MODELLING OF
NONLINEAR COMPONENTS
1. NONLINEAR ELEMENTS

a) Piecewise Linear

b) Continuously Nonlinear

\[ \lambda \]

\( L_0 \) \hspace{2cm} \( L (i) \)

\( i \)

(Symmetrical Curve) \hspace{2cm} (Arbitrary Curve)
2. PIECEWISE LINEAR R

- Varistors
- Metal-Oxide Arrester (MOAs) My Suggestion

\[ V(t) \]

Region 0: \( V(t) = R_0 i(t) \)
Region 1: \( V(t) = V_{k1} + R_1 i(t) \)
Region 2: \( V(t) = V_{k2} + R_2 i(t) \)

a) Check for region (e.g., monitor \( V(t) \)).
b) Use correct model for region
c) Polarity of DC source \( V_{k1} \) changes when
   polarity of \( V(t) \), \( i(t) \) changes.
d) Overshoot from \( (t-\Delta t) \) to \( t \) is not a problem
   because next segment is anchored at \( V_{k1} \)
   and solution at \( (t+\Delta t) \) is forced to fall on the
   correct characteristic.

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**Piecewise Linear R (Cont.)**

*(Traditional)*

**EMTP Model for Piecewise Linear R**

Model is built with two components: 1) Normal R, 2) Saturated R'

\[ V_{sat} = \frac{V}{R_0} + \frac{Vsat}{R_1} \]

Solve for R' and specify as TYPE 92 element

\[ \frac{1}{R_0} + \frac{1}{R'} = \frac{1}{R_1} \]

\[ \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_2} \]

\[ \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{R_3} \]

N slopes can be built

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3. SURGE ARRESTER WITH SPARK GAP

Branch 1 conduction starts when $V > V_{\text{gap}}$
Conduction stops when current reverses direction

* More general arrester characteristics can be achieved with the continuous nonlinear model.
4. PIECEWISE LINEAR INDUCTANCE

\[ \lambda \]
\[ \lambda(0) \quad \lambda(1) \quad (2) \]
\[ L_0 \quad L_1 \quad L_2 \]
\[ i(t) \]
\[ K \rightarrow OM \]
\[ N(t) \]

**Micota\'s Technique (EDDA)**

Anchor characteristics on vertical axis, \( \lambda_i \).

**EMTP**

Specify knee values \( \lambda_{sat} \).

From the characteristic:

\[ \lambda(t) = \lambda_{k1} + L_1 i(t) \]

Replacing this \( \lambda(t) \) in eqn. above,

\[ \lambda_{k1} + L_1 i(t) = \frac{\Delta t}{2} U(t) + \lambda(t-\Delta t) + \frac{\Delta t}{2} U(t-\Delta t) \]

\[ U(t) = \frac{2L_1}{\Delta t} i(t) + \left\{ -U(t-\Delta t) - \frac{\Delta t}{2} \lambda(t-\Delta t) + \frac{\Delta t}{2} \lambda_{k1} \right\} \]

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Piecewise Linear Inductance (Cont.)

\[ U(t) = \frac{2L_1}{\Delta t} i(t) + E_{kl}(t) \cdot K \quad \frac{2L_1}{\Delta t} E_{kl}(t) \quad \text{for Region 1} \]

\[ E_{kl}(t) = -V(t-\Delta t) - \frac{2}{\Delta t} \lambda(t-\Delta t) + \frac{2}{\Delta t}\lambda k_1 \]

- Similar models apply to the other regions.
- We can use \( \lambda(t) \) or \( i(t) \) to identify operating region.

\[ \lambda \]

- Return to the characteristic after the overshoot at \( t \) would produce oscillations in the voltage with the trapezoidal rule of integration.
- This problem is eliminated with the CDA solution procedure.
Piecewise Linear Inductance (Cont. 2)

EMTP MODEL

Switches close at $\lambda \geq \lambda_{\text{sat}_2}$ etc.

Specify: $L_0, \lambda_{\text{residual}}$

\[
\frac{1}{L_0} + \frac{1}{L_1'} = \frac{1}{L_1} \Rightarrow L_1', \lambda_{\text{sat}_1}
\]

\[
\frac{1}{L_1} + \frac{1}{L_2'} = \frac{1}{L_2} \Rightarrow L_2', \lambda_{\text{sat}_2}
\]

\[
\frac{1}{L_2} + \frac{1}{L_3'} = \frac{1}{L_3} \Rightarrow L_3', \lambda_{\text{sat}_3}
\]
5. CONTINUOUSLY NONLINEAR ELEMENTS

Nonlinear Element

Thevenin Equivalent

a) Linear network is solved before connecting nonlinear element

b) Nonlinear element is connected to thevenin equivalent pulled from
c) Current i(t) is injected to full linear network with sources geroed for incremental solution

d) Total Solution = (a) + (c).
6. NONLINEAR RESISTANCE

Multiport Case:

In general we will have $N$ nonlinearities and the Thevenin will be multiport. The intersection Thevenin—nonlinear elements will be the solution of a system of nonlinear equations.

Varistor Equation:

$U = f(i)$ some curve \hfill (a)

Network Equation:

$U = \frac{E_{th} - R_{th} \cdot I}{line \ with \ negative \ slope}$ \hfill (b)

Intersection of (a) and (b) gives solution point. We can use Newton or some other zero-finding procedure.
Nonlinear Resistance (Cont.)

EMTP:
- Specified point by point
- Points are interpolated linearly in the solution
- Characteristic must be monotonically increasing

Surge Arrester Option:
- In addition to the \((U, i)\) points, we can specify \(U_{spark}\) over which conduction begins.
- \(U < U_{spark} \Rightarrow i = 0\)
- \(U > U_{spark} \Rightarrow i = f(U)\)

Conduction stops when current reverses direction

E.g. Zinc-Oxide arresters can be described by
\[
i = \left(\frac{|U|}{C}\right)\alpha (\text{sign of } U)
\]
\(\alpha, C\) parameters.
7. NONLINEAR INDUCTANCE

\[ \lambda = f(i) \]

\(a) \) Convert \((\lambda, i)\) curve to \((v, i)\) curve.

\[ U(t) = \frac{d\lambda(t)}{dt} \]

Integrating with trapezoidal,

\[ \lambda(t) = \frac{\Delta t}{2} U(t) + \lambda(t-\Delta t) + \frac{\Delta t}{2} U(t-\Delta t) \]

\[ = \frac{\Delta t}{2} U(t) + h\lambda(t) \]

Substituting \(\lambda(t)\) in \(\lambda = f(i)\),

\[ \frac{\Delta t}{2} U(t) + h\lambda(t) = f(i) \]

\[ U(t) = (\frac{2}{\Delta t}) [f(i) - h\lambda(t)] \]
Nonlinear Inductance (Cont.)

EHM:
- Specified point by point
- Points are interpolated linearly
- Must be monotonically increasing
- CDCA is applied when changing segments.

Conversion VRMS/IRMS to $\lambda/i$

Program CONVERT from Microtran.

Trick to get $\lambda$ versus $i$ in transient simulation

1) Want $\lambda_1$ versus $i_1$ in inductance.
2) Connect large linear $L_2$ in parallel.
3) Plot $i_2$ versus $i_1$ with proper scaling for $i_2$

\[ V = \frac{d\lambda_1}{dt} = \frac{d\lambda_2}{dt} \Rightarrow \lambda_1 = \lambda_2, \quad C = L_2 \]

but \[ \lambda_2 = L_2 \cdot i_2 \]

Then plot of $(L_2, i_2)$ vs. $i_1$ is the same as plot of $\lambda_1$ vs. $i_1$.