Spatial and Temporal Dependencies of Velocities of Underwater Drifting Nodes

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Abstract—Ocean currents influence the motion of nodes in underwater networks. In this work, we investigate spatial and temporal dependencies of these ocean currents. As a measure for these dependencies we choose to use the comparison between estimated and true velocity of a tracked underwater drifting node. Different methods are used to estimate velocities, combining the velocities of a number of anchor nodes to a velocity estimate. These methods are nearest neighbor, a weighted superposition of anchor node velocities, and a deterministic method based on a linear equation system, as well as variations of these methods.

Trajectories of drifting nodes collected during sea trials and numerical ocean current models are used to evaluate these spatial dependencies. Regarding the use of these dependencies to assist tracking algorithms, the reliability of velocity estimates is investigated and a confidence index to detect unreliable velocity estimates is designed.

Our results suggest that ocean currents show strong horizontal spatial and temporal dependencies and could be useful as prior information for tracking algorithms.

I. INTRODUCTION

Spatial and temporal dependencies of ocean current velocity fields could be used to assist localization and tracking underwater. To assess and illustrate these dependencies, we consider an underwater acoustic communication network consisting of \( K \) anchor nodes and one tracked node, and utilize available information of velocity \( v_k \) of anchor nodes \( k = 1 \ldots K \) to estimate the velocity of a tracked node \( v_{TR} \). Since acoustic ranging and position information of anchor nodes is required for underwater acoustic localization [1] anyhow, the velocity of anchor nodes can be extracted at the tracked node from past position information.

In [2] long-range temporal correlations of ocean currents of the order of hours to several months are found. Spatial dependency has been assumed in [3] where an Acoustic Doppler Current Profiler is used to measure ocean currents at different depths in order to estimate velocity for dead reckoning (DR) navigation. Alternatively, in [4] a collaborative localization algorithm for fleets of vertically sinking drifters is presented, relying on the temporal correlation of ocean currents.

In this paper, we consider ocean current-induced node motion, and assume a slowly changing average current velocity field [5]. However, self-propelled motion is also allowed by adding the resulting propelled velocity to the estimated velocity \( \hat{v}_{TR} \).

The remainder of this paper is organized as follows: In Section II we introduce our system model. Section III is split into the description of velocity estimation methods, a simple temporal filter, and the proposal of a confidence index for velocity estimates. The proposed methods are evaluated using model-based simulations and sea trial data in Section IV before we draw our conclusions in Section V.

II. SYSTEM MODEL

Our setup consists of \( K \) anchor nodes with known location, which frequently share location information with a tracked node via acoustic communication, and thereby velocity. In addition, using acoustic ranging, the tracked node is assumed aware of approximate distance \( d_k \) to each of the \( K \) anchor nodes. In this paper, we limit our methods to two dimensions. However, extension is straightforward.

For \( v_{TR} \) and \( \hat{v}_{TR} \) being the true and estimated velocity of the tracked node, respectively, our objective is to minimize the relative velocity estimation error

\[
e_v = \frac{\|v_{TR} - \hat{v}_{TR}\|}{\|v_{TR}\| + \|\hat{v}_{TR}\|},
\]

which is known as \( p \)-relative distance [6] and is 1 if the vectors point in opposite directions and 0 if the vectors are the same. We choose this error measure because it is a relative measure, includes direction as well as speed errors, and is valid if one of the velocities is zero. In the following, we present some algorithms for estimating \( v_{TR} \).

III. METHOD

A. Velocity Estimation

As mentioned before, we use ranging information to anchor nodes. A noisy version of this range is available to the tracked node as part of the localization. However, as the estimation error is expected to be much larger than ranging accuracy, the effect of ranging noise on our approach is small.

