Dynamic Spectrum Management for Multiple-Antenna Cognitive Radio Systems: Designs with Imperfect CSI

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Abstract

In this paper, we study the problem of resource allocation and optimization for multiple-input multiple-output (MIMO) cognitive radio (CR) systems under the assumption of imperfect channel state information (CSI) of the channels between the secondary users (SUs) and the primary users (PUs) at the SUs. We formulate robust design optimization problems for CR systems with one or more SUs communicating over a single or multiple frequency carriers in the presence of multiple PUs. We propose a linear matrix inequality (LMI) transformation that facilitates proper treatment of channel uncertainty at the SU transmitter and we provide solutions to the design problems based on convex optimization and Lagrange dual decomposition techniques. Finally, we demonstrate the importance of the proposed formulations and the implications of ignoring channel uncertainties when designing for CR systems.

Index Terms

Spectrum Management, Cognitive Radio Systems, Multiple Carrier Systems, Partial Channel Knowledge, Convex Optimization.

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I. INTRODUCTION

Cognitive radio (CR) systems have received a growing attention from the research community in the last few years due to the increased interest in solving the scarcity problem of the radio frequency (RF) resource. CR solutions rely on the well established fact that the allocated frequency spectrum is highly underutilized [1, 2]. Thus, a CR system, consisting of one or more secondary users (SUs), can be allowed to operate within a pre-assigned spectrum band as long as it guarantees no harmful disturbance to the operations by the spectrum owners (a.k.a. primary users (PUs)). To this end, the CR system has to utilize intelligent algorithms for sensing and prediction of traffic by the PUs. Once a spectrum hole is detected, a secondary transmission can take place under some restrictions to avoid causing any significant interference to neighboring active primary transmissions[1].

Most of the work in the CR-related literature falls under one of the following two categories. The first set of work deals with the problem of measuring, sensing and prediction of the unused resources, cf. e.g. [5, 6]. The other set of work deals with the problem of efficient utilization of the discovered resources, cf. e.g. [7–11]. While both of these categories are of great importance, in this paper, we focus our attention to the latter.

Assuming perfect knowledge of the channels between the SU transmitter and PUs at the SU transmitter, the design of CR systems subject to constraints on the interference caused at PUs has been studied in [7, 8]. However, in practical scenarios, perfect knowledge of the channels to PUs may not be possible. Under the assumption of partial knowledge of these channels at the SU transmitter, robust design of a single-user CR system was considered in [9]. In [9], Zhang et al. considered a CR system with multiple antennas at the SU transmitter that communicates with a single antenna SU receiver in the presence of a single PU with a single receive antenna.

[1] In CR systems, either an ON/OFF model can be adopted in which no secondary transmission is allowed in the presence of primary activity [3], or a limited interference model in which the maximum amount of interference generated at the primary receiver is specified [4].

The assumption of a single antenna at the SU and PU receivers allowed the robust design to be formulated as a second order cone programming (SOCP) problem. Furthermore, while this manuscript was under review, robust designs maximizing the worst case signal-to-interference ratio (SINR) of the SUs subject to interference constraints to the PUs for secondary and primary systems with single-antenna receivers appeared in [12] and [13].

In our work, we study different design problems for CR systems with partial knowledge of the CSI of PUs channels at the SU transmitter. We first consider general point-to-point systems with multiple antennas at both the transmitter and receiver of the CR system in the presence of multiple PUs with multiple receive antennas each. We formulate the robust mutual information (MI) maximization of such system, and we show its equivalence to a convex optimization problem that can be efficiently solved. Different from previous and concurrent works on robust designs mentioned above, we distinguish between single and multiple-carrier systems. The latter makes different constraints on interference caused to PUs applicable and renders a Lagrange dual decomposition approach attractive to solve the resulting optimization problems. We also demonstrate how the convex optimization formulation can be applied to solve other related problems such as the robust minimization of the CR transmitted power subject to rate constraints. Finally, we study extensions of these design problems to scenarios with multipoint-to-point CR systems with multiple carriers.

Organization: The rest of this paper is organized as follows. In Section II we present the system and channel uncertainty models to be used in this paper. In Section III we formulate and discuss the optimization problems for the point-to-point, single and multiple-carrier CR case. We extend the results from Section III to the multipoint-to-point CR case in Section IV. Finally, in Section V we present performance results and we conclude the paper in Section VI.

Notation: In this paper, we use bold upper case and lower case letters for matrices and vectors, respectively. $[\cdot]^T$, $[\cdot]^H$, and $[\cdot]^{-1}$ denote transposition, Hermitian transposition, and matrix inversion, respectively. $\mathrm{Tr}(\cdot)$ refers to the trace of a matrix, $\mathrm{vec}(\boldsymbol{A})$ converts the matrix $\boldsymbol{A}$ into a column vector, and $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix. $\otimes$ denotes the Kronecker product of two matrices, $\log\det(\boldsymbol{A})$ refers to the base 2 logarithm of the determinant of the matrix $\boldsymbol{A}$ and $\Re(\cdot)$ refers to the real part of a complex number. Finally, given a square matrix $\boldsymbol{A}$, inequalities of the form $\boldsymbol{A} \geq \boldsymbol{0}$ denote that the matrix $\boldsymbol{A}$ is positive semidefinite, while inequalities of the form $\boldsymbol{a} \geq \boldsymbol{0}$ are treated element wise.

II. System Model

In this section, we first introduce the CR transmission model and then motivate and formulate the adopted channel uncertainty model.

A. *Point-to-Point and Multipoint-to-Point CR Systems*

We consider CR communication systems that share a group of N parallel channels, i.e., multiple-carrier systems, with a number, L, of PUs which are the licensed owners of the bandwidth. We repeatedly specialize to the case $N = 1$, which we refer to as single-carrier case in the following. Furthermore, we distinguish between point-to-point and multipoint-to-point CR systems, where in the latter K SU transmitters communicate with a SU receiver. The general multipoint-to-point CR transmission system is depicted in Figure 1

Let us first consider the point-to-point CR system, i.e., $K = 1$. We assume that the SU transmitter is equipped with $M_{t,SU}$ transmit antennas and communicates with a SU receiver that is equipped with $M_{r,SU}$ receive antennas. In this multiple-input multiple-output (MIMO) system, the vector of signals received at the SU receiver at the n^{th} carrier[2] can be written as

$$\boldsymbol{y}_n = \boldsymbol{H}_n \boldsymbol{x}_n + \boldsymbol{n}_n, \tag{1}$$

where $\boldsymbol{H}_n \in \mathbb{C}^{M_{r,SU} \times M_{t,SU}}$ is the channel matrix between the SU transmitter and receiver over the n^{th} carrier, $\boldsymbol{x}_n$ is the vector of transmitted signals from the SU transmitter at carrier n whose covariance matrix is $\mathbb{E}\left\{\boldsymbol{x}_n \boldsymbol{x}_n^H\right\} = \boldsymbol{Q}_n$, and $\boldsymbol{n}_n$ is the i.i.d. circularly symmetric complex Gaussian

[2]Note that we drop the carrier index n in the following when we specialize to the single-carrier case $N = 1$.

noise vector at carrier n at the SU receiver whose covariance matrix is $\mathbb{E}\left\{\boldsymbol{n}_n\boldsymbol{n}_n^H\right\} = \boldsymbol{I}_{M_{r,SU}}, \forall n$ The transmission from the CR system will also be received by each PU because of the shared bandwidth and the vector of interference signals observed in the n^{th} carrier at the ℓ^{th} PU receiver can be expressed as

$$\boldsymbol{y}_{\text{PU},\ell,n} = \boldsymbol{G}_{\ell,n}\boldsymbol{x}_n, \quad \ell \in \{1,\ldots,L\}, n \in \{1,\ldots,N\}, \tag{2}$$

where $\boldsymbol{G}_{\ell,n} \in \mathbb{C}^{M_{r,PU}\times M_{t,SU}}$ is the channel matrix between the SU transmitter and the ℓ^{th} PU receiver over the n^{th} carrier. The received signal from the CR transmitter, $\boldsymbol{y}_{\text{PU},\ell,n}$, will result in added interference to the different PUs and typically constraints that limit this interference are enforced to protect the rights of the PUs as licensed owners of the bandwidth. We will consider two types of interference constraints. The first is a per-carrier interference constraint

$$\text{Tr}\left\{\mathbb{E}\left\{\boldsymbol{y}_{\text{PU},\ell,n}\,\boldsymbol{y}_{\text{PU},\ell,n}^H\right\}\right\} = \text{Tr}\left(\boldsymbol{G}_{\ell,n}\boldsymbol{Q}_n\boldsymbol{G}_{\ell,n}^H\right) \le I_{\ell,n}, \quad \ell \in \{1,\cdots,L\}, n \in \{1,\cdots,N\}, \tag{3}$$

in which we set a maximum limit $I_{\ell,n}$ on the interference from each carrier. The second type of constraints is a total interference constraint

$$\sum_{n=1}^{N} \text{Tr}\left(\boldsymbol{G}_{\ell,n}\boldsymbol{Q}_n\boldsymbol{G}_{\ell,n}^H\right) \le I_{\ell}, \quad \ell \in \{1,\ldots,L\}. \tag{4}$$

That is, the sum of CR interference from all carriers is assumed to be below the maximum allowable interference level I_ℓ at the ℓ^{th} PU receiver resulting from the CR system transmission. The maximum allowable interference level is often referred to as the *interference temperature* [4]. The interference temperature is usually set at a level that satisfies the quality-of-service (QoS) requirement of the PU. We assume that this maximum level is known at the SU transmitter. This is possible through public regulatory guidelines that govern the use of the frequency band of interest.

