Compact Modeling of High-Frequency, Small-Dimension Bipolar Transistors

D. L. Pulfrey, A. R. St. Denis, and M. Vaidyanathan

Abstract— **Recent progress in the development of compact models for high-speed bipolar transistors with quasi-ballistic base widths (on the order of a mean-free path length) is summarized. The correctness of basing such models on the drift-diffusion equation is examined by comparing results with solutions to the Boltzmann transport equation. Useful and well-founded compact expressions are presented for important dc and ac device parameters.**

Keywords— **Compact models, bipolar transistors, quasi-ballistic transport, maximum oscillation frequency.**

I. Introduction

COMPACT models for device design are sets of analytical expressions that relate the terminal behavior of a device to its composition and layout. Such models provide insight into key factors determining device performance, and, in a transistor, for example, allow rapid estimation of the dc bias current and the small-signal parameters.

For bipolar transistors, compact models have long been based on the drift-diffusion equation (DDE). Twenty-five years ago, an advanced technology, such as Fairchild's Isoplanar II process, yielded homojunction devices (BJTs) with a base width W_B of approximately 3500 Å and a peak f_T of about 5 GHz [1]. Now, prototype heterojunction devices (HBTs) have $W_B = 500$ Å, $f_T \approx 150$ GHz, and $f_{\max}$ values approaching 0.5 THz [2]. Can compact models for these and other small, fast transistors still be built on the DDE?

The starting point for answering the above question is to review how the DDE is obtained from the Boltzmann transport equation (BTE). The one-dimensional form of the BTE can be written in the form

$$\frac{\partial f}{\partial t} + v_z \frac{\partial f}{\partial z} - \frac{q\mathcal{E}}{m^*}\frac{\partial f}{\partial v_z} = \mathcal{C}_{\text{in}} - \mathcal{C}_{\text{out}} \qquad (1)$$

where z is position, t is time, $\mathcal{E}$ is the electric field (assumed to lie in the z-direction), m^* is the electron effective mass, v_z is the z-component of the electron velocity $\mathbf{v}$, $f(z,\mathbf{v},t)$ is the electron "velocity distribution function," q is the electronic charge, and $\mathcal{C}_{\text{in}}$ and $\mathcal{C}_{\text{out}}$ are the "collision integrals." The first two "moments" of the BTE can be found [3, Ch. 5] by integration of (1), and yield the following equations for the carrier concentration n and the current density J_n:

$$\frac{n}{\tau_{\text{rec}}} = \frac{1}{q}\frac{\partial J_n}{\partial z} - \frac{\partial n}{\partial t} \qquad (2)$$

and

$$J_n = q\mu_n n\mathcal{E} + qD_n\frac{\partial n}{\partial z} + 2\mu_n n\frac{\partial u}{\partial z} - \tau_{\text{sc}}\frac{\partial J_n}{\partial t} \qquad (3)$$

where τ_{rec} is the recombination lifetime, and where τ_{sc} is an average time related to scattering processes, μ_n is the mobility, D_n is the diffusivity, and u is the average z-directed kinetic energy, defined by the following expressions:

$$\frac{1}{\tau_{\text{sc}}} = \frac{\int [v_z/\tau^\star(\mathbf{v})] f(z,\mathbf{v},t)\,d\mathbf{v}}{\int v_z f(z,\mathbf{v},t)\,d\mathbf{v}} \qquad (4)$$

$$\mu_n = \frac{q\tau_{\text{sc}}}{m^*} \qquad (5)$$

$$D_n = \left[\frac{2u}{q}\right]\mu_n \qquad (6)$$

$$u = \frac{\int [m^* v_z^2/2] f(z,\mathbf{v},t)\,d\mathbf{v}}{\int f(z,\mathbf{v},t)\,d\mathbf{v}} \qquad (7)$$

where $\tau^\star(\mathbf{v})$ is the "momentum relaxation time" for an electron with velocity $\mathbf{v}$. Equation (2) is just the usual continuity equation. The drift-diffusion equation is obtained from (3) by ignoring the terms involving $\partial u/\partial z$ and $\partial J_n/\partial t$ to write

