Transformer Modelling
1. TRANSFORMER MODELLING

Low-Frequency Model

No capacitance. Constant parameters. Valid to ≤ 2-3 kHz

Traditional EMTP versions use the coupled impedances model, or \([Z]\) matrix model to represent the transformer. MicroTran's EMTP includes an \([L]^{-1}\) option ("INVERSE") that offers more accuracy and flexibility in the representation.

\([Z]\) Model

\[
\begin{bmatrix}
V_{13} \\
V_{24}
\end{bmatrix} = \begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{12} & Z_{22}
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} = \left( \frac{N_2}{N_1} \right) \begin{bmatrix}
\frac{N_2}{N_1} Z_{12} \\
\frac{N_2}{N_1} Z_{22}
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

This follows directly from the transformer circuit:

\[
V_{13} = I_1 Z_1 + (I_1 + I_2') Z_{12}
\]

\[
V_{24} = I_2 Z_2 + E_2 = I_2 Z_2 + \left( \frac{N_2}{N_1} \right) (I_1 + I_2') Z_{12}
\]

and

\[
I_2' = \frac{N_2}{N_1} I_2
\]

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Transformer Modelling (Cont.)

\[
[Z] = \begin{bmatrix}
\frac{Z_1 + Z_m N_2}{N_1} Z_m \\
\frac{N_2}{N_1} Z_m \\
Z_2 + \left(\frac{N_2}{N_1}\right)^2 Z_m
\end{bmatrix}
\]

Notice that in the internal EHTP representation the R's and the L/A are separated.
\(R_{11}, L_{11}\)
\(R_{22}, L_{22}\)
\(R_{12}, L_{12}\)

\[Z_{11} = R_{11} + jX_{11}\]
\[L_{11} = \frac{X_{11}}{2\pi \times 60}\]

e tc.

Main Problems:

- This model requires a finite value of magnetizing impedance \(Z_m\).
  If no magnetizing branch is given \(Z_m = \infty\) and the model would not work.

- Since \(Z_m (pu) \gg Z_1 (pu), Z_2 (pu)\), all the elements in \([Z]\) are dominated by \(Z_m\), while it is \(Z_1\) and \(Z_2\) that are more important during surge transfer through the transformer.

Example from Theory Book:

\([Z] = [R] + j[X] = \begin{bmatrix}
0.0025 & 0 \\
0 & 0.0025
\end{bmatrix} + j \begin{bmatrix}
100 & 99.95 \\
99.95 & 100
\end{bmatrix}\) p.u.

On per unit.

- The EHTP network solution is based on an admittance formulation:
  \([Y] = [Z]^{-1}\). Since all elements of \([Z]\) are dominated by \(Z_m\), the matrix is almost singular and the inversion is ill-conditioned.
MicroTrans's version of the ENTP includes the \([L]^{-1}\) option ("inverse") that avoids the problems of the \([Z]\) representation.

Magnetizing branch not required. It can be added later at the terminals.

**[Y] Model**

\[
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{12} & Y_{22}
\end{bmatrix}
\begin{bmatrix}
V_{11}' \\
V_{22}'
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{Z_S} & -\frac{Y_{11}}{Y_{22}} & \frac{1}{Z_S} \\
-\frac{Y_{12}}{Y_{22}} & \frac{1}{Z_S} & \frac{Y_{11}}{Y_{22}}
\end{bmatrix}
\begin{bmatrix}
V_{11}' \\
V_{22}'
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

The derivation follows from the equivalent circuit.

\[Z_S = Z_1 + \left(\frac{N_1}{N_2}\right)^2 Z_2\]

\[
\begin{bmatrix}
\frac{1}{Z_{S1}} & -\frac{1}{Z_{S2}} \\
-\frac{1}{Z_{S2}} & \frac{1}{Z_{S1}}
\end{bmatrix}
\begin{bmatrix}
V_{11}' \\
E_1'
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2'
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{V_{11}'}{N_1 N_2} \\
\frac{E_1'}{N_1}
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2'
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{Z_{S1}}{Z_{S1}} & \frac{Z_{S2}}{Z_{S2}} \\
\frac{Z_{S2}}{Z_{S2}} & \frac{Z_{S1}}{Z_{S1}}
\end{bmatrix}
\begin{bmatrix}
V_{11}' \\
E_1'
\end{bmatrix}
=
\begin{bmatrix}
I_1 \\
I_2'
\end{bmatrix}
\]

and \([Y]\) above is obtained by substitution.
The $[L]^{-1}$ option in MicroTran is based on the $[Y]$ matrix model with the simplification of extracting the series resistance and inverting only the inductance part (this is valid if $WL > R$; $WL/R = 15$ at 60 Hz and higher beyond). ($R = kV_0$, $L$ = constant, $\frac{WL}{R} = \frac{W}{kV_0} = \frac{V_0}{R} \frac{k}{k}$)

To use this option, the coupled-branch model is used with the keyword "INVERSE" to indicate that $[L]^{-1}$ is entered instead of $[L]$. The data matrices are:

$$[R] = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \text{ and } [L]^{-1} = \begin{bmatrix} \frac{1}{L_S} & -\frac{N_1}{N_2} \frac{1}{L_S} \\ -\frac{N_1}{N_2} \frac{1}{L_S} & \left(\frac{N_1}{N_2}\right)^2 \frac{1}{L_S} \end{bmatrix} (\mu H)^{-1}$$

$$Z_1 = R_1 + jX_1 \quad X_5 = X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2 = WL_5$$
$$Z_2 = R_2 + jX_2 \quad L_5 = \frac{X_5}{2\pi f 60} \text{ for } f = 60 Hz$$

MicroTran's data entry processor outputs data uses the program BC750 to convert the transformer input data into a file with the $[L]^{-1}$ data format.
3. **THREE-PHASE CONNECTIONS**

Suppose the external connections among coils \( Y, \Delta \), etc. have not yet been made.

1) Voltages and currents are positive sequence.

   We get an equivalent:

\[
\begin{align*}
V_a^+ & \\
V_b^+ & \\
V_c^+ & \\
\end{align*}
\]

\[
\[ Y^+ \] = \begin{bmatrix} Y_{11}^+ & Y_{12}^+ \\ Y_{12}^+ & Y_{22}^+ \end{bmatrix}
\]

2) Similarly, we get an equivalent for negative sequence (= positive sequence) and for zero sequence.

\[
\begin{align*}
\Rightarrow V_a^- & \\
\Rightarrow V_b^- & \\
\end{align*}
\]

\[
\begin{align*}
Y^o & = \begin{bmatrix} Y_{11}^o & Y_{12}^o \\ Y_{12}^o & Y_{22}^o \end{bmatrix} \\
Y^- & = Y^+ \\
\end{align*}
\]

\[
\begin{align*}
Y_s & = \frac{1}{3} \left[ Y^o + 2 [Y^+] \right] = \frac{1}{3} \left[ (Y_{11}^o + 2 Y_{11}^+) (Y_{12}^o + 2 Y_{12}^+) \right] \\
Y_{11}^o & = \frac{1}{3} \left[ Y^o - [Y^+] \right] = \frac{1}{3} \left[ (Y_{11}^o - Y_{11}^+) (Y_{12}^o - Y_{12}^+) \right]
\end{align*}
\]

\[
\begin{align*}
Y_{12}^o & = \frac{1}{3} \left[ Y^o - [Y^+] \right] = \frac{1}{3} \left[ (Y_{12}^o - Y_{12}^+) (Y_{22}^o - Y_{22}^+) \right]
\end{align*}
\]

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Three-Phase Connections (Cont.)

The full-blown \([Y_{ph}]\) matrix will be \([6 \times 6]\)

\[
[Y_{ph}] = \begin{bmatrix}
    a & A & b & B & c & C \\
    a & \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) & Y_{ab} & Y_{ab} & Y_{ab} & Y_{ab} \\
    \frac{1}{3} (Y_{ii} - Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & Y_{ab} & Y_{ab} & Y_{ab} & Y_{ab} \\
    \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & Y_{ab} & Y_{ab} & Y_{ab} \\
    \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & Y_{ab} & Y_{ab} & Y_{ab} \\
    \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & \frac{1}{3} (Y_{ii} - Y_{ii}^+) & Y_{ab} & Y_{ab} & Y_{ab} \\
\end{bmatrix}
\]

This is the matrix that will be input to the \([L]^{-1}\) model of coupled branches.

where,

\[
Y_{aa} = \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) \quad Y_{ab} = \frac{1}{3} (Y_{ii} - Y_{ii}^+) \\
Y_{AA} = \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) \quad Y_{AB} = \frac{1}{3} (Y_{ii} - Y_{ii}^+) \\
Y_{AA} = \frac{1}{3} (Y_{ii} + 2Y_{ii}^+) \quad Y_{AB} = \frac{1}{3} (Y_{ii} - Y_{ii}^+)
\]

A similar result is obtained for the \([Z]\) matrix input option. In MicroTran these matrices are built automatically by the outData program using the BcRevI routine.
4. EXTERNAL CONNECTIONS

The model for the disconnected coils, e.g., \([Z]\) or \([L]^{-1}\) model, is what is given to the EMTP. In addition, node names are specified. It is in specifying these node names that the particular connection (Y, Δ, etc) is achieved.

In the example, 1', 2', 3' would all be "ground"; 4', 5 would have the same name, etc.
More than two coils per phase

The modelling procedure indicated can be directly extended to three-phase $N$-coil transformers.

For example, for a three-phase three-winding transformer,

$$\begin{bmatrix} Y^+ \end{bmatrix} = \begin{bmatrix} Y_{11}^+ & Y_{12}^+ & Y_{13}^+ \\ Y_{12}^+ & Y_{22}^+ & Y_{23}^+ \\ Y_{13}^+ & Y_{23}^+ & Y_{33}^+ \end{bmatrix}$$

and similarly for $\begin{bmatrix} Y_0 \end{bmatrix} = [3 \times 3]$

Conversion to phase coordinates will still give

$$\begin{bmatrix} Y_{ph} \end{bmatrix} = \begin{bmatrix} [Y_3] & [Y_m] & [Y_m] \\ [Y_m] & [Y_5] & [Y_m] \\ [Y_m] & [Y_m] & [Y_5] \end{bmatrix}$$

with the augmented dimension

$$\begin{bmatrix} Y_{ph} \end{bmatrix} = [9 \times 9]$$

$$[Y_5] = \frac{1}{3} \left[[Y_0] + 2[Y^+]\right] = [3 \times 3]$$

$$[Y_m] = \frac{1}{3} \left[[Y_0] - [Y^+]\right] = [3 \times 5]$$
6. UNBALANCED STRUCTURES

The indicated modelling procedure is not restricted to balanced structures and symmetrical components. In fact, сыf cores are not perfectly balanced (even though the balanced assumption is still reasonable).

For a general unbalanced structure \([Y_0], [Y^+], [Y^-]\) would simply become \([Y_1], [Y_2], [Y_3]\) in the \([Y_{node}]\) description and instead of \([Y_s]\) and \([Y_u]\) in \([Y_{phase}]\) we would have \([Y_{phase}] = T [Y_{node}] T^{-1}\).

Where \(T\) is the transformation matrix that decouples the unbalanced system.
7. MAGNETIZING BRANCH

In the \([L]^{-1}\) model the magnetizing branch \(Z_m\) can be connected externally to the model, at the transformer terminals. This allows one to model the saturation of the core using the nonlinear \(L\) models available in the EMTP.

\[ Z_m \]

Core losses

\((\lambda, i_m)\) curve

\((\lambda, i_m)\) curve can be obtained from manufacturer's (\(V_{rms}, I_{rms}\)) characteristic using MicroTrans' support routine \(UCONVERT\).

Best to connect across winding closest to the core (usually the low voltage winding).

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Magnetizing Branch (Cont.)

Hysteresis Loop

Modern high-voltage transformers have cores of grain-oriented steel and hysteresis loops are relatively narrow and their accurate representation is not usually critical.

Core Losses

\[
P_{\text{core}} = P_{\text{hysteresis}} + P_{\text{eddy}}
\]

From Theory Book,

\[
P_{\text{hysteresis}}/P_{\text{eddy}} = 3 \text{ for silicon steel}
\]

\[
P_{\text{hysteresis}}/P_{\text{eddy}} = \frac{1}{3} \text{ to } \frac{2}{3} \text{ for grain-oriented steel}
\]

\(R_c\) is frequency dependent = \(R(w)\) but this effect is usually not too critical in system-level transients.

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7. HIGH-FREQUENCY MODEL

The coupled-coils constant-parameter model is valid only to 1 to 3 kHz or so. For higher frequencies the capacitances become important and the skin effect on the losses becomes more important.

Chimkhai-Martí Interpretation.
- $Z_{winding}(\omega)$ is obtained from short-circuit tests with a variable-frequency supply.

- The outside capacitances can be measured directly with appropriate tests.
High-Frequency Model (Cont.)

Z winding can be synthesized by equivalent network of constant parameters.

\[ Z_{\text{winding}} = \text{synthesized by equivalent network of constant parameters} \]

\[ Z_{\text{winding}} = \frac{1}{R_0 L_0} + \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_3 C_3} \]

Detailed model to first peak

2nd peak

3rd peak

\[ f (\text{Hz}) \]

\[ Z_{\text{winding}} \]

Constant R, L 60 Hz value

The other capacitances can be added to the external terminals as indicated in the previous slide's diagram.
FREQUENCY DOMAIN RESULTS

Positive Sequence
FREQUENCY DOMAIN RESULTS

Zero Sequence
SHORT-CIRCUIT TESTS

(a) Positive Sequence Test
(b) Zero Sequence Test

Slide 16
Interwinding Capacitance and Capacitance to Ground
SIMULATION OF FAULT INTERRUPTION

(a) Frequency-dependent model.

(b) Constant-parameter model.
8. OTHER HIGH-FREQUENCY MODELS

Conventional high-frequency models try to synthesize directly the elements of the $[Y]$ matrix model.

$$[Y(\omega)] = \begin{bmatrix} Y_{11}(\omega) & Y_{12}(\omega) & Y_{13}(\omega) & Y_{14}(\omega) & Y_{15}(\omega) & Y_{16}(\omega) \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ Y_{61}(\omega) & Y_{62}(\omega) & \cdots & \cdots & \cdots & Y_{66}(\omega) \end{bmatrix}_{6\times6}$$

Problems

1) Off diagonal elements, e.g., $Y_{12}(\omega)$, are not minimum phase functions and are very difficult to synthesize with stable functions.

2) Large number of functions, each with a large number of poles, are required.

The 2-winding $(\omega)$ model is absolutely stable and requires a minimum of poles. Also it fits directly the short circuit impedances which are more critical functions for the overall accuracy of the model.

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Other High-Frequency Models (Cont.)

\[ Y(\omega) \] versus Zwinding models. Number of frequency functions

<table>
<thead>
<tr>
<th>3φ - 2 wdg</th>
<th>[ Y(\omega) ] full</th>
<th>[ Y(\omega) ] on</th>
<th>Zwinding</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>6</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>12</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

\[ Y(\omega) \] = simplified Ontario Hydro
fewer functions than \[ Y(\omega) \] full but still
has the problem of fitting mutual elements
with non-minimum phase shift functions

Zwinding Only requires the fitting of minimum
phase shift short-circuit impedance functions
\[ \Rightarrow \text{Absolutely stable and much cheaper.} \]