As will be shown in Section III, the design formulation with the per-carrier interference constraints (3) is more restrictive than that with the total interference constraint (4). Hence, the latter gives more flexibility to the operation of CR systems. On the other hand, per-carrier interference constraints (3) can be considered as a more fine-grained approach to interference limitation better suited to take into account spectral masks in which different carriers have different levels of interference tolerance [14]. Furthermore, as we will demonstrate in Section III-B, design problems with per-carrier interference constraints can have lower complexity than the corresponding ones with a total interference constraint.

The formulations of the interference constraints in (3) and (4) can also be used to impose constraints on the interference signal received by each antenna at the receiver of the PU, or by different subsets of these antennas, cf. [7, Sec. II]. Furthermore, it is worth mentioning that the presence of multiple antennas at the receiver of the PU allows (spatial) processing of the received signals $\boldsymbol{y}_{\mathrm{PU},\ell,n}$ into $\overline{\boldsymbol{y}}_{\mathrm{PU},\ell,n} = \boldsymbol{U}_{\ell,n}\boldsymbol{y}_{\mathrm{PU},\ell,n} = \boldsymbol{U}_{\ell,n}\boldsymbol{G}_{\ell,n}\boldsymbol{x}_n$. Thus, one could impose interference constraints on the processed signal $\overline{\boldsymbol{y}}_{\mathrm{PU},\ell,n}$. However, this would require the availability of more information at the SU. More specifically, each PU's receiving matrices $\boldsymbol{U}_{\ell,n}$ must be known to the SU. This extra knowledge is only justifiable in CR systems with a higher level of collaboration between SU and PU systems (i.e., feedback from PUs to the SU system), but it is not obtainable with an SU listening to PU signals only as described in Section II-B and "far less appealing from a practical standpoint" [10, Sec. II]. Nevertheless, if such knowledge is assumed, the approaches that are described in the following sections can still be applied to these constraints as well.

For the extension to multipoint-to-point systems (see Figure 1 for $K > 1$), we will use $k = 1, \ldots, K$ as the SU transmitter index. $\boldsymbol{H}_{k,n}$ is the the channel matrix between the k^{th} SU transmitter and the SU receiver over the n^{th} carrier, $n = 1, \ldots, N$, and $\boldsymbol{G}_{\ell,k,n}$ represents the channel matrix between the k^{th} SU transmitter and the ℓ^{th} PU receiver over the n^{th} carrier.

B. Channel Uncertainty Model

For readability, let us consider the point-to-point CR system (the same model described here is directly applicable to the multipoint-to-point system on a per-SU basis). We make the common assumption that the channels between the SU transmitter and receiver, $\boldsymbol{H}_n$, are perfectly known at the SU transmitter (cf. e.g. [7–9]). However, we only assume partial channel knowledge of the channels $\boldsymbol{G}_{\ell,n}$ between the SU transmitter and PUs. The discrepancy in quality of channel state information is clearly motivated by the fact that SU transmitter and receiver belong to the same communication system, which provides a number of possibilities for refining channel estimation, while the SU and PUs do not cooperate to facilitate estimation of the channels between SU transmitter and PU receivers. One possible scenario in which the SU may obtain an estimate of the channels $\boldsymbol{G}_{\ell,n}$ is when both the PUs and SU are sharing the same frequency band and the PU systems are time division duplex (TDD). Through the SU's knowledge of the PUs' pilot symbols, it can estimate the channels $\boldsymbol{G}_{\ell,n}$, e.g., [7, 9, 10]. Hence, we assume

$$\boldsymbol{G}_{\ell,n} = \hat{\boldsymbol{G}}_{\ell,n} + \boldsymbol{E}_{\ell,n} \tag{5}$$

where $\hat{\boldsymbol{G}}_{\ell,n}$ is the estimate of $\boldsymbol{G}_{\ell,n}$ and $\boldsymbol{E}_{\ell,n}$ is the estimation error matrix.

Using this estimate and a confidence region that is selected based on the accuracy of the estimation method used, the SU can obtain knowledge of these channels with a *bounded uncertainty model* [9], [10, Sec. IV].[3] Applying this model, the uncertainty in the channel estimation is described by the bounded region

$$\mathcal{U}\left(\epsilon_{\ell,n}\right) = \left\{\boldsymbol{G}_{\ell,n} | \boldsymbol{G}_{\ell,n} = \hat{\boldsymbol{G}}_{\ell,n} + \boldsymbol{E}_{\ell,n}, \mathrm{Tr}\left(\boldsymbol{E}_{\ell,n}\boldsymbol{R}_{\ell,n}\boldsymbol{E}_{\ell,n}^{H}\right) \leq \epsilon_{\ell,n}^{2}\right\}, \tag{6}$$

where $\boldsymbol{R}_{\ell,n}$ is a Hermitian positive definite shaping matrix that describes the shape of the uncertainty region $\mathcal{U}\left(\epsilon_{\ell,n}\right)$. For example, assuming $\boldsymbol{E}_{\ell,n}$ as zero-mean with covariance matrix $\mathbb{E}\left\{\boldsymbol{E}_{\ell,n}^{H}\boldsymbol{E}_{\ell,n}\right\} = \boldsymbol{\Sigma}_{\boldsymbol{E}\ell,n}$, $\boldsymbol{R}_{\ell,n} = \left(\boldsymbol{\Sigma}_{\boldsymbol{E}\ell,n}\right)^{-1}$ has been used in [9, Sec. II] for the special case $M_{r,PU} = 1$. Similar models were also considered in the context of single user MIMO systems (e.g. [15, 16]), and in the context of multiple-user systems (e.g. [17]).

III. Point-to-Point CR Systems

In this section, we consider the CR system with a single secondary transmitter and receiver. We distinguish between the special case of $N = 1$, i.e., single-carrier case, and the multiple-carrier case $N > 1$, since the two cases lead to different robust optimization problems.

[3]A bounded uncertainty model for SU-to-PU channels was also adopted in [12, 13], which appeared while this manuscript was under review.

A. Single-Carrier Case

For convenience, we drop the carrier index n in this section.

1) Mutual Information (MI) Maximization Subject to a Power Constraint: We will start with the MI maximization problem subject to a total transmitter power constraint and constraints to limit interference with each of the L PUs. For the bounded channel uncertainty model in (6), we will design CR systems such that the interference constraints are enforced for every $\boldsymbol{G}_\ell \in \mathcal{U}(\epsilon_\ell)$. This problem can be formulated as:

$$\max_{\boldsymbol{Q}} \quad \log\det\left(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right) \tag{7a}$$

$$\text{s.t.} \quad \text{Tr}\left(\boldsymbol{Q}\right) \leq P, \tag{7b}$$

$$\text{Tr}\left(\boldsymbol{G}_\ell \boldsymbol{Q} \boldsymbol{G}_\ell^H\right) \leq I_\ell, \quad \forall\, \boldsymbol{G}_\ell \in \mathcal{U}(\epsilon_\ell),\ \ell \in \{1, \cdots, L\}, \tag{7c}$$

$$\boldsymbol{Q} \geq \boldsymbol{0}, \tag{7d}$$

where P is the total allowable power for the secondary transmitter. While the problem formulation in (7) is convex, the inequality in (7c) represents L sets with infinitely many constraints, one for each $\boldsymbol{G}_\ell$ in $\mathcal{U}(\epsilon_\ell)$[4]. However, each of these infinite sets of constraints can be exactly characterized by a single LMI constraint. This result is summarized in the following theorem.