$$J_n = q\mu_n n\mathcal{E} + qD_n\frac{\partial n}{\partial z}. \qquad (8)$$

Conventional compact models for BJTs and HBTs are based on a solution of (2) and (8) in the base region, with values of the transport parameters $\mu_n \equiv \mu_{n0}$ and $D_n \equiv D_{n0}$ found by assuming that the velocity distribution $f(z,\mathbf{v},t)$ retains a form close to its equilibrium, Maxwellian shape.[1] The validity of this approach for high-speed, small-dimension devices, namely, devices where the neutral base width W_B is comparable to the average scattering (or mean-free path) length l_{sc}, will hence depend on whether f does indeed retain a Maxwellian form, as well as on whether the last two terms of (3) can indeed be neglected; in other words, it will depend on the extent to which the true moment equation (3) reduces to the near-equilibrium form of the DDE:

$$J_n = q\mu_{n0} n\mathcal{E} + qD_{n0}\frac{\partial n}{\partial z}. \qquad (9)$$

In what follows, an overview of the above issue is presented, with examples drawn from recent work; further details are available in the cited references. BJTs are considered first, followed by HBTs. In all cases, the base is assumed to be field free and the device is assumed to be working in the forward-active mode.

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This work was partially supported by NSERC, NORTEL, and MICRONET, Canada, and by the UBC Killam program.

[1] For a Maxwellian shape, $f(z,\mathbf{v},t) \propto \exp[-m^* v^2/2k_BT]$.

II. Compact Modeling of Small-Dimension BJTs

A. Static Characteristics

Classical approaches to computing the dc collector current in BJTs originate from the seminal work of Shockley [4]. In this case, equations (2) and (9) are solved with boundary conditions corresponding to the well-known "law of the junction": $n(0) = n_E^* \equiv (n_i^2/N_B)\exp(V_{BE}/k_BT)$ and $n(W_B) = 0$, where n_i is the intrinsic carrier concentration, N_B is the base doping, V_{BE} is the applied base-emitter voltage, k_B is Boltzmann's constant, T is the temperature, and the neutral base lies between $z = 0$ and $z = W_B$. The first step towards improving compact models for J_C in short-base devices is to note that these boundary conditions must be updated. At $z = 0$, it is important to recognize that electrons are actually injected from the emitter into the base in the form of a "unidirectional" or "hemi"-Maxwellian distribution, consisting of a number of carriers $n_E^*/2$ all moving in the forward (positive) direction; the backward-going distribution at $z = 0$ is then determined by those carriers which are backscattered from the base. While in a large device, the latter distribution is also a hemi-Maxwellian consisting of $n_E^*/2$ carriers, such that the Shockley boundary condition of a total number of carriers $n(0) = n_E^*$ applies, this is not the case in short-base devices. Similarly, at $z = W_B$, it is important to recognize that $n(W_B)$ cannot be identically zero, as some finite concentration is needed to carry the current; as the base width diminishes, this concentration grows, and the Shockley assumption $n(W_B) = 0$ becomes increasingly invalid. Roulston [5], [6] was the first to recognize that Shockley's boundary conditions needed to be updated; his modification focused on $n(W_B)$, and provides only a partial correction [7]. More recently, and by carefully considering the actual fluxes of electrons injected into and flowing out of the base at $z = 0$ and $z = W_B$, Hansen [8] came up with the following new boundary conditions:

$$n(0) = \frac{J_n(0)}{2qv_R} + n_E^* \tag{10}$$

$$n(W_B) = \frac{-J_n(W_B)}{2qv_R} \tag{11}$$

where $v_R = \sqrt{k_BT/2\pi m}$ is a thermal velocity associated with the Maxwellian velocity distribution. Solving (2) and (9) with (10) and (11) yields a new expression for the collector current:

$$J_C = \frac{-qD_{n0}n_E^*}{W_B + D_{n0}/v_R} \tag{12}$$

where $\tau_{\rm rec} \to \infty$ has been assumed for simplicity. Equation (12) differs from the classical, Shockley result due to the D_{n0}/v_R term in the denominator; this term limits the current to its "ballistic" value of $-qn_E^*v_R$ when $W_B \to 0$. Hansen's approach also leads to a new expression for transit time:

$$\tau_B = \frac{W_B^2}{2D_{n0}} + \frac{W_B}{2v_R}. \tag{13}$$

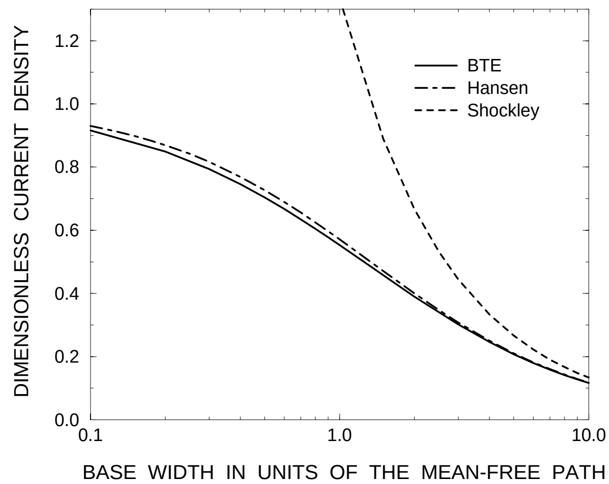

Fig. 1. Dimensionless dc collector current $-J_C/qn_E^*v_R$ versus dimensionless base width $W_B/l_{\rm sc}$ for a homojunction device. The solid line shows results from a BTE solution [11], the stippled line shows values from Hansen's result (12), and the dashed line shows values from Shockley's classical expression $J_C = -qD_{n0}n_E^*/W_B$.

This expression differs from the classical result due to the $W_B/2v_R$ term, which is the value of the transit time in the ballistic limit. It should be noted here that Tanaka and Lundstrom [9] also derived (12) and (13), using a technique which is essentially equivalent to that employed by Hansen [10].

Figure 1 shows a plot of Hansen's result (12) for the dc current along with results from a solution of the BTE [11]. It is surprising that a correction of the boundary conditions is the only thing required in order to properly predict J_C, but as shown in Fig. 1, this modification does lead to a remarkable agreement with results from the BTE, even at very narrow base widths. Shown also in Fig. 1, for contrast, are the values of current predicted by Shockley's classical expression, $J_C = -qD_{n0}n_E^*/W_B$; these values are far too high. As will be shown (see Fig. 4), Hansen's result (13) for the transit time also closely matches BTE results for thin-base homojunction devices.

It should be noted here that the agreement in Fig. 1 does *not* mean that (9) is rigorously valid on a microscopic level when $W_B \sim l_{\rm sc}$. A more careful examination [12] shows that the assumptions (regarding the point value of D_n and the neglect of u) on which (9) is based do indeed begin to break down when the base width is small. While the errors are not too great for $W_B \sim l_{\rm sc}$ [13], and while they have a limited impact on the predicted value of J_C in BJTs (provided Hansen's boundary conditions are employed), one should not hasten to draw conclusions regarding the general validity of (9) in small-dimension devices based solely on the agreement in Fig. 1. The examples which follow will serve to emphasize this point.

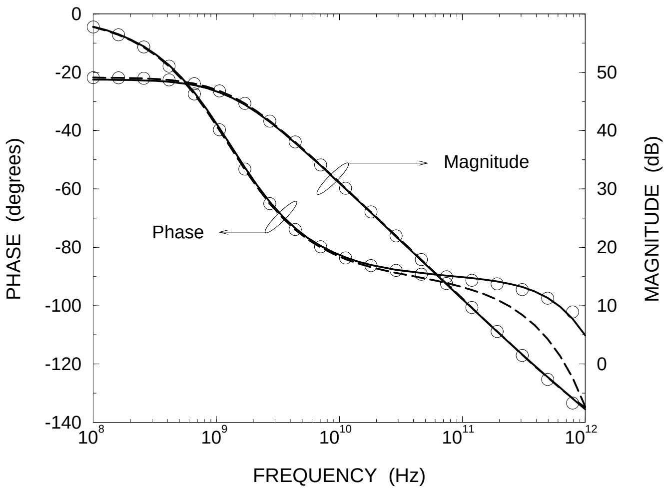

Fig. 2. Common-emitter current gain versus frequency for a homojunction device with $W_B = 1l_{sc}$. The solid line shows values from a BTE solution [15], the dashed line shows values from the one-flux approach [14], and the open circles are values from the Thomas-Moll form (15).

