EECE 360 – Systems & Control

Electrical & Computer Engineering
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http://www.ece.ubc.ca/~leos/e360.html

Systems

\[ x(t) \xrightarrow{G} y(t) \]

Watt’s flyball governor
Systems

Example 1

\[ y(t) = \frac{d}{dt} x(t) \]

Example 2

\[ r(t) = \frac{v(t)}{R} + C \frac{dv(t)}{dt} + \frac{1}{L} \int_0^t v(t) dt \]

Laplace Transform

Laplace transform is an integral transform:

\[ F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st} dt \]

Inverse Laplace transform:

\[ f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st} ds \]
Laplace Transform

Region of convergence

If \( |f(t)| < Me^{-\alpha t} \) for all positive \( t \), then \( \int_0^\infty |f(t)| e^{-\alpha t} dt < \infty \) will converge for \( \sigma > \alpha \).

Some important Laplace Transform Pairs:

<table>
<thead>
<tr>
<th>( F(s) )</th>
<th>( f(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{s} )</td>
<td>( \delta(t) )</td>
</tr>
<tr>
<td>( \frac{1}{s^2} )</td>
<td>( 1(t) )</td>
</tr>
<tr>
<td>( \frac{a}{s^2 + \omega^2} )</td>
<td>( e^{-at} )</td>
</tr>
<tr>
<td>( \frac{\omega}{s^2 + \omega^2} )</td>
<td>( \sin(\omega t) )</td>
</tr>
<tr>
<td>( \frac{\omega}{s^2 + 2\omega^2} )</td>
<td>( \cos(\omega t) )</td>
</tr>
</tbody>
</table>

\[
\frac{F(s)}{s^2 + \omega^2} = e^{-at} \sin(\omega t) \]
\[
\frac{F(s)}{(s+a)^2 + \omega^2} = e^{-at} \cos(\omega t) \]
\[
\frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta \omega t} \sin(\omega \sqrt{1-\zeta^2} t) \]

See Table 2.3, pp. 59, for more pairs.

Properties

<table>
<thead>
<tr>
<th>Item no.</th>
<th>Theorem</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>( \mathcal{L}[f(t)] = F(s) = \int_0^\infty f(t)e^{-st} dt )</td>
<td>Definition</td>
</tr>
<tr>
<td>2.</td>
<td>( \mathcal{L}[k(t)] = kF(s) )</td>
<td>Linearity theorem</td>
</tr>
<tr>
<td>3.</td>
<td>( \mathcal{L}[f(t) + g(t)] = F(s) + G(s) )</td>
<td>Linearity theorem</td>
</tr>
<tr>
<td>4.</td>
<td>( \mathcal{L}[e^{-at}f(t)] = F(s + a) )</td>
<td>Frequency shift theorem</td>
</tr>
<tr>
<td>5.</td>
<td>( \mathcal{L}[f(t-T)] = e^{-st}F(s) )</td>
<td>Time shift theorem</td>
</tr>
<tr>
<td>6.</td>
<td>( \mathcal{L}[f(at)] = \frac{1}{a}F\left(\frac{s}{a}\right) )</td>
<td>Scaling theorem</td>
</tr>
<tr>
<td>7.</td>
<td>( \mathcal{L}\left{ \frac{df}{dt}\right} = sF(s) - f(0-) )</td>
<td>Differentiation theorem</td>
</tr>
<tr>
<td>8.</td>
<td>( \mathcal{L}\left{ \frac{d^2f}{dt^2}\right} = s^2F(s) - sf(0-) - f(0-) )</td>
<td>Differentiation theorem</td>
</tr>
<tr>
<td>9.</td>
<td>( \mathcal{L}\left{ \frac{d^n f}{dt^n}\right} = s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0-) )</td>
<td>Differentiation theorem</td>
</tr>
<tr>
<td>10.</td>
<td>( \mathcal{L}\left{ \int_0^t f(t) dt \right} = \frac{F(s)}{s} )</td>
<td>Integration theorem</td>
</tr>
</tbody>
</table>
| 11. | \( f(\infty) = \lim_{s\to 0} sF(s) \) | Final value theorem
| 12. | \( f(0+) = \lim_{s\to \infty} sF(s) \) | Initial value theorem

1 For this theorem to yield correct finite results, all roots of the denominator of \( F(s) \) must have negative real parts and no more than one can be at the origin.
2 For this theorem to be valid, \( f(t) \) must be continuous or have a step discontinuity at \( t = 0 \) (i.e., no impulses or their derivatives at \( t = 0 \)).
Laplace Transform

Back to example 2

\[ r(t) = \frac{v(t)}{R} + C \frac{dv(t)}{dt} + \frac{1}{L} \int_0^t v(t) dt \]

Applying Laplace transform (assume that the initial voltage of capacitor is zero).

\[ R(s) = \frac{V(s)}{R} + sCV(s) + \frac{V(s)}{sL} \]

\[ Output \quad V(s) \quad R(s) \quad G(s) \]
\[ Transfer \quad RLs \quad RLCs^2 + Ls + R \quad Function \]

Transfer Function

\[ \frac{R(s)}{C(s)} = \frac{b_ms^m + b_{m-1}s^{m-1} + \ldots + b_0}{a_ns^n + a_{n-1}s^{n-1} + \ldots + a_0} \]
Transfer Function

\[ r(t) \rightarrow g(t) \rightarrow y(t) = r(t)g(t) \]

\[ R(s) \rightarrow G(s) \rightarrow Y(s) = R(s)G(s) \]

Block Diagram Transformations

Combining blocks in cascade:

\[ X_1 \rightarrow G_1(s) \rightarrow X_2 \rightarrow G_2(s) \rightarrow X_3 \]

or

\[ X_1 \rightarrow G_1G_2 \rightarrow X_3 \]

or

\[ X_1 \rightarrow G_2G_1 \rightarrow X_3 \]
Block Diagram Transformations

Moving a summing point behind a block:

How about moving a summing point ahead of a block? Try on your own.

Block Diagram Transformations

Moving a pickoff point ahead of a block:
Block Diagram Transformations

Moving a pickoff point behind a block:

Eliminating a feedback loop:

Very Important!
Block Diagram Transformations

Example:

![Block Diagram Example](image1)

Block Diagram Transformations

Example (Cont.):

![Block Diagram Example (Cont.)](image2)
Example (Cont.):

\[ Y(s) = \frac{G_2 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2} \]

Example (Cont.):

\[ Y(s) = \frac{G_1 G_2 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3} \]