Theorem 1: For the bounded channel uncertainty model in (6), the optimization problem in (7) is equivalent to

$$\max_{\boldsymbol{Q}, \boldsymbol{S}_\ell, \mu_\ell} \quad \log\det\left(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right) \tag{8a}$$

$$\text{s.t.} \quad \text{Tr}\left(\boldsymbol{Q}\right) \leq P, \tag{8b}$$

$$\text{Tr}\left(\hat{\boldsymbol{G}}_\ell \boldsymbol{Q} \hat{\boldsymbol{G}}_\ell^H\right) + \text{Tr}\left(\hat{\boldsymbol{G}}_\ell \boldsymbol{S}_\ell \hat{\boldsymbol{G}}_\ell^H\right) + \mu_\ell \epsilon_\ell^2 \leq I_\ell, \quad \ell \in \{1, \cdots, L\}, \tag{8c}$$

[4]In case that the interference constraint is dropped, the problem in (7) can be solved efficiently in closed form by taking the SVD of the channel matrix $\boldsymbol{H}$ and allocating the power using water-filling over the resulting parallel beams [7]. When interference constraints are added as in (7), no closed form solution is usually available and numerical techniques are utilized to find an optimal solution.

$$\begin{bmatrix} \boldsymbol{S}_\ell & \boldsymbol{Q} \\ \boldsymbol{Q} & \mu_\ell \boldsymbol{R}_\ell - \boldsymbol{Q} \end{bmatrix} \succeq \boldsymbol{0}, \ \mu_\ell \geq 0, \qquad \ell \in \{1, \cdots, L\}, \tag{8d}$$

$$\boldsymbol{Q} \succeq \boldsymbol{0}. \tag{8e}$$

Proof: See Appendix A. ■

The above optimization problem is convex and can be efficiently solved using interior point methods [18–20].

We observe from the design formulation in (8) that the uncertainty of the PUs' channels results in an increased interference that is created at the PUs. This is evident from the extra interference terms on the left hand side of the constraint (8c) compared to the corresponding constraints in the case of perfect channel state information of the PUs' channels.

2) Power and Interference Minimization Subject to Rate Constraint: We now demonstrate that the design approach presented in the previous section can be applied to obtain design formulations for other related CR problems. One of these problems is the robust design of the CR transmitter in a way to minimize the transmitted power subject to achieving a minimum rate constraint, C_{tar}, for the secondary receiver and subject to satisfying each PU's maximum interference constraint for every channel $\boldsymbol{G}_\ell \in \mathcal{U}(\epsilon_\ell)$. This problem is of a particular interest to CR systems with battery-powered mobile SU transmitters. Minimizing the transmitted power from these terminals translates directly to a longer terminal and network lifetime. Using an approach similar to the one presented in the previous section, this robust design problem is equivalent to the following convex optimization problem:

$$\min_{\boldsymbol{Q}, \boldsymbol{S}_\ell, \mu_\ell} \quad \text{Tr}\,(\boldsymbol{Q}) \tag{9a}$$

$$\text{s.t.} \quad \log\det\left(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^H\right) \geq C_{\text{tar}}, \tag{9b}$$

and (8c)-(8e). Similarly, one may design the CR transmitter in a way to minimize the total, or weighted sum, interference to the PUs subject to a minimum rate constraint. By limiting the interference generated at the different PUs to the minimum possible level, the CR system may leave some room for other CR systems to use available resource. In other cases, the frequency band the PU is using might be unlicensed or leased by the secondary network. In such a case, there is no solid interference threshold that needs to be strictly preserved and causing the lowest interference is mutually beneficial. The formulation for this design problem can be obtained by extending the design variables in (9) to include I_ℓ and replacing the objective in (9a) by the (weighted) sum of I_ℓ.

It is worth mentioning that while the MI maximization problem is always feasible (since $\boldsymbol{Q} = \boldsymbol{0}$ is always a feasible point), the power and interference minimization problems subject to rate constraint may not be always feasible for arbitrary C_{tar} and I_ℓ. We will provide numerical examples regarding feasibility of this problem in Section V.

B. Multiple-Carrier Case

We now consider the scenario in which the SU and PUs share N parallel channels such as in multiple-carrier communication systems. We focus on sum-rate maximization, but note that robust the power minimization problem (9) can be extended to the multiple-carrier case too (see results in Section V).

1) Robust Optimization Problem: Given the channel uncertainty model in (6), the robust sum rate maximization problem can be formulated as

$$\max_{\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}} \quad \sum_{n=1}^{N} \log\det\left(\boldsymbol{I} + \boldsymbol{H}_n \boldsymbol{Q}_n \boldsymbol{H}_n^H\right) \tag{10a}$$

$$\text{s.t.} \quad \sum_{n=1}^{N} \operatorname{Tr}\left(\boldsymbol{Q}_n\right) \le P, \tag{10b}$$

$$\left\{ \begin{array}{l} \sum_{n=1}^{N} \left[\operatorname{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n}\boldsymbol{Q}_n\hat{\boldsymbol{G}}_{\ell,n}^H\right) + \operatorname{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n}\boldsymbol{S}_{\ell,n}\hat{\boldsymbol{G}}_{\ell,n}^H\right) + \mu_{\ell,n}\epsilon_{\ell,n}^2\right] \le I_\ell, \quad \forall \ell, \\ \text{or} \\ \operatorname{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n}\boldsymbol{Q}_n\hat{\boldsymbol{G}}_{\ell,n}^H\right) + \operatorname{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n}\boldsymbol{S}_{\ell,n}\hat{\boldsymbol{G}}_{\ell,n}^H\right) + \mu_{\ell,n}\epsilon_{\ell,n}^2 \le I_{\ell,n}, \quad \forall \ell,\ \forall n. \end{array} \right\} \tag{10c}$$

$$\boldsymbol{A}_{\ell,n}(\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}) = \begin{bmatrix} \boldsymbol{S}_{\ell,n} & \boldsymbol{Q}_n \\ \boldsymbol{Q}_n & \mu_{\ell,n}\boldsymbol{R}_{\ell,n} - \boldsymbol{Q}_n \end{bmatrix} \ge \boldsymbol{0}, \quad \forall \ell,\ \forall n \tag{10d}$$

$$\mu_{\ell,n} \geq 0, \qquad \forall \ell,\ \forall n, \tag{10e}$$

$$\boldsymbol{Q}_n \geq \boldsymbol{0}, \qquad \forall n, \tag{10f}$$

where the first and second line of (10c) are applied for the case of total (4) and per-carrier (3) interference constraints, respectively. Similar to the design formulation in (8), the optimization problem in (10) is convex and can be solved efficiently.

2) Lagrange Dual Problem: An alternative approach to solving (10) is through deriving the Lagrange dual of the robust optimization problem in (10). The Lagrange dual decomposition approach has been used extensively in the literature to solve different resource allocation and optimization problems [21–25]. As we demonstrate below, this approach results in decomposing the optimization problem in (10) into N independent and similar sub-problems that can be solved in parallel.

Let us first consider the case of a total interference constraint (4). Defining the convex regions

$$\mathcal{B}_n = \left\{ (\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}) \mid \boldsymbol{Q}_n \geq \boldsymbol{0},\ \mu_{\ell,n} \geq 0,\ \boldsymbol{A}_{\ell,n}(\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}) \geq \boldsymbol{0},\ \forall \ell \in \{1, \cdots, L\} \right\}, \tag{11}$$

the Lagrange dual function of the primal problem in (10) can be written as

$$\begin{aligned} g(\lambda, \boldsymbol{\eta}) = & \max_{\substack{\{\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \\ \mu_{\ell,n}\} \in \mathcal{B}_n}} \sum_{n=1}^{N} \log\det\left(\boldsymbol{I} + \boldsymbol{H}_n \boldsymbol{Q}_n \boldsymbol{H}_n^H\right) - \lambda \left(\sum_{n=1}^{N} \mathrm{Tr}\left(\boldsymbol{Q}_n\right) - P \right) \\ & - \sum_{\ell=1}^{L} \eta_\ell \left\{ \sum_{n=1}^{N} \left(\mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n} \boldsymbol{Q}_n \hat{\boldsymbol{G}}_{\ell,n}^H \right) + \mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n} \boldsymbol{S}_{\ell,n} \hat{\boldsymbol{G}}_{\ell,n}^H \right) + \mu_{\ell,n} \epsilon_{\ell,n}^2 \right) - I_\ell \right\} \\ = & \sum_{n=1}^{N} g_n(\lambda, \boldsymbol{\eta}) + \lambda P + \sum_{\ell=1}^{L} \eta_\ell I_\ell, \end{aligned} \tag{12}$$

where the functions $g_n(\lambda, \boldsymbol{\eta})$ are defined as

$$\begin{aligned} g_n(\lambda, \boldsymbol{\eta}) = & \max_{\{\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}\} \in \mathcal{B}_n} \log\det\left(\boldsymbol{I} + \boldsymbol{H}_n \boldsymbol{Q}_n \boldsymbol{H}_n^H\right) - \lambda \mathrm{Tr}\left(\boldsymbol{Q}_n\right) \\ & - \sum_{\ell=1}^{L} \eta_\ell \left\{ \mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n} \boldsymbol{Q}_n \hat{\boldsymbol{G}}_{\ell,n}^H \right) + \mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n} \boldsymbol{S}_{\ell,n} \hat{\boldsymbol{G}}_{\ell,n}^H \right) + \mu_{\ell,n} \epsilon_{\ell,n}^2 \right\}. \end{aligned} \tag{13}$$

For a given λ and $\boldsymbol{\eta}$, the functions $g_n(\lambda, \boldsymbol{\eta})$ represent a convex optimization problem. Furthermore, they are independent and can be simultaneously evaluated. Finally, the dual problem can be written as

$$\min_{\lambda,\boldsymbol{\eta}} \quad g(\lambda,\boldsymbol{\eta}) \qquad \text{s.t.} \quad \lambda \geq 0, \quad \boldsymbol{\eta} \geq \mathbf{0}, \tag{14}$$

which can be solved using gradient based approaches such as the ellipsoid method [21].