Several methods for the estimation of the velocity of a tracked node have been investigated and the most promising variants are presented in this section. For the selection of these methods the factors computational complexity, accuracy, and robustness against noisy range and velocity measurements are of importance. All methods try to make use of spatial dependencies in the ocean current velocity field. Generally, similarities to particle tracking [7] can be noticed, however, with fewer particles, or in our case nodes.
1) Nearest Neighbor (NN): The simplest method to estimate $\hat{v}_{TR}$ is by choosing
$$
\hat{k} = \arg \min_{k=1 \ldots K} (d_k),
$$
and setting
$$
\hat{v}_{TR} = v_k.
$$
The major drawback of the NN method is that velocity information of anchor nodes, other than $\hat{k}$, is not considered, which makes it sensitive to noisy velocity measurements at anchor nodes.

2) Weighted Superposition (WSP): As first proposed by [8], in the WSP method velocities of anchor nodes are superimposed such that
$$
\hat{v}_{TR} = \frac{\sum_{k=1}^{K} w(d_k) \cdot v_k}{\sum_{k=1}^{K} w(d_k)},
$$
where $w(d_k)$ is a weight function. We choose $w(d_k) = d_k^{-\lambda}$ as a pragmatic choice in the absence of a statistical model for spatial dependencies. The power $\lambda$ of the weight function should be adapted to the noise level from $\lambda = 0$ for high noise levels to $\lambda = 4$ for no noise, which has been found by iterative optimization for different noise levels.

3) WSP with direction information: One drawback of the WSP method in the form of (4) is that only distance information is considered. This means, if some anchor nodes are spaced densely in a small region and some other anchor nodes are sparsely distributed around the tracked node, the velocities of the anchor nodes spaced densely will be weighted too high. Therefore, it would be desirable to take complete locations of anchor nodes $r_k = (x_k, y_k)^T$ and the tracked node $r_{TR} = (x_{TR}, y_{TR})^T$ into consideration in order to compensate for this effect. A simple extension of WSP to also use direction information, instead of only range information in the former case, is proposed in [8] where the direction term of the weight function is given by
$$
t_k = \sum_{j=1}^{K} \frac{1}{d_j} \left( 1 - \frac{(r_k - r_{TR}) \cdot (r_j - r_{TR})}{d_k \cdot d_j} \right) / \sum_{j=1}^{K} \frac{1}{d_j}. \tag{5}
$$
The weight function $w(d_k)$ for a squared inverse distance weighting becomes then
$$
w(d_k) = d_k^{-2} \cdot (1 + t_k). \tag{6}
$$

4) Least Squares (LS): Assuming that ocean currents change linearly over space, estimating $\hat{v}_{TR}$ can also be interpreted as estimating two planes, one each for the $x$- and $y$-coordinate of the velocity. We formulate the problem as
$$
\begin{pmatrix}
x_1 & y_1 & 1 \\
x_2 & y_2 & 1 \\
\vdots & \vdots & \vdots \\
x_K & y_K & 1
\end{pmatrix} \begin{pmatrix}
a_x \\
b_x \\
a_y \\
b_y
\end{pmatrix} = \begin{pmatrix}
v_{x,1} \\
v_{y,1} \\
v_{x,2} \\
v_{y,2} \\
\vdots \\
v_{x,K} \\
v_{y,K}
\end{pmatrix},
$$
with anchor position matrix $P$ (in Cartesian coordinates), anchor velocity matrix $V$, and plane coefficients in matrix $C$, wherein the coefficients $a_{(x,y)}$, $b_{(x,y)}$, and $c_{(x,y)}$ describe the plane for the $x$- and $y$-coordinate of the velocity, respectively. Solving for $C$ yields
$$
C = (P^T \cdot P)^{-1} \cdot (P^T \cdot V), \tag{8}
$$
and
$$
\hat{v}_{TR} = (x_{TR} \ y_{TR} \ 1) \cdot C. \tag{9}
$$
Note that for the LS method at least three anchor nodes are required. Due to the matrix inversion in (8), the LS method is sensitive to short distances between anchor nodes and noisy velocity measurements $v_k$.