B. High-Frequency Characteristics

For the purposes of predicting the intrinsic small-signal characteristics of thin-base BJTs, in addition to updating the boundary conditions as suggested by Hansen, it is apparent that one should add the $-\tau_{sc}(\partial J_n/\partial t)$ term to (9), as it may become important at the very high frequencies at which thin-base devices can operate. Doing this with the choice $\tau_{sc} \equiv D_{n0}/4v_R^2$ yields an augmented, near-equilibrium DDE:

$$J_n = q\mu_{n0}n\mathcal{E} + qD_{n0}\frac{\partial n}{\partial z} - \left[\frac{D_{n0}}{4v_R^2}\right]\frac{\partial J_n}{\partial t}. \quad (14)$$

One might hope that a solution of (2) and (14) in the base region with Hansen's boundary conditions—an approach that can be shown [10] to be essentially equivalent to employing the so-called "one-flux method" [14]—would yield accurate compact expressions for the intrinsic, small-signal characteristics of small-dimension BJTs. However, this is not the case. An example of the type of discrepancy that can occur is shown in Fig. 2, where the magnitude and phase of the common-emitter current gain β from the one-flux approach are compared with values obtained from a solution of the BTE [15]; the plot is for a device with $W_B = 1l_{sc}$. As shown, the one-flux approach correctly predicts the magnitude of β, but incorrectly predicts its phase at high frequencies. This is an important deficiency, as the phase of β at high frequencies directly determines high-speed circuit model parameters (such as `PTF` in SPICE) and influences the performance of many high-speed circuits [16]. In addition to the error in the phase of β, as pointed out in [15], the flux approach also erroneously predicts the values of other small-signal parameters.

The main reason for the failure of the augmented DDE (14) in predicting the high-frequency characteristics lies in its assumption that the velocity distribution function, namely, $f(z, \mathbf{v}, t)$, always retains a Maxwellian shape, an assumption that is not valid for the small-signal part of $f(z, \mathbf{v}, t)$ at high frequencies [15]. When $W_B \sim l_{sc}$, many electrons traverse the base without experiencing any collisions, and the arrival time of these electrons then depends solely on their initial speed and angle of injection into the base. As explained in [15], the resulting variation in base transit times directly influences the small-signal part of the carrier distribution function, significantly removing it from a Maxwellian shape at high frequencies, and thereby invalidating (14).

While (14) does fail at high frequencies,[2] a compact model for the small-signal characteristics of thin-base devices can still be found. For example, the current gain can be expressed using the well-known Thomas-Moll form [18]:

$$\beta = \frac{\alpha_0 \exp\{j[(K-1)/\sqrt{K}](\omega\tau_B/\nu)\}}{(1-\alpha_0)\{1 + j[\omega\tau_B/K(1-\alpha_0)\nu]\}} \quad (15)$$

where ω is the radian frequency, α_0 is the low-frequency value of the common-base current gain, ν is a fitting factor, and $K \equiv 1/\alpha_0\nu$. The parameter values to use in (15) are as follows: α_0 can be found by using (14) at *low* frequencies, together with Hansen's boundary conditions, which yields

$$\alpha_0 = \left[\cosh\left(\frac{W_B}{L_n}\right) + \frac{D_{n0}}{2v_R L_n}\sinh\left(\frac{W_B}{L_n}\right)\right]^{-1} \quad (16)$$

where $L_n \equiv \sqrt{D_{n0}\tau_{\rm rec}}$; τ_B can be found using (13); and the value of ν can be found from the BTE solutions in [15] as $\nu \approx 1.12$ for $W_B \sim l_{sc}$. As shown in Fig. 2, equation (15) with these values is successful in matching the BTE results. This modeling approach can be extended to find compact expressions for all the small-signal parameters [15].

III. Compact Modeling of Small-Dimension HBTs

A. Static Characteristics

In abrupt-junction HBTs fabricated in the $Al_xGa_{1-x}As$ system, the electrons injected into the base from the emitter have a velocity distribution that is substantially distorted from a Maxwellian form, a consequence of thermionic emission and quantum-mechanical tunneling which occurs at the band spike located at the emitter-base junction. It is unlikely that models based on the use of the near-equilibrium DDE (9) would apply with such a "hot-electron" injected distribution. Recent BTE solutions [19], [20] for transport across the base of such HBTs (accounting carefully for the effects of the band spike and the types of scattering found in these devices) demonstrate that this is indeed the case, as illustrated by the results in Figs. 3 and 4.