While design formulation with the per-carrier interference constraints (3) result in a larger number of constraints in the primal problem (see (10c)), the dual problem can be significantly simplified. Defining the convex regions $\mathcal{C}_n$

$$\mathcal{C}_n = \Big\{(\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}) \mid \boldsymbol{Q}_n \geq \mathbf{0},\ \mu_{\ell,n} \geq 0,\ \boldsymbol{A}_{\ell,n}(\boldsymbol{Q}_n, \boldsymbol{S}_{\ell,n}, \mu_{\ell,n}) \geq \mathbf{0}, \\ \mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n}\boldsymbol{Q}_n\hat{\boldsymbol{G}}_{\ell,n}^H\right) + \mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,n}\boldsymbol{S}_{\ell,n}\hat{\boldsymbol{G}}_{\ell,n}^H\right) + \mu_{\ell,n}\epsilon_{\ell,n}^2 \leq I_{\ell,n},\ \ell \in \{1,\cdots,L\}\Big\}, \tag{15}$$

the Lagrange dual function becomes (cf. [7])

$$\begin{aligned} g(\lambda) &= \max_{\{\boldsymbol{Q}_n,\boldsymbol{S}_n,\mu_{\ell,n}\}\in\mathcal{C}_n} \sum_{n=1}^{N} \log\det\left(\boldsymbol{I} + \boldsymbol{H}_n\boldsymbol{Q}_n\boldsymbol{H}_n^H\right) - \lambda\left(\sum_{n=1}^{N}\mathrm{Tr}\left(\boldsymbol{Q}_n\right) - P\right) \\ &= \sum_{n=1}^{N} g_n(\lambda) + \lambda P, \end{aligned} \tag{16}$$

where

$$g_n(\lambda) = \max_{\boldsymbol{Q}_n,\boldsymbol{S}_{\ell,n},\mu_{\ell,n}\in\mathcal{C}_n} \quad \log\det\left(\boldsymbol{I} + \boldsymbol{H}_n\boldsymbol{Q}_n\boldsymbol{H}_n^H\right) - \lambda\mathrm{Tr}\left(\boldsymbol{Q}_n\right). \tag{17}$$

The dual problem can then be written as

$$\min_{\lambda} \quad g(\lambda) \qquad \text{s.t.} \quad \lambda \geq 0. \tag{18}$$

Compared to (14), the problem in (18) is a single variable optimization problem and since $g(\lambda)$ is convex in λ, bisection search over λ can be used to obtain the optimal solution [18, page 249].

3) A Comparison Between Primal and Dual Approaches: We would like to highlight some aspects of the primal and dual approaches for solving the optimization problem for point-to-point multiple-carrier CR systems. Consider the total interference constraint problem in Section III-B1. The primal optimization problem in (10) has $N(L+1)$ positive semidefinite matrices for $\boldsymbol{Q}_n$ and $\boldsymbol{S}_{\ell,n}$, and each of these matrices consists of $\frac{1}{2}M_{t,SU}(M_{t,SU}+1)$ (distinct) variables. The problem also includes another NL real variables for $\mu_{\ell,n}$. The number of LMI constraints for this problem is NL constraints of size $2M_{t,SU}$ in (10d), and N constraints of size $M_{t,SU}$ in (10f). While the size of the variables and constraints in (10) scales linearly with the number of the carriers, N, the complexity of the solving (10) (as log determinant maximization problem subject to LMI constraints) will grow as polynomial of N whose order is significantly higher than the first degree (e.g., [26]). In contrast to this, the dual problem for the total interference constraint problem provides an alternative approach whose complexity grows linearly with N. To see this, consider the dual formulation in (14) for the total interference constraint problem in Section III-B1. Each iteration will require the evaluation of $g(\lambda, \boldsymbol{\eta})$ in (12), which is mainly solving N instances of the optimization problem in (13). While each instance of the optimization problem in (13) has similar structure to the primal design formulation, it has a smaller number of variables and constraints, and, what is more important, these numbers are independent of N. Hence, the complexity of the dual approach scales linearly with N. A similar argument holds for the dual approach of optimization problem with per-carrier interference constraints in the previous section. Similar advantages of the dual approach (linear growth with number of carriers) have been previously shown for other multi-carrier systems such as DSL systems, e.g. [21], and in the downlink of wireless OFDMA systems, e.g. [24].

IV. Multipoint-to-Point CR Systems

Finally, we proceed to the general case in which we consider a CR system with multiple SUs in an uplink multiple-carrier communication scenario. We consider processing at the base station, to which necessary parameters are fed back by users, which distributes transmission covariance matrices $\boldsymbol{Q}_{k,n}$ to users. In order to avoid interference between the SUs themselves, we assume that each carrier is assigned, at most, to one SU at any time. That is, if the carrier of index $n = 1, \cdots, N$ is assigned to user $\hat{k}$ ($\boldsymbol{Q}_{\hat{k},n} \neq \mathbf{0}$), then $\boldsymbol{Q}_{k,n} = \mathbf{0}$ for every $k \neq \hat{k}$. Given a set of maximum allowable powers $\mathcal{P} = \{P_1, \ldots, P_K\}$ for each SU $k = 1, \ldots, K$, the joint problem of carrier assignment and robust resource allocation for sum rate maximization under a total interference constraint can be formulated as

$$\max_{\substack{\boldsymbol{Q}_{k,n},\boldsymbol{S}_{\ell,k,n},\\ \mu_{\ell,k,n}}} \quad \sum_{k=1}^{K}\sum_{n=1}^{N} \log\det\left(\boldsymbol{I} + \boldsymbol{H}_{k,n}\boldsymbol{Q}_{k,n}\boldsymbol{H}_{k,n}^{H}\right), \tag{19a}$$

$$\text{s.t.} \quad \sum_{n=1}^{N} \mathrm{Tr}\left(\boldsymbol{Q}_{k,n}\right) \leq P_k, \quad \forall k, \tag{19b}$$

$$\sum_{k=1}^{K}\sum_{n=1}^{N}\left[\mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,k,n}\boldsymbol{Q}_{k,n}\hat{\boldsymbol{G}}_{\ell,k,n}^{H}\right) + \mathrm{Tr}\left(\hat{\boldsymbol{G}}_{\ell,k,n}\boldsymbol{S}_{\ell,k,n}\hat{\boldsymbol{G}}_{\ell,k,n}^{H}\right) + \mu_{\ell,k,n}\epsilon_{\ell,k,n}^{2}\right] \leq I_\ell,\ \forall \ell, \tag{19c}$$

$$\boldsymbol{A}_{\ell,k,n}(\boldsymbol{Q}_{k,n}, \boldsymbol{S}_{\ell,k,n}, \mu_{\ell,k,n}) = \begin{bmatrix} \boldsymbol{S}_{\ell,k,n} & \boldsymbol{Q}_{k,n} \\ \boldsymbol{Q}_{k,n} & \mu_{\ell,k,n}\boldsymbol{R}_{\ell,k,n} - \boldsymbol{Q}_{k,n} \end{bmatrix} \geq \boldsymbol{0},\ \mu_{\ell,k,n} \geq 0,$$

$$\forall\, \ell, \forall\, k, \forall\, n, \tag{19d}$$

$$\boldsymbol{Q}_{k,n} \geq \boldsymbol{0}, \quad \forall\, k, \forall\, n, \tag{19e}$$

$$\text{if } \boldsymbol{Q}_{\hat{k},n} \neq \boldsymbol{0} \text{ then } \boldsymbol{Q}_{k,n} = \boldsymbol{0}, \boldsymbol{S}_{\ell,k,n} = \boldsymbol{0}, \mu_{\ell,k,n} = 0 \quad \forall\, k \neq \hat{k}, \forall\, \ell, \forall\, n. \tag{19f}$$

Alternative formulations using per-carrier interference constraints, per-user interference constraints, or both can be easily derived and the solution to the new formulations should, in general, be less complex.