5) Weighted Least Squares (WLS): It might be known that velocities of some anchor nodes are more accurate than other velocities. In addition, it might be desirable that anchor nodes closer to the tracked node get a higher weight to estimate the velocity field near the tracked node more accurately. This can be achieved by introducing a weight matrix $W$ into (8) which leads to
$$
C = (P^T \cdot W \cdot P)^{-1} \cdot (P^T \cdot W \cdot V). \tag{10}
$$
The weight matrix $W$ is a diagonal matrix whose elements are usually set to the reciprocal of the variance of the corresponding measurement. In this work other considerations also influence the design of the weight matrix. This is the distance of the anchor node to the tracked node. The method is then called weighted least squares (WLS). Again, the pragmatic choice for the weights on the diagonal $w_{k,k} = d_k^{-2}$ is made in the absence of a statistical model.

### B. Using Temporal Dependencies

Temporal dependencies of the ocean current field can be used to reduce the influence of velocity measurement noise and random velocity fluctuations due to wave motion. The velocity estimate $\bar{v}_{TR}[n]$ of a velocity estimation method at time step $n$ is fed into an infinite impulse response (IIR) filter described by
$$
\bar{v}_{TR}[n] = \alpha \cdot \bar{v}_{TR}[n] + (1 - \alpha) \cdot \bar{v}_{TR}[n-1], \tag{11}
$$
where $\bar{v}_{TR}[n]$ is the filtered estimate. We investigate this filter with different settings of $\alpha$ for the WSP method and sea trial data in Section IV-B2.

### C. Confidence Index of Estimates

Even the most sophisticated estimation method is worth nothing if its reliability is unknown. Therefore, it is important to have a confidence index for velocity estimates. If more than one anchor node is present, the similarity of the current velocities at these different anchor nodes can give an impression how homogeneous the velocity field is and, therefore, how reliable the estimate might be.

We use the normalized variance of the anchor node velocities as a measure for the homogeneity of the current velocity field around a tracked node. Since most presented velocity
estimation algorithms use a weighting with the distance between tracked node and anchor nodes, this weighting is also introduced in the equation of the confidence index. Otherwise, anchor nodes at a large distance contribute significantly to the confidence index and might cause the estimate to be rejected. But since they contribute almost nothing to the estimation, the estimate might still be quite good. This weighted normalized variance is calculated in two steps: First the weighted mean velocity vector is calculated by

$$\mu_w = \frac{1}{K} \sum_{k=1}^{K} v_k \cdot w(d_k).$$  \hspace{1cm} (12)

Then, the weighted variance vector is computed by

$$\sigma_{w}^2 = \frac{1}{K} \sum_{k=1}^{K} (v_k \cdot w(d_k) - \mu_w)^2,$$  \hspace{1cm} (13)

where the square is element-wise. The weighted normalized variance is then achieved by

$$\sigma_{wn}^2 = \frac{\|\sigma_w^2\|}{\|\mu_w^2\|}.$$  \hspace{1cm} (14)

The weighted normalized variance $\sigma_{wn}^2$ is an indicator for the relative variability of the velocity field around the tracked node. A large variability means that the velocity estimate for the tracked node might be inaccurate. However, a low value for the variability indicates that the estimate should be reliable.

Another indicator for the reliability of an estimate might simply be the distance of the closest anchor node to the tracked node given as

$$d_{\text{min}} = \min_{1 \leq k \leq K} d_k.$$  \hspace{1cm} (15)

To provide a measurement for the reliability of a velocity estimate we try to predict the relative velocity estimation error from (1) of an estimate. This is achieved as follows: One quarter of the trajectories simulated by an ocean current model (OCM) are used in a training phase to estimate the parameters $a^T$ of the error prediction function.