Figure 3 shows a plot of the BTE solutions for the dc collector current as a function of base width. Curves are shown for an abrupt-junction AlGaAs/GaAs HBT and a GaAs homojunction device. As illustrated, the behavior

[2] It is worth noting that (14) can be improved to some extent by adjusting the value of the $-\tau_{sc}(\partial J_n/\partial t)$ term [17, Sec. 3.3].

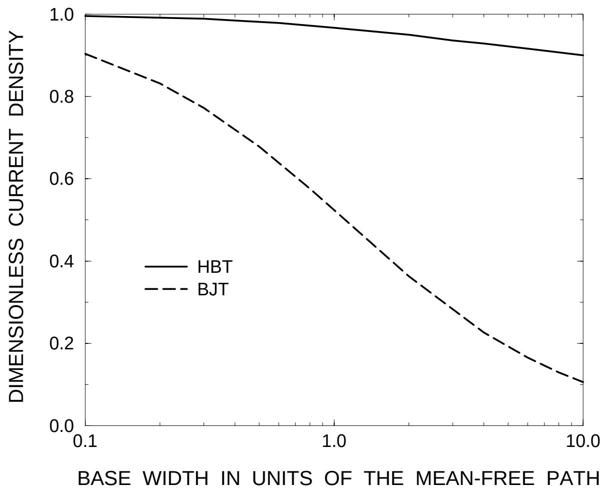

Fig. 3. BTE values [19], [20] of dimensionless dc collector current $J_C/(-qF^+)$ versus dimensionless base width W_B/l_{sc} for an abrupt-junction AlGaAs/GaAs HBT and a GaAs homojunction device, where F^+ refers to the flux of electrons injected from the emitter to the base. The solid line shows results for the HBT, and the dashed line shows results for the homojunction device.

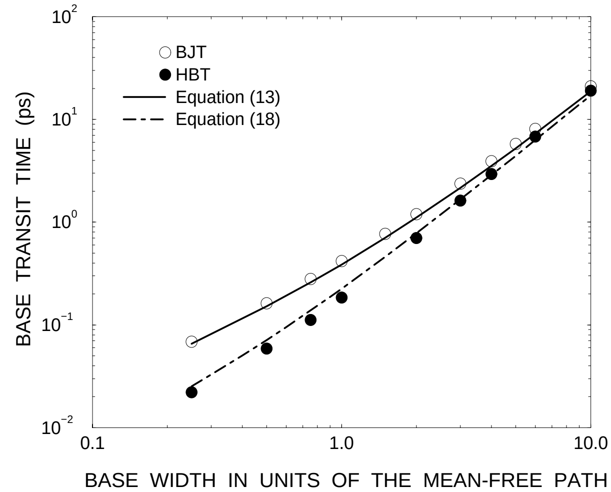

Fig. 4. Base transit time τ_B versus dimensionless base width W_B/l_{sc} for an abrupt-junction AlGaAs/GaAs HBT and a GaAs homojunction device. The open and solid circles are values for the HBT and homojunction device, respectively, as found from a solution to the BTE [19]. The solid line shows values from (13), and the stippled line shows values from (18).

of J_C as a function of W_B is decidedly different in the two cases. While the homojunction curve resembles those shown earlier in Fig. 1, the same is not true for the abrupt-junction HBT curve, where the current is sustained at a value close to that arising from the injected flux, even at large base widths.

Figure 4 shows results for the base transit time (found simply as the ratio of the static base charge over the static collector current) for the two devices. The values of τ_B in the HBT case are substantially lower, a direct consequence of the hot-electron nature of the injected distribution.

Compact models for J_C and τ_B in the HBT case can be obtained by making a few readily apparent observations. For J_C, as mentioned, the current is sustained at a value close to that arising from the injected flux, even at wide base widths (at least up to $10 l_{\text{sc}}$), and it may therefore simply be written as

$$J_C \approx -qF^+ \equiv -q\, n^+_{\text{TTE}}(V_{BE})\, v^+_{\text{TTE}}(V_{BE}) \qquad (17)$$