Problem (19) is no longer convex due to the exclusivity constraint in (19f). Thus, the primal problem can no longer be solved using convex optimization tools, and an exhaustive search over all possible carrier assignments is needed to obtain a globally optimal solution. However, this approach can be prohibitively complex, especially when the number of carriers, N, is large. One approach to overcome this prohibitive complexity is to use a Lagrange dual decomposition approach similar to the one presented in Section III-B and used in [24, 25]. It is worth mentioning that while the primal problem in the point-to-point multiple-carrier case in (10) is convex, the multi-user problem with the exclusivity constraint in (19) is not, and generally the duality gap is non-zero[5]. Thus, the solution obtained by using the Lagrange dual decomposition approach is an upper bound for the optimal solution of (19). Nevertheless, the approach can significantly reduce the complexity of the problem in (19).

It is also worth mentioning that the design problem of a downlink robust CR system with multiple-carriers can be solved in an analogous way after replacing the per-user power constraints in (19b) for the uplink system by the single power constraint $\sum_{k=1}^{K}\sum_{n=1}^{N}\operatorname{Tr}\left(\boldsymbol{Q}_{k,n}\right) \leq P$. In this case, the vector of dual variables $\boldsymbol{\lambda}$ in the uplink dual problem reduces to a single variable. This, in turn, helps reduce the complexity of finding the optimal Lagrange multipliers for the dual optimization problem.

V. Simulation Results

In this section we present some numerical results to evaluate the performance of the proposed algorithms. We consider CR systems in which SU transmitters and receivers are equipped with two antennas each, $M_{t,SU} = M_{r,SU} = 2$. Similarly, each PU receiver is equipped with two antennas, $M_{r,PU} = 2$. The coefficients of the channels $\boldsymbol{H}_n$ between any SU transmitter and its corresponding receiver are modeled as i.i.d. circularly symmetric complex Gaussian random variables with unit variance. The coefficients of $\hat{\boldsymbol{G}}_{\ell,n}$ are modeled similarly with a variance equal to[6] 10^{-3} and $\boldsymbol{R}_{\ell,n} = \boldsymbol{I}_{M_{r,PU}}$. Average plots are generated using Monte Carlo simulations over 5000 channel sets and CVX toolbox for Matlab [20] was used to solve the problems under consideration. On average, it took CVX 1.52 and 4.92 seconds to solve the single and multiple carrier problems of Sections V-A1 and V-A2 below on a Xeon E5405 processor running at 2.00 GHz clock speed. Given the generic nature of the optimization toolbox and the underling

[5]For the case of single antenna users in the non-CR context, it was shown that the duality gap vanishes as N grows [21, 24, 25, 27].

[6]The choice of channels variances represents a typical scenario in which the PUs are farther from the SU transmitter than the SU receiver. Nevertheless, absolute values are of little significance for the ensuing performance evaluation, as a change to the variance of the coefficients of $\boldsymbol{G}_\ell$ would be compensated by a change of I_ℓ.

software platform used, we believe that these figures can be significantly enhanced in any practical implementation of the design problems under consideration.

A. Point-to-Point CR System

1) MI maximization for a single-carrier CR system ($N = 1$): We first consider a CR system with a single point-to-point link sharing the bandwidth with a single PU $(L = 1)$, and we study the robust MI maximization problem in (7). In Figure 2, we plot the average MI of the CR system versus the maximum allowable level of interference at the PU for different levels of bounded channel uncertainty following the model in (6). We also compare the proposed design with the case of perfect knowledge of the channel $\boldsymbol{G}$ (i.e., $\epsilon = 0$) which was studied in [7]. As one might expect, when I increases, the SU transmitter is allowed to transmit more power and the average MI increases. When I reaches a sufficiently large level, the transmitted power $(\text{Tr}(\boldsymbol{Q}))$ reaches the maximum level P and the MI remains constant. This is true for all levels of channel uncertainty.

In order to show how important it is to take the channel uncertainty into account when designing for CR systems, let us assume that the design process took place under the assumption of perfect CSI at the SU transmitter while in fact there was some uncertainty associated with the CSI used in the design problem. Figure 3 plots the average of the relative interference threshold violation $((\text{Tr}(\boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^H) - I)^+/I)$ versus the level of uncertainty associated with the CSI that was not taken into account in the design process. We observe that the channel uncertainty, when not considered in the design process, leads to frequent violations of the interference constraints. Hence, design formulations and solutions as presented in this paper are needed to avoid harmful interference in practical CR systems.

2) Sum rate maximization for a multiple-carrier CR system: In this experiment, we consider a point-to-point CR system with $N = 8$ carriers and a single PU $(L = 1)$ with a total interference constraint (cf. (4)). In Figure 4, we plot the average of the maximum sum rate versus I, and in Figure 5 we plot the average of the total transmitted power over all carriers $(\sum_{n=1}^{N} \text{Tr}(\boldsymbol{Q}_n))$

versus I for different values of ϵ in (6). By comparing Figures 4 and 5, we observe that there exists an interval over which the average sum rate increases with I even though the average total transmitted power has already reached its maximum limit P (compare Figures 4 and 5 for $I \in [-7, -4]$ dB for $\epsilon = 0.02$ and for $I \in [-14, -6]$ dB for $\epsilon = 0)$. This can be attributed to the ability of the joint design problem to allocate resources over different carriers to satisfy the total interference constraint.

3) Transmit power minimization for a multiple-carrier CR system: In this experiment, we consider the robust power minimization problem in the multiple-carrier communication scenario considered above (extension of the robust design formulation in (9) to the multiple-carrier case). For this design problem, the target sum rate is 10 bps/Hz, and we study the feasibility of this robust design formulation for different levels of channel uncertainty, ϵ, in (6). The problem becomes infeasible if no set $\{\boldsymbol{Q}_n\}$ can be found that achieves the target sum rate while satisfying the interference constraints for every channel realization $\boldsymbol{G}_{\ell,n}$ within the bounded $\mathcal{U}(\epsilon)$. In this case, we assume an outage event has occurred. We generated 13,000 random sets of channels $\{\boldsymbol{H}_n, \hat{\boldsymbol{G}}_{\ell,n}\}$ and used them to plot the probability of outage versus I in Figure 6. As expected, the probability of outage decreases as I increases for a fixed ϵ, and increases with ϵ for a given value of I. To capture the average sum power versus I relationship, we selected all feasible sets of channels for which all the compared designs with different ϵ were feasible at $I = -19$ dB, and for these 1895 selected sets we plotted the average power versus I. It can be seen from Figure 7 that a higher channel uncertainty level mandates additional transmit power to achieve a target rate in the CR system since the robust design problem can be viewed as a design problem with perfect channel information with more restrictive interference constraints.

B. Multipoint-to-Point CR System

Finally, we consider a CR system with two SUs $(K = 2)$, four carriers $(N = 4)$, and a single PU $(L = 1)$ in a system setup similar to the one presented in Figure 1. In Figure 8, we plot the average of the maximum sum rate against the maximum interference I. We compare the maximum rate obtained by the solution of the dual problem to the one obtained by solving the primal problem in (19) by employing an exhaustive search over all possible combinations of carrier allocation amongst the two SUs. It is clear from Figure 8 that even for a CR system with as little as $N = 4$ carriers, the difference in sum rate is almost negligible (i.e. duality gap is almost zero). This demonstrates the effectiveness and usability of the proposed dual technique for practical CR systems design.

VI. Conclusion

In this paper, we introduced new algorithms for robust resource optimization in MIMO CR systems. We studied several optimization problems and provided solutions based on an LMI transformation that facilitated efficient treatment of the imperfect CSI of the PUs' channels at the SU transmitters. Systems with multiple MIMO PUs and multiple MIMO SUs utilizing multiple frequency bands were considered and efficient solutions were provided for each case. We presented numerical results that demonstrated the effectiveness and robustness of the proposed algorithms. We also demonstrated the detrimental effect of designing for CR systems without taking channel uncertainties into account on the level of interference at the PUs.

