2} = V_2 I_{G2}^* = (0.916 \angle 10.2^\circ)(0.224 \angle 24.2^\circ)
\]

\[
S_{G2} = 0.1986 - 0.0495i
\]

\[
P_{G2} = 0.1986
\]

\[
Q_{G2} = -0.0495 \quad \text{(this generator absorbs Q = 0.0495 p.u. and still generates P = 0.1987 p.u., therefore this machine acts as a generator)}
\]
Power Balance (continued)

Absorbed Power
Load:
\[ P_{\text{Load}} = \frac{(V3)^2}{R_{\text{Load}}} = (0.920)^2 \times 0.1 = 0.0846 \text{ p.u.} \]

Real Power dissipated in lines:
line 1-2: \[ P = I_{12}^2 \times R_{\text{line12}} = (0.129)^2 \times 0.03 = 0.0005 \]
line 1-3: \[ P = I_{13}^2 \times R_{\text{line13}} = (0.019)^2 \times 0.05 = 0.00002 \]
line 2-3: \[ P = I_{23}^2 \times R_{\text{line23}} = (0.096)^2 \times 0.05 = 0.00046 \]

The power balance is:

\[ P_{\text{generated}} = P_{\text{absorbed}} \]
\[ P_{G2} = P_{\text{Load}} + P_{\text{line12}} + P_{\text{line13}} + P_{\text{line23}} + P_{\text{absorbed}} - G_1 \]
\[ 0.1986 = 0.0846 + 0.0005 + 0.00002 + 0.0004 + 0.113 = 0.1986 \]

Note: Nodal analysis has been used to find the operating conditions of the system before the fault. In practice, a Load-flow solution would have been used instead.
Step 2. Fault at Bus 2 (Thévenin Contribution)

To simulate a Fault at Bus 2, we will add the pre-fault response to the Thévenin Contribution.

We use a source equal to the pre-fault voltage at point 2 and set all the original sources to zero. To solve this network for the transient period, we require the use of transient values for the machines impedances.

The machine impedances for the transient period are:
\[ z_{g1}' = 0.01 + j 0.25 , \quad z_{g2}' = 0.03 + j 0.4 \]

Including transformers:
\[ z_{gt1}' = (0.01 + 0.01) + j(0.25 + 0.15) = 0.02 + j 0.40 \]
\[ z_{gt2}' = (0.02 + 0.01) + j(0.40 + 0.20) = 0.03 + j 0.60 \]

The matrix \([Y_{BUS}]\) and its inverse \([Z_{BUS}]\) become:

\[
\begin{bmatrix}
0.509 - j6.960 & -0.187 + j2.486 & -0.198 + j1.980 \\
-0.187 + j2.486 & 0.468 - j6.129 & -0.198 + j1.980 \\
-0.198 + j1.980 & -0.198 + j1.980 & 0.496 - j3.960
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.020 + j0.275 & 0.014 + j0.186 & 0.023 + j0.229 \\
0.014 + j0.186 & 0.024 + j0.319 & 0.025 + j0.251 \\
0.023 + j0.229 & 0.025 + j0.251 & 0.061 + j0.487
\end{bmatrix}
\]
Fault current and changes in voltages are now obtained in the following way:

\[
\begin{bmatrix}
\Delta V_1 \\
\Delta V_2 \\
\Delta V_3
\end{bmatrix} = 
\begin{bmatrix}
\Delta V_1 \\
-Vf \\
\Delta V_3
\end{bmatrix} = Z_{BUS} \begin{bmatrix}
0 \\
-If \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\Delta V_1 \\
-Vf \\
\Delta V_3
\end{bmatrix} = \begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix} \begin{bmatrix}
0 \\
-If \\
0
\end{bmatrix}
\]

from where:

\[
\Delta V_1 = Z_{11} \cdot 0 + (-Z_{12}) If + Z_{13} \cdot 0 = -Z_{12} If
\]

\[
\Delta V_2 = -Vf = Z_{12} \cdot 0 + (-Z_{22}) If + Z_{23} \cdot 0 = -Z_{22} If
\]

\[
\Delta V_3 = Z_{31} \cdot 0 + (-Z_{32}) If + Z_{33} \cdot 0 = -Z_{32} If
\]

*Note that only elements from column 'P' are needed.

From the second equation:

\[
If = \frac{-Vf}{-Z_{22}} = \frac{Vf}{Z_{22}} = \frac{Vf}{Z_{Thev}}
\]

where \(Z_{22} = Z_{Thev}\) is the Thevenin impedance for a fault at node 2.
Fault current and changes in voltages are now calculated.
The Thévenin Impedance for a fault at bus 2 is:

\[ Z_{22} = Z_{\text{Thev}} = 0.024 + j0.319 \]

Using the pre-fault voltage at node 2:
\[ V_f = V_2 = 0.916 \angle 10.2^\circ \]

we find the fault current:
\[ I'_f = \frac{V_f}{Z_{22}} = \frac{0.916 \angle 10.2^\circ}{0.024 + j0.319} = \frac{0.916 \angle 10.2^\circ}{0.320 \angle 85.6^\circ} = 2.862 \angle -75.4^\circ \]

The voltage changes at the other nodes are found from:
\[ \Delta V_1 = Z_{12} (-I'_f ) = (0.014 + j0.186)(2.862 \angle -75.4^\circ + 180^\circ) \]
\[ \Delta V_2 = Z_{22} (-I'_f ) = (0.024 + j0.319)(2.862 \angle -75.4^\circ + 180^\circ) \]
\[ \Delta V_3 = Z_{32} (-I'_f ) = (0.025 + j0.251)(2.862 \angle -75.4^\circ + 180^\circ) \]

\[ \Delta V_1 = Z_{12} (-I'_f ) = 0.533 \angle -169.7^\circ \]
\[ \Delta V_2 = Z_{22} (-I'_f ) = 0.916 \angle -169.8^\circ = -V_f \]
\[ \Delta V_3 = Z_{32} (-I'_f ) = 0.723 \angle -171.2^\circ \]
Adding results from steps 1 and 2, we obtain the faulted voltages at each node:

\[ V_1' = V_1 + \Delta V_1 = 0.929 \angle 7.1^\circ + 0.533 \angle -169.7^\circ = 0.398 \angle 2.7^\circ \]

\[ V_2' = V_2 + \Delta V_2 = 0.916 \angle 10.2^\circ + 0.916 \angle -169.8^\circ = 0 \]

\[ V_3' = V_3 + \Delta V_3 = 0.920 \angle 7.2^\circ + 0.723 \angle -171.2^\circ = 0.199 \angle 1.3^\circ \]

The current contributions from the lines during the fault are:

\[ \Gamma_{12f}' = y_{12} (V_{1f} - V_{2f}) = (0.187 - j2.486)(0.398 \angle 2.7^\circ - 0) = 0.992 \angle -82.9^\circ \]

\[ \Gamma_{32f}' = y_{32} (V_{3f} - V_{2f}) = (0.198 - j1.980)(0.398 \angle 2.7^\circ - 0) = 0.395 \angle -83.0^\circ \]

The generator contribution is found by Kirchhoff Currents Law at the faulted node:

\[ \Gamma_{g2f}' = \Gamma_f' - \Gamma_{12f}' - \Gamma_{32f}' = 1.497 \angle -68.5^\circ \]

All quantities have been calculated in per unit. Results are for phase ‘a’.