For each velocity estimate $1 \leq n \leq N$ we formulate a vector $c_n$ of reliability indicators as

$$c_n = (\sigma_{wn,n}^2 \cdot d_{\text{min},n}^2 \quad d_{\text{min},n} \quad \sqrt{d_{\text{min},n}^2 + \sigma_{wn,n}^2} \quad \sigma_{wn,n}^2 \quad \sigma_{wn,n})^T.$$  \hspace{1cm} (16)

For the training of the overdetermined equation system

$$a^T = (C^T \cdot C)^{-1} \cdot (C^T \cdot \varepsilon^T)$$  \hspace{1cm} (17)

is solved with error vector $\varepsilon = (e_{v,1} \ldots e_{v,n} \ldots e_{v,N})$, where $e_{v,n}$ is the relative velocity estimation error introduced in (1), and reliability indicator matrix

$$C = \begin{pmatrix} c_1^T & \ldots & c_n^T & \ldots & c_N^T \end{pmatrix}^T.$$  \hspace{1cm} (18)

The parameters $a^T$ of the error prediction function are stored and the error $\hat{\varepsilon}_n$ of a new sample $\tilde{n}$ can be predicted to

$$\hat{\varepsilon}_n = c_\tilde{n} \cdot a^T.$$  \hspace{1cm} (19)

This method will be applied to the three quarters of the remaining trajectories derived by OCM simulation and also to sea trial data with the parameters trained on simulated trajectories.

IV. RESULTS

For most simulations trajectories of nodes simulated with OCMs are used. However, as a proof of concept for real applications, we also use data recorded during two sea trials. In this paper, we investigate the influence of the number of anchor nodes on different methods, the classifier operating characteristic (COC) for our confidence index, estimation results and the confidence index in the time-domain for a sea trial, and the performance gain when using temporal filtering to conquer noisy velocity measurements.

A. Model-based Results

Several OCMs have been developed over time and most of them are designed for large basins to the scale of whole oceans and long time periods of days and months. In this work, we are concerned about small regions extending to the communication range of underwater (UW) modems, shallow waters, and on time steps as small as a few seconds. For these conditions only few OCMs are available. One of these is the Shallow Water Hydrodynamic Finite Element Model (SHYFEM) [9] as it is designed to resolve the hydrodynamic equations for coastal seas and other small basins. In this section, we present model-based simulation results by using SHYFEM to simulate time-varying current velocity fields. These are used to generate trajectories of anchor and tracked nodes, initially placed uniformly in an area of $2 \times 2 \text{km}^2$.

We use a set of 100 scenarios with random bathymetry and randomly deploy 100 tracked nodes and a varying number of anchor nodes. In this paper we present some of our findings.

1) Influence of the Number of Anchor Nodes: First, in Fig. 1 we show how the number of anchor nodes influences the different proposed methods. The NN method and both variants of the WSP method are equal for one anchor node. For two anchor nodes a significant improvement can be observed for all three methods. With increasing number of anchor nodes the

![Fig. 1. Influence of the number of anchor nodes on different velocity estimation methods.](image-url)
WSP methods have an advantage over the simple NN method, with the WSP method with direction information being slightly superior to the version without direction information. The LS and WLS method require at least three anchor nodes and their performance is worse than that of the three other methods for low numbers of anchor nodes. The WLS method approaches the error performance of the other three methods with increasing number of anchor nodes. The LS method also improves, but does not reach the performance of the other methods even for eight anchor nodes. The reason is that all anchor nodes are used with the same weight, whereas all other methods give a higher weight to closer anchor nodes. That is an indication that the spatial dependency decreases with range.

2) Performance of the Confidence Index: In Fig. 2 we investigate the proposed confidence index for different levels of measurement noise of anchor node velocities with error threshold \( \eta \) as a parameter by showing the COC. We distinguish the hypothesis good for errors \( \varepsilon_v \leq 0.1 \) and bad for \( \varepsilon_v > 0.1 \) both given by (1). A good estimate as accepted \( (\hat{\varepsilon}_v \leq \eta) \) is defined a detection and contributes to the detection rate \( P_D \). But accepting a bad estimate \( (\hat{\varepsilon}_v > \eta) \) is a false alarm, contributing to the false alarm rate \( P_F \). In Fig. 2 we give the curves of the COC for three different standard deviations of velocity measurement noise of anchor nodes \( \sigma_v \) and also three different thresholds \( \eta \). The performance decreases with increasing noise standard deviation \( \sigma_v \). However, for even lower noise levels, we have not found a better performance, since the remaining estimation error of our velocity estimation methods would lead to some false alarm, especially if the threshold \( \eta \) is set to a high level \( (\eta > 0.2) \).