Appendix A

Proof of Theorem 1

Proof: Each of the L constraints in (7c) can be written as

$$I_\ell - \boldsymbol{e}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}}\right) \boldsymbol{e}_\ell - 2\Re\left\{\hat{\boldsymbol{g}}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}}\right) \boldsymbol{e}_\ell\right\} - \hat{\boldsymbol{g}}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}}\right) \hat{\boldsymbol{g}}_\ell \geq 0$$
$$\forall \quad \epsilon_\ell^2 - \boldsymbol{e}_\ell^H \left(\boldsymbol{R}_\ell^T \otimes \boldsymbol{I}_{M_{r,PU}}\right) \boldsymbol{e}_\ell \geq 0, \quad (20)$$

where $\hat{\boldsymbol{g}}_\ell = \text{vec}(\hat{\boldsymbol{G}}_\ell)$, and $\boldsymbol{e}_\ell = \text{vec}(\boldsymbol{E}_\ell)$. Similar to the inequality (7c), the inequality (20) represents an infinite set of constraints, one for each $\boldsymbol{e}_\ell$ satisfying $\epsilon_\ell^2 - \boldsymbol{e}_\ell^H \left(\boldsymbol{R}_\ell^T \otimes \boldsymbol{I}_{M_{r,PU}}\right) \boldsymbol{e}_\ell \geq 0$. However, the quadratic forms in (20) enable us to obtain a single constraint that is precisely equivalent to the infinite constraints in (20). In particular, using the results of S-lemma [28], one can show that the set of constraints in (20) holds if and only if there exists $\mu_\ell \geq 0$ such that

$$\begin{bmatrix} \mu_\ell \left(\boldsymbol{R}_\ell^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) - \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) & -\left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) \hat{\boldsymbol{g}}_\ell \\ -\hat{\boldsymbol{g}}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) & I_\ell - \hat{\boldsymbol{g}}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) \hat{\boldsymbol{g}}_\ell - \mu_\ell \epsilon_\ell^2 \end{bmatrix} \geq \mathbf{0}. \tag{21}$$

Furthermore, one can obtain an equivalent LMI of smaller size by employing the Schur Complement Theorem [29] that is presented in the following lemma

Lemma 1 (Schur Complement Theorem): Consider the Hermitian matrix $\boldsymbol{X} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B}^H \\ \boldsymbol{B} & \boldsymbol{C} \end{bmatrix}$, such that $\boldsymbol{C} > \mathbf{0}$. Then the constraint $\boldsymbol{X} \geq 0$ is satisfied if and only if $\boldsymbol{A} - \boldsymbol{B}^H \boldsymbol{C}^{-1} \boldsymbol{B} \geq \mathbf{0}$.

Applying Lemma 1 to the LMI in (21), we obtain the following equivalent inequality:

$$\begin{aligned} & I_\ell - \hat{\boldsymbol{g}}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) \hat{\boldsymbol{g}}_\ell - \mu_\ell \epsilon_\ell^2 \\ & \quad - \hat{\boldsymbol{g}}_\ell^H \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) \left[\left(\mu_\ell \boldsymbol{R}_\ell^T - \boldsymbol{Q}^T \right) \otimes \boldsymbol{I}_{M_{r,PU}} \right]^{-1} \left(\boldsymbol{Q}^T \otimes \boldsymbol{I}_{M_{r,PU}} \right) \hat{\boldsymbol{g}}_\ell \geq \mathbf{0}, \end{aligned} \tag{22}$$

which can be written in terms of $\boldsymbol{G}_\ell$ as

$$I_\ell - \operatorname{Tr}\left(\hat{\boldsymbol{G}}_\ell \boldsymbol{Q} \hat{\boldsymbol{G}}_\ell^H \right) - \mu_\ell \epsilon_\ell^2 - \operatorname{Tr}\left(\hat{\boldsymbol{G}}_\ell \boldsymbol{Q} \left(\mu_\ell \boldsymbol{R}_\ell - \boldsymbol{Q} \right)^{-1} \boldsymbol{Q} \hat{\boldsymbol{G}}_\ell^H \right) \geq \mathbf{0}. \tag{23}$$

Finally, to obtain an LMI representation of (23), one observes that (23) can be written as

$$\operatorname{Tr}\left(\hat{\boldsymbol{G}}_\ell \boldsymbol{Q} \hat{\boldsymbol{G}}_\ell^H \right) + \operatorname{Tr}\left(\hat{\boldsymbol{G}}_\ell \boldsymbol{S}_\ell \hat{\boldsymbol{G}}_\ell^H \right) + \mu_\ell \epsilon_\ell^2 \leq I_\ell, \tag{24a}$$

$$\boldsymbol{S}_\ell \geq \boldsymbol{Q} \left(\mu_\ell \boldsymbol{R}_\ell - \boldsymbol{Q} \right)^{-1} \boldsymbol{Q}. \tag{24b}$$

The constraint in (24b) is indeed equivalent to the LMI $\begin{bmatrix} \boldsymbol{S}_\ell & \boldsymbol{Q} \\ \boldsymbol{Q} & \mu_\ell \boldsymbol{R}_\ell - \boldsymbol{Q} \end{bmatrix} \geq \mathbf{0}$, by applying the result of Lemma 1. ■

REFERENCES

[1] G. Staple and K. Werbach, "The end of spectrum scarcity [spectrum allocation and utilization]," *IEEE Spectr.*, vol. 41, no. 3, pp. 48–52, Mar. 2004.

[2] Federal Communications Commission Spectrum Policy Task Force, "Report of the Spectrum Efficiency Working Group," *FCC, Tech Report*, Nov. 2002.

[3] P. Setoodeh and S. Haykin, "Robust transmit power control for cognitive radio," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 915 –939, May 2009.

[4] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[5] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE Commun. Surveys & Tutorials*, vol. 11, no. 1, pp. 116–130, Mar. 2009.

[6] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, "Next generation/dynamic spectrum access/cognitive radio wireless networks: a survey," *Comput. Netw. (Elsevier)*, vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[7] R. Zhang and Y.-C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE J. Select. Topics Signal Processing*, vol. 2, no. 1, pp. 88–102, Feb. 2008.

[8] G. Bansal, M. Hossain, and V. Bhargava, "Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4710–4718, Nov. 2008.

[9] L. Zhang, Y.-C. Liang, Y. Xin, and H. Poor, "Robust cognitive beamforming with partial channel state information," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4143–4153, Aug. 2009.

[10] K. Phan, S. Vorobyov, N. Sidiropoulos, and C. Tellambura, "Spectrum sharing in wireless networks via QoS-aware secondary multicast beamforming," *IEEE Trans. Signal Processing*, vol. 57, no. 6, pp. 2323–2335, June 2009.

[11] L. Zhang, Y.-C. Liang, and Y. Xin, "Joint beamforming and power allocation for multiple access channels in cognitive radio networks," *IEEE J. Select. Areas Commun.*, vol. 26, no. 1, pp. 38–51, Jan. 2008.

[12] G. Zheng, K.-K. Wong, and B. Otterston, "Robust cognitive beamforming with bounded channel uncertainties," *IEEE Trans. Signal Processing*, vol. 47, pp. 4871–4881, Dec. 2009.

[13] E. Gharavol, Y.-C. Liang, and K. Mouthaan, "Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information," *IEEE Trans. Veh. Technol.*, vol. 59, pp. 2852–2860, Jul. 2010.

[14] G. Scutari, D. Palomar, and S. Barbarossa, "Asynchronous iterative water-filling for Gaussian frequency-selective interference channels," *IEEE Trans. Inform. Theory*, vol. 54, no. 7, pp. 2868 –2878, July 2008.

[15] A. Abdel-Samad, A. Gershman, and T. Davidson, "Robust transmit beamforming based on

imperfect channel feedback," in *IEEE Veh. Technol. Conf.*, vol. 3, Sep. 2004, pp. 2049–2053.

[16] J. Wang and D. Palomar, "Worst-case robust MIMO transmission with imperfect channel knowledge," *IEEE Trans. Signal Processing*, vol. 57, no. 8, pp. 3086–3100, Aug. 2009.

[17] M. Shenouda and T. Davidson, "On the design of linear transceivers for multiuser systems with channel uncertainty," *IEEE J. Sel. Areas Commun.*, vol. 26, pp. 1015–1024, Aug. 2008.

[18] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, March 2004.

[19] J. F. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," 1999.

[20] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming (web page and software). http://stanford.edu/˜boyd/cvx," June 2009.

[21] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Trans. Commun.*, vol. 54, no. 7, pp. 1310–1322, Jul. 2006.

[22] W. Yu, "A dual decomposition approach to the sum power Gaussian vector multiple access channel sum capacity problem," in *Proc. Conf. Inf. Sciences and Systems*, 2003.

[23] L. Xiao, M. Johansson, and S. Boyd, "Simultaneous routing and resource allocation via dual decomposition," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1136–1144, July 2004.

[24] K. Seong, M. Mohseni, and J. Cioffi, "Optimal resource allocation for OFDMA downlink systems," in *Proc. IEEE Int. Symp. Inform. Theory*, Jul. 2006, pp. 1394–1398.

[25] I. Wong and B. Evans, "Optimal downlink OFDMA resource allocation with linear complexity to maximize ergodic rates," *IEEE Trans. Wireless Commun.*, vol. 7, no. 3, pp. 962–971, Mar. 2008.

