Distributed Transmit Power Allocation for Relay-Assisted Cognitive-Radio Systems

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Cognitive-Radio (CR) Systems

- Utilize **unused** or **partially occupied** frequency bands in **adaptive**, **unlicensed** fashion \(\Rightarrow\) More **efficient** spectrum utilization
- **Spectrum sensing** \(\Rightarrow\) Dynamically adjust **transmission parameters** (carrier frequency, bandwidth, transmit power, ...)
- CR capabilities will be relevant, e.g., for **UWB systems**

Relay-Assisted CR Systems

- CR systems will naturally operate at **low transmit powers** to limit interference experienced by **primary users**
- **Example:** Transmit power of UWB devices **limited** by FCC spectral mask
  \(\Rightarrow\) **Relay assistance** attractive to improve coverage and performance of CR systems
Introduction

Cognitive-Radio (CR) Systems

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- Spectrum sensing ⇒ Dynamically adjust transmission parameters (carrier frequency, bandwidth, transmit power, ...)
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Relay-Assisted CR Systems

- CR systems will naturally operate at low transmit powers to limit interference experienced by primary users
- Example: Transmit power of UWB devices limited by FCC spectral mask
  