B. Sea Trial

While in our simulations we used a reliable model for ocean current, it is clear that spatial dependency greatly depends on the structure of the channel. Moreover, apart from ranging noise, anchor node velocity measurements may also experience noise. For this reason, to validate our numerical results in the following we present results from sea trials. The sea trials were conducted off the shores of Israel (see description in [10]) and Singapore.

1) Time Series Results: The trajectories of four drifting yachts given in Fig. 3 have been measured using Global Positioning System (GPS) receivers during the Israel sea trial. For the evaluation of our proposed methods, one of the yachts has been chosen as the tracked node, while the other three served as anchor nodes. Fig. 4 shows the time-series of measured and estimated speed (a) and direction (b) using the three methods NN, WLS, and LS presented in Section III-A. While the LS method seems best in most cases, the average results shown in Table I with magnitude \( v \) and angle \( \phi \) of \( v \) in polar coordinates, imply that the WSP method outperforms both NN and LS methods. This is because the NN method does not use all the provided velocity information and the LS method is sensitive to scenarios with anchor nodes close to each other.

<table>
<thead>
<tr>
<th>Measure</th>
<th>NN</th>
<th>WSP</th>
<th>LS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E[</td>
<td>\varepsilon_v</td>
<td>] )</td>
<td>0.114</td>
</tr>
<tr>
<td>( E[</td>
<td>\varepsilon_\phi</td>
<td>] )</td>
<td>0.371</td>
</tr>
</tbody>
</table>

In the lower part (c) of Fig. 4 measured relative velocity estimation error \( \varepsilon_v \) and predicted relative velocity error \( \hat{\varepsilon}_v \) are compared for the same sea trial and the WSP method. Generally, the predicted error is too low. The main reason is that the training was performed on simulated trajectories and no adaption to the conditions during the sea trial have been performed. But as a tendency the predicted error assumes its largest values when the measured error is also large.

2) Using Temporal Dependencies: To show temporal dependency, ocean currents we use sea trial data collected near Singapore with trajectories of two drifting nodes sampled every second for more than an hour with GPS receivers. Fig. 5 (a) shows the trajectories of the two nodes, one being used as anchor node and the other as tracked node. We apply the NN method, simply assuming the velocity of the anchor node as velocity estimate of the tracked node. In Fig. 5 we vary...
The filter coefficient \( \alpha \) of the IIR filter given by (11). We observe that the error performance gets better as we decrease from \( \alpha = 1 \) towards \( \alpha = 0.002 \), especially for high velocity measurement noise standard deviations \( \sigma_v \). However, if we continue to decrease \( \alpha \), the responsiveness of the filter gets worse and it looses its ability to follow changes of speed and direction as are found in the trajectories.

V. CONCLUSION

In this paper we proposed different approaches for velocity estimation of tracked underwater drifting nodes to show spatial and temporal dependencies of the current velocity field. With OCM simulations we tested the influence of the number of anchor nodes, that showed improved error performance for larger numbers of anchor nodes for all methods. The proposed confidence index was able to detect a lot of estimates exceeding a selected threshold by only missing a few good estimates with appropriate threshold settings. For sea trial data we showed the performance of the velocity estimation methods in the time-domain and also how a simple filter can use temporal dependencies in order to suppress noisy velocity measurements of anchor nodes. Results suggest strong spatial and temporal dependencies, that could be used to assist localization and tracking systems. Therefore, in future work these dependencies shall be fed into a tracking algorithm as prior information.

REFERENCES