[26] L. Vandenberghe, S. Boyd, and S. Wu, "Determinant maximization with linear matrix inequality constraints," *SIAM Journal on Matrix Analysis and Applications*, vol. 19, pp. 499–533, Dec. 1998.

[27] Z.-Q. Luo and S. Zhang, "Duality gap estimation and polynomial time approximation for optimal spectrum management," *IEEE Trans. Signal Processing*, vol. 57, no. 7, pp. 2675–2689, July 2009.

[28] V. A. Yakubovich, "S-procedure in nonlinear control theory," *Vestnik Leningrad University*, vol. 4, no. 1, pp. 73–93, 1977.

[29] F. Zhang, *The Schur complement and its applications*. Springer, 2005.

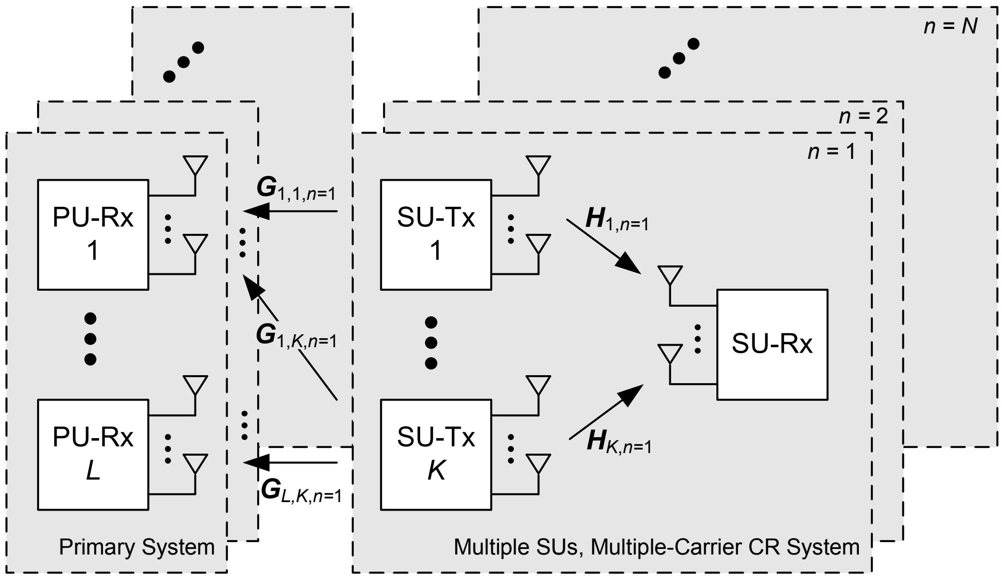

Fig. 1: Multipoint-to-point CR system with K SUs, L PUs, and N carriers. $K = 1$ corresponds to the case of point-to-point CR transmission. $N = 1$ corresponds to the special case of single-carrier systems.

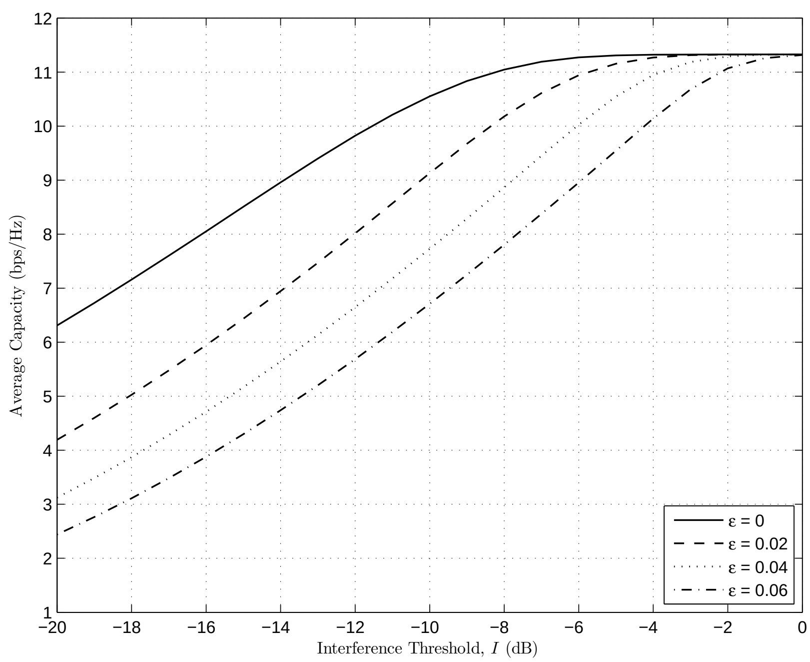

Fig. 2: Average of the maximum link MI of a single-carrier, point-to-point CR system in the presence of a single PU versus the maximum allowable interference at the PU for different values of ϵ. Maximum allowable power at the SU transmitter is $P = 20$ dB.

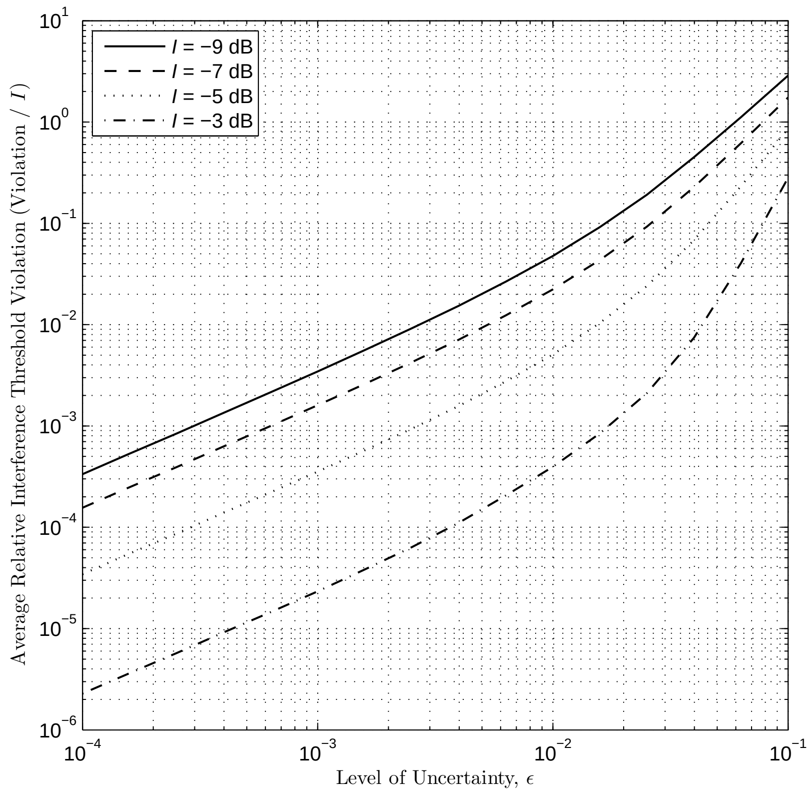

Fig. 3: Average of the relative interference threshold violation $((\mathrm{Tr}(\boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^H)-I)^+/I)$ versus the level of uncertainty associated with the CSI that was not taken into account in the design process. A single-carrier, point-to-point CR system is considered in the presence of a single PU with a maximum allowable power at the SU transmitter of $P=20$ dB.

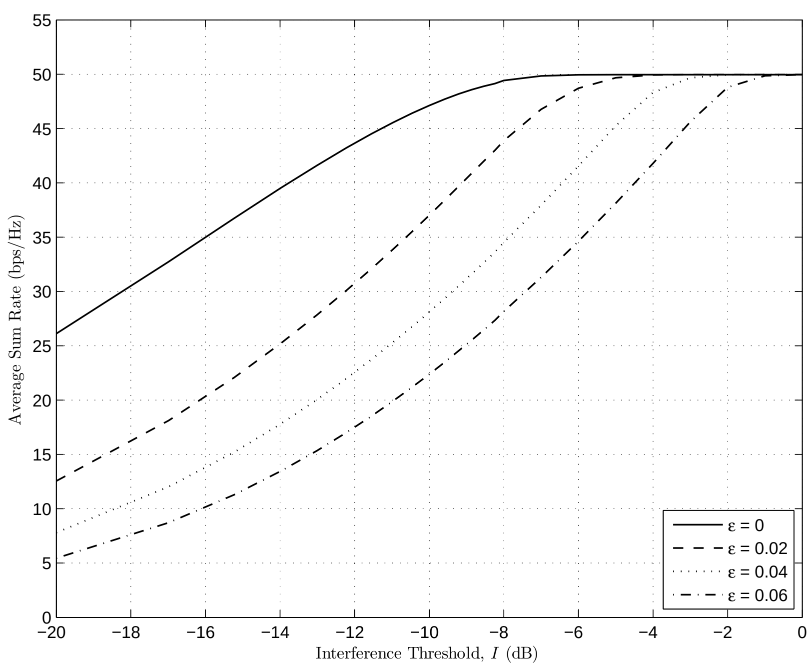

Fig. 4: Average of the maximum sum rate of a multiple-carrier $(N=8)$, point-to-point CR system in the presence of a single PU versus the maximum allowable interference I at the PU for different values of ϵ. Maximum allowable power at the SU transmitter is $P=20$ dB.