where $n^+_{\text{TTE}}(V_{BE})$ is the concentration of injected carriers arising from tunneling and thermionic emission, and $v^+_{\text{TTE}}(V_{BE})$ is their average velocity. An expression for τ_B can be found by recognizing that, in the ballistic limit ($W_B/l_{\text{sc}} \to 0$), τ_B must approach a value $W_B/v^+_{\text{TTE}}(V_{BE})$. The hot-electron nature of the injected distribution in an abrupt-junction HBT makes $v^+_{\text{TTE}}(V_{BE})$ differ from the homojunction value of $2v_R$. Given that this is the case, a reasonable approach is to simply replace the ballistic limit of $W_B/2v_R$ in (13) by the value $W_B/v^+_{\text{TTE}}(V_{BE})$, yielding, for HBTs, the following equation:

$$\tau_B(V_{BE}) = \frac{W_B^2}{2D_{n0}} + \frac{W_B}{v^+_{\text{TTE}}(V_{BE})}. \qquad (18)$$

Expressions for the bias-dependent quantities $n^+_{\text{TTE}}(V_{BE})$ and $v^+_{\text{TTE}}(V_{BE})$ can be obtained by utilizing the analysis in [21]. These expressions, as well as a more detailed discussion of the results in Figs. 3 and 4, will be reported elsewhere [22]. For now, the success of the above approach can be demonstrated by simply noting that v^+_{TTE} is $3.7 \times 2v_R$ for the bias value considered in Figs. 3 and 4, and plotting, for example, the resulting values of (18). As shown in Fig. 4, equation (18) is successful in matching the HBT results. Shown also are values from (13), which applies in the homojunction case.

B. High-Frequency Characteristics

One of the consequences of shrinking device dimensions has been a corresponding increase in base doping densities. As a result, the base resistance r_{bb} is no longer much larger than the emitter and collector resistances, r_{ee} and r_{cc}, and the dynamic resistance $1/g_m$. Thus, in addition to issues related to the employment of the DDE, conventional compact models which assume $r_{bb} \gg r_{ee}, r_{cc}, 1/g_m$ are not likely to be valid for modern devices. One important example is provided by the expression for the maximum oscillation frequency ($f_{\max}$), an important high-speed figure of merit.

Recent work [23] has shown that it is still possible to use a compact expression for $f_{\max}$ of the familiar form,

$$f_{\max} = \sqrt{\frac{f_T}{8\pi (RC)_{\text{eff}}}} \qquad (19)$$

provided that $(RC)_{\text{eff}}$ accounts for all the device parasitics. As shown in [23], for $f_{\max}$ as defined by extrapolation of Mason's unilateral power gain [24], the value of $(RC)_{\text{eff}}$ is given by[3]

$$(RC)_{\text{eff}} = (r_b C_c)_{\text{eff}} + [\omega_T r_{cc} C_{jc}] \left(r_{ee} + \frac{1}{g_m} \right) C_{jc} \quad (20)$$

where $(r_b C_c)_{\text{eff}}$ accounts for the distributed nature of the base-collector network and is specified in [23], and C_{jc} is the total collector-base junction capacitance.

Equations (19) and (20) allow for the rapid identification of speed-limiting parasitics in modern transistors, and, in general, these will include elements other than the base resistance and collector-base junction capacitance, the parasitics conventionally assumed to be the limiters of $f_{\max}$.

IV. Conclusions

This paper has presented a brief overview of recent work related to the compact modeling of small-dimension bipolar transistors and heterojunction bipolar transistors. The main points can be summarized as follows:

- **Small-Dimension BJTs**
  - The DDE with proper boundary conditions gives a good description of the dc collector current and base transit time of BJTs at all base widths.
  - At high frequencies, the small-signal part of the velocity distribution function $f(z, \mathbf{v}, t)$ is significantly removed from a Maxwellian form to invalidate DDE approaches.
  - A compact model for the high-frequency characteristics of thin-base BJTs can be found by utilizing the Thomas-Moll forms and carefully choosing parameter values.
- **Small-Dimension HBTs**
  - BTE solutions for the dc collector current and base transit time have been obtained. These solutions show features related to the "hot-electron" nature of the injected distribution, and demonstrate that DDE-based models for these quantities do not apply.
  - Preliminary expressions for J_C and τ_B in thin-base HBTs can be obtained by making readily apparent observations from the BTE solutions.
  - Compact expressions for the $f_{\max}$ of HBTs have been found and should aid in identifying speed-limiting parasitics in modern devices.

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