Mismatched Bit-Interleaved Coded Noncoherent Orthogonal Modulation

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***Abstract*—We consider several low-complexity decoding metrics for bit-interleaved coded modulation (BICM) with orthogonal signaling and noncoherent detection. Using the generalized mutual information (GMI) as the relevant performance measure, we demonstrate that only a small performance degradation is entailed by using these mismatched metrics instead of the maximum likelihood (matched) metric. Furthermore, for coded transmission with sum-product symbol-by-symbol (SBS) decoding, we illustrate that metric scaling with a constant factor can improve the throughput performance of these metrics.**

***Index Terms*—Bit-interleaved coded modulation (BICM), mismatched decoding, metric correction, orthogonal modulation, noncoherent detection, symbol-by-symbol decoding.**

I. Introduction

Orthogonal modulation with noncoherent detection is attractive due to its low-complexity detector implementation. This type of signaling, e.g., in the form of frequency shift-keying (FSK) and pulse-position modulation (PPM), has been used in scenarios where coherent detection is impossible or expensive to employ.

A popular and practical technique for coded transmission is bit-interleaved coded modulation (BICM) [1]. In this letter, we consider several approximate BICM metrics which would reduce the detection complexity. We make use of recent advances in the study of BICM with general mismatched decoding metrics [2]–[4]. Using the generalized mutual information (GMI) as the performance measure, we find that these mismatched metrics can attain a GMI close to that of the maximum-likelihood (ML), i.e. matched, metric. For sum-product symbol-by-symbol (SBS) decoding, which is used in a number of state-of-the-art error-control coding systems such as LDPC and Raptor codes, we illustrate that metric scaling with a constant factor [4] can improve the throughput performance of these metrics.

II. BICM Metrics for Noncoherent Orthogonal Modulation

A. Noncoherent Orthogonal Modulation

We consider a discrete-time memoryless channel with input random variable X from the M-ary orthogonal constellation $\mathcal{X}$ and output random variable Y from the alphabet $\mathcal{Y}$. Each orthogonal transmit symbol can be represented by a vector $x = [x_0 \dots x_{M-1}]$ of M elements where only one element is one and all the others are zero. Let $h = a\,\mathrm{e}^{j\phi}$ be the complex channel gain and n be the length-M complex AWGN vector with variance N_0 per element. The received sample is a length-M complex vector such that

$$y = a\,\mathrm{e}^{j\phi}\, x + n\,. \tag{1}$$

Noncoherent detection requires only knowledge of the magnitude of the channel gain and the received sample. With noncoherent detection, the channel transition probability is [5]

$$p_{Y|X}(y|x) \propto I_0\left(\frac{2a|y_e(x)|}{N_0}\right)\,, \tag{2}$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, and $|y_e(x)|$ is the magnitude of the element of y at the position of the non-zero element of x.

B. BICM Metrics

Let $m = \log_2 M$ be the number of bits in the label of each transmit symbol. Furthermore, let $b_i(x)$ be the i-th bit in the label of x. For level $i = 0, \dots, m-1$, the matched BICM log-likelihood ratio (LLR) metric is

$$\Lambda_i^{\mathrm{matched}}(y) = \ln \frac{\sum_{x\in\mathcal{X}_i^0} p_{Y|X}(y|x)}{\sum_{x\in\mathcal{X}_i^1} p_{Y|X}(y|x)}$$

$$= \ln \sum_{x\in\mathcal{X}_i^0} I_0\left(\frac{2a|y_e(x)|}{N_0}\right) - \ln \sum_{x\in\mathcal{X}_i^1} I_0\left(\frac{2a|y_e(x)|}{N_0}\right), \tag{3}$$

where $\mathcal{X}_i^b = \{x \in \mathcal{X} : b_i(x) = b\}$, for $b = 0, 1$.

We now consider several low-complexity mismatched metrics. The popular max-log LLR metric is

$$\Lambda_i^{\mathrm{max-log}}(y) = \max_{x\in\mathcal{X}_i^0} \ln I_0\left(\frac{2a|y_e(x)|}{N_0}\right) - \max_{x\in\mathcal{X}_i^1} \ln I_0\left(\frac{2a|y_e(x)|}{N_0}\right). \tag{4}$$

Since both functions $\ln(\cdot)$ and $I_0(\cdot)$ are monotonic with positive input, the max-log LLR can be presented as

$$\Lambda_i^{\mathrm{max-log}}(y) = \ln I_0\left(\frac{2a}{N_0}\max_{x\in\mathcal{X}_i^0}|y_e(x)|\right) - \ln I_0\left(\frac{2a}{N_0}\max_{x\in\mathcal{X}_i^1}|y_e(x)|\right). \tag{5}$$

That is, the detector can search for the maximum values of $|y_e(x)|$ before computing $\ln I_0(\cdot)$.

We consider the approximation of the function $\ln I_0(\alpha)$ with positive $\alpha \in \mathbb{R}$ as illustrated in Figure 1. When α is large, $I_0(\alpha)$ can be asymptotically approximated by $e^{\alpha}/\sqrt{2\pi\alpha}$ and $\ln I_0(\alpha)$ can be approximated by α. From (5), this approximation leads to the new LLR

$$\Lambda_i^{\mathrm{a-max-log}}(y) = \frac{2a}{N_0}\left(\max_{x\in\mathcal{X}_i^0}|y_e(x)| - \max_{x\in\mathcal{X}_i^1}|y_e(x)|\right). \tag{6}$$

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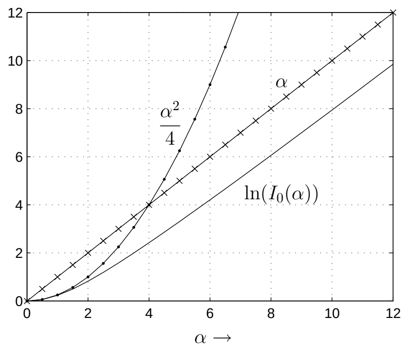


Fig. 1. Approximation to the function $\ln(I_0(\cdot))$.

A similar metric has been considered in [6].

When α is small, $I_0(\alpha)$ can be approximated by the first two terms in its power series expansion $1+\alpha^2/4$, and $\ln I_0(\alpha)$ can be approximated by $\alpha^2/4$. The corresponding LLR is

$$\Lambda_i^{\mathrm{ps-max-log}}(y) = \frac{a^2}{N_0^2}\left(\max_{x\in\mathcal{X}_i^0}|y_e(x)|^2 - \max_{x\in\mathcal{X}_i^1}|y_e(x)|^2\right) . \tag{7}$$

Metrics (6) and (7) have lower computational complexity than the max-log metric (5), which in turn has lower computational complexity than the matched metric (3).

If we drop the factor $2a/N_0$ in (6) and a^2/N_0^2 in (7), we have the parameter-free metrics

$$\Lambda_i^{\mathrm{pf-a-max-log}}(y) = \max_{x\in\mathcal{X}_i^0}|y_e(x)| - \max_{x\in\mathcal{X}_i^1}|y_e(x)| , \tag{8}$$

$$\Lambda_i^{\mathrm{pf-ps-max-log}}(y) = \max_{x\in\mathcal{X}_i^0}|y_e(x)|^2 - \max_{x\in\mathcal{X}_i^1}|y_e(x)|^2 . \tag{9}$$

Both channel gain and noise power estimation are not needed in the calculation of these metrics. This simplification further reduces the detection complexity. The parameter-free power series max-log metric (9) has been considered in [7] for BICM with iterative decoding (BICM-ID). The use of BICM-ID for noncoherent orthogonal signaling has also been considered in several other papers, e.g. [8].

III. GMI Performance

A. Review: Generalized Mutual Information (GMI)

The GMI has been proposed as a performance measure for BICM in [2], [3]. It is the largest achievable rate that is inferred from the random coding argument with word decoding. In this letter we consider only equiprobable input. With this distribution, for level i and LLR metric $\Lambda_i(y)$, the corresponding binary I-curve function $I_i(s)$ can be presented as [4, Eqn. (18)]

$$I_i(s) = 1 - \mathcal{E}_{X,Y}\left\{\log_2(1+\exp(-\operatorname{sgn}(b_i(X))\Lambda_i(Y)s))\right\} , \tag{10}$$

where the function $\operatorname{sgn}(\cdot)$ is defined for the labeling bits as $\operatorname{sgn}(0)=1$ and $\operatorname{sgn}(1)=-1$. The BICM I-curve is the sum of the binary I-curves of the levels [4],

$$I(s) = \sum_{i=0}^{m-1} I_i(s) . \tag{11}$$

For orthogonal constellations, all levels will have the same binary I-curve if the same metric is applied. Thus, the BICM I-curve is simply m times the binary I-curve of any level. In particular,

$$I(s) = mI_0(s) . \tag{12}$$

Finally, the BICM GMI is the peak value of this BICM I-curve,

$$I^{\mathrm{gmi}} = \max_{s>0} I(s) . \tag{13}$$

B. Numerical Results

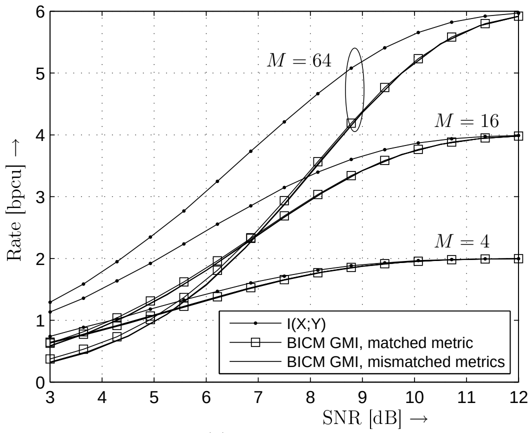
We obtain the binary I-curves for level $i=0$ by Monte-Carlo integration using (10) and the BICM I-curve by (12). We consider the cases $M = 4, 16$, and 64 over AWGN and Rayleigh fading channels. Figure 2 shows the BICM GMI with the matched metric (3) and the five mismatched metrics (5), (6), (7), (8), and (9) at different SNR values. The SNR is defined as $\mathcal{E}\{a^2\}/N_0$. It is interesting, and perhaps surprising, to observe that all the five mismatched metrics attain practically the same GMI across the whole SNR range. Furthermore, this GMI is very close to that of the matched metric. The mutual information $I(X;Y)$ is also included in Figure 2 as a reference. In communication systems where Gray labelings are used, BICM could achieve a rate close to $I(X;Y)$ [1]. However, for orthogonal constellations, all labelings are equivalent and are not Gray for $M>2$, and we observe a significant degradation between the BICM GMI and $I(X;Y)$.

Before the BICM mismatched decoding framework was introduced in [2], [3], $I(1)$ (the value of $I(s)$ at $s=1$) may have been used as a performance measure, e.g. in [9]. The value $I(1)$ is less than or equal to the GMI I^{gmi}, and equality holds only if the I-curve $I(s)$ peaks at $s=1$, see (13). We note that the I-curve of the matched metric always peaks at $s=1$. For an example with $M=16$ and Rayleigh fading channel, Figure 3 illustrates the difference between $I(1)$ and the GMI of the mismatched metrics. Interestingly, the value $I(1)$ of the power series max-log (7) is in fact negative over the SNR range (lower part of Figure 3). Overall, we can see notable differences between $I(1)$ and I^{gmi}, which is optimized with respect to s.

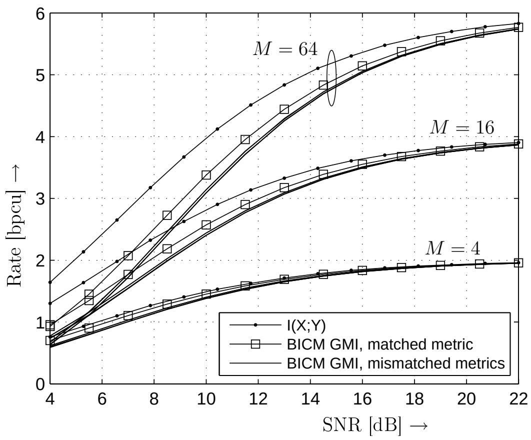
IV. SBS Throughput and Metric Scaling

If we scale the LLR by a factor $c>0$, we arrive at a new I-curve whose GMI remains unchanged, but the critical point (the s-coordinate of the peak) becomes multiplied by $1/c$ [4]. It was demonstrated in [4] that LLR scaling with the factor which shifts the critical point of the I-curve to $s=1$ can improve the throughput performance of sum-product SBS decoding.

Figure 4 shows the BICM I-curve for the case $M=16$ and Rayleigh fading channel at a moderate SNR of 8.5 dB, where the GMIs of the mismatched metrics are approximately 2.0 bits per channel use (bpcu). The BICM I-curve of the max-log (5), asymptotic max-log (6), power series max-log (7), parameter-free asymptotic max-log (8), and parameter-free power series max-log (9) peaks at $s = 0.46$, 0.44, 0.086, 6.3, and 4.4, respectively. Below the peak of each I-curve, we present the



(a) AWGN channel



(b) Rayleigh fading channel

Fig. 2. BICM GMI of noncoherent orthogonal transmission.

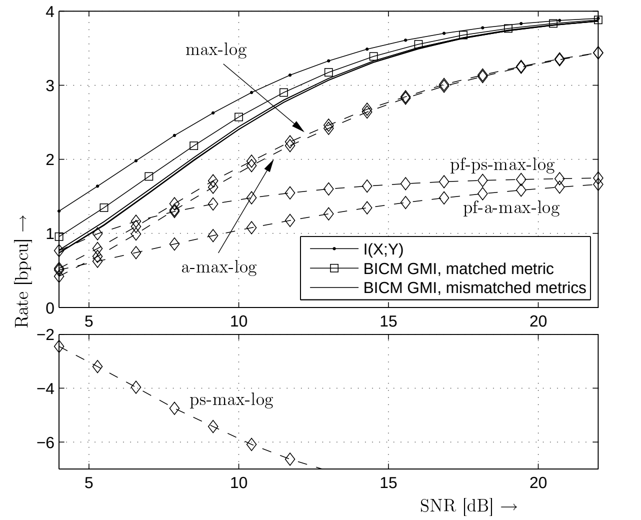


Fig. 3. $I(1)$ (dashed lines with diamond markers) of mismatched metrics for the case $M = 16$ over Rayleigh fading channel.

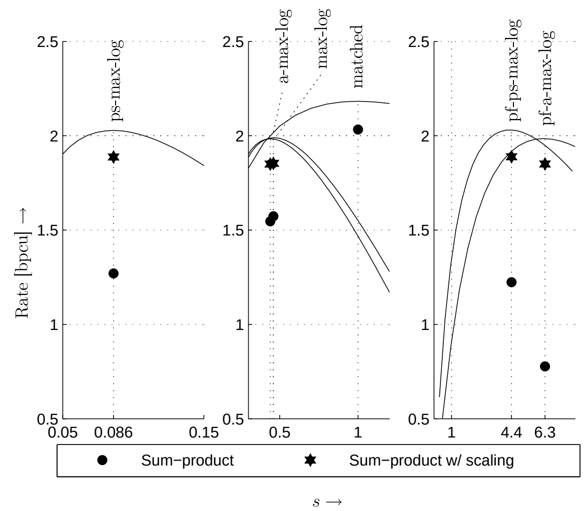


Fig. 4. BICM I-curve and coded throughput of different metrics. $M = 16$ over Rayleigh fading channel at a moderate SNR of 8.5 dB.

average throughput obtained by simulation with an off-the-shelf Raptor code and sum-product SBS decoding. The code is described in [4, Example 1]. We see that while the mismatched GMIs are roughly the same, the throughput values are notably different. We apply constant LLR scaling with the factor 0.46, 0.44, 0.086, 6.3, and 4.4 to the metric (5), (6), (7), (8), (9), respectively. Now, the scaled mismatched metrics achieve throughput performances close to their associated GMIs. We note that the scaling factor depends on the average SNR but not on the instantaneous channel parameters. Thus, for sum-product SBS decoding and transmission over a Rayleigh fading channel, the parameter-free metrics with scaling still offer computational complexity saving compared to their parameter-dependent counterparts.

V. Conclusion

We have considered several mismatched decoding metrics for BICM with noncoherent orthogonal modulation. We have shown a perhaps surprising result that these low-complexity metrics can attain a GMI similar to that of the matched metric. We have also illustrated the application of these metrics with sum-product SBS decoding and metric scaling.

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