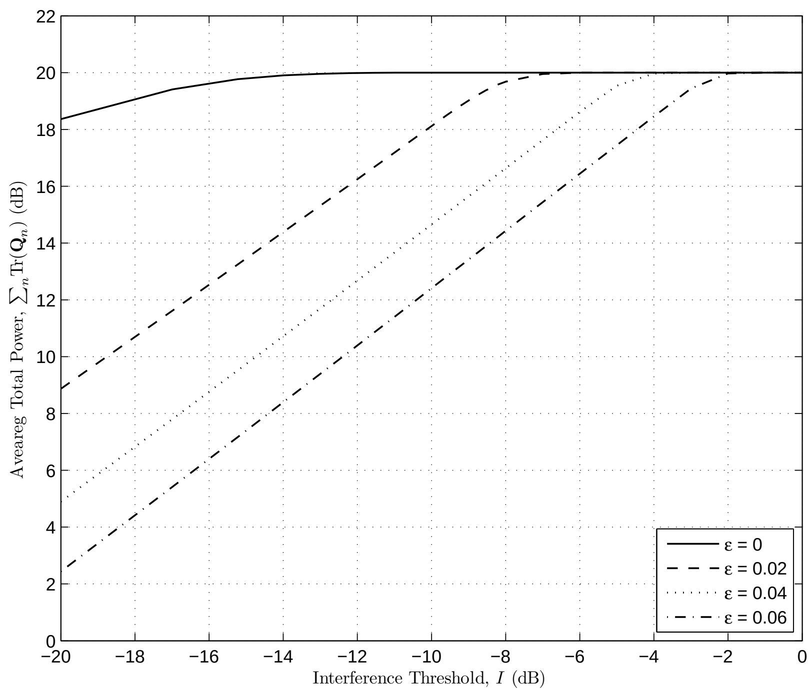

Fig. 5: Average of the total transmitted power $\sum_n \mathrm{Tr}(\boldsymbol{Q})$ from the SU transmitter when optimizing for the link MI of a multiple-carrier $(N = 8)$, point-to-point CR system in the presence of a single PU versus the maximum allowable interference I at the PU for different values of ϵ. Maximum allowable power at the SU transmitter is $P = 20$ dB.

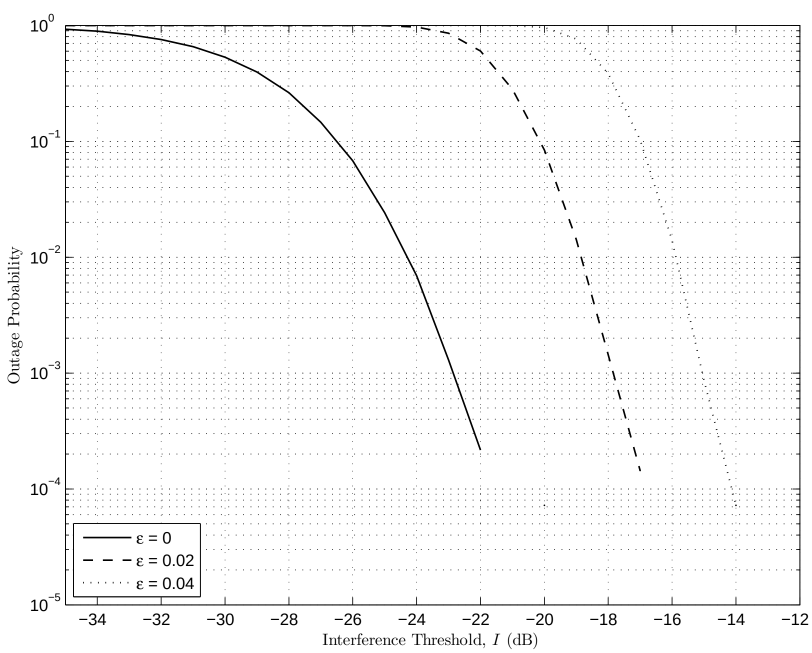

Fig. 6: Outage probability of the robust power minimization for a multiple-carrier $(N = 8)$, point-to-point CR system in the presence of a single PU versus the maximum allowable interference I at the PU for different values of ϵ. The target sum rate is 10 bps/Hz

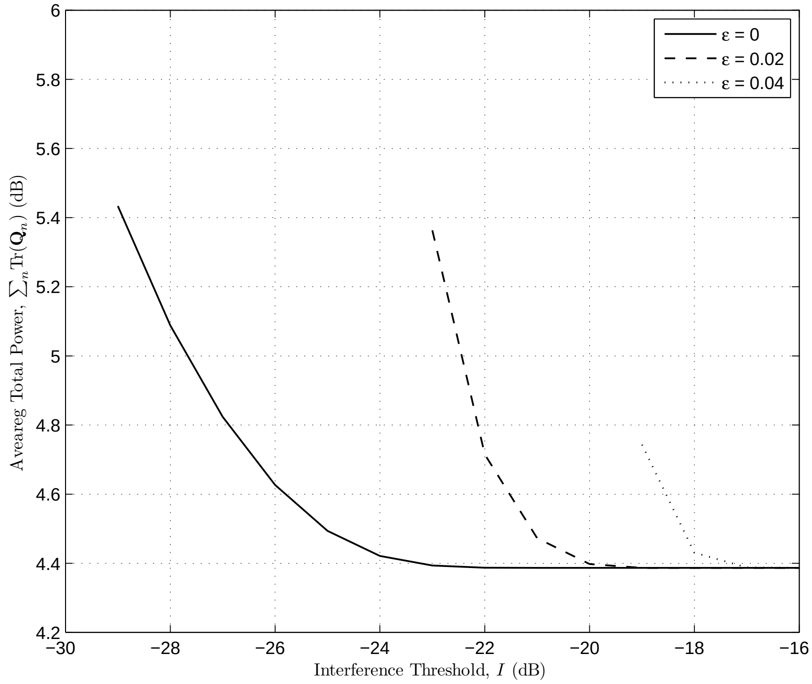

Fig. 7: Average minimum transmit power from the SU of a multiple-carrier $(N = 8)$, point-to-point CR system in the presence of a single PU versus the maximum allowable interference I at the PU for different values of ϵ. The target sum rate is 10 bps/Hz

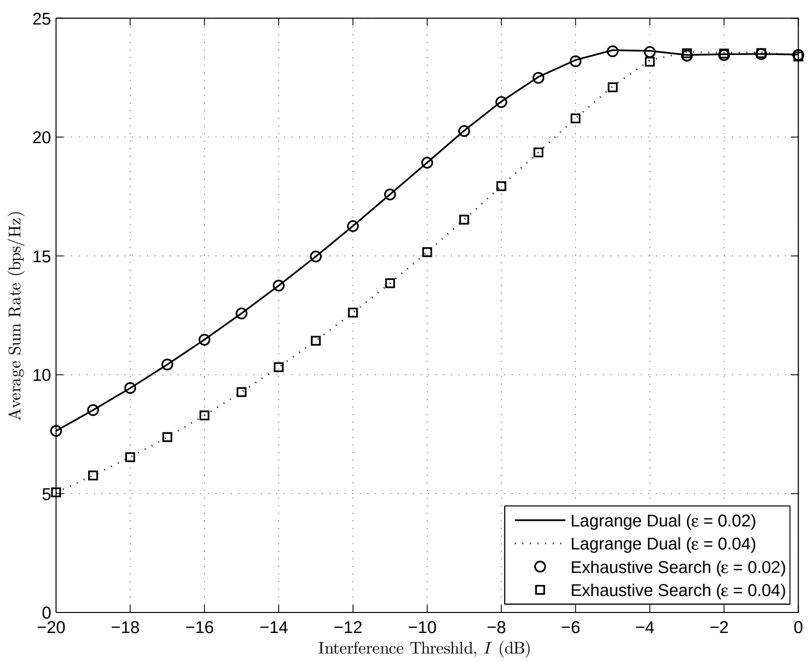

Fig. 8: Average of the maximum sum rate obtained from solving the primal and dual optimization problems of a multiple-carrier $(N = 4)$, multipoint-to-point $(K = 2)$ CR system in the presence of a single PU versus the maximum allowable interference I at the PU for different values of ϵ. Maximum allowable power at each SU transmitter is $P = 17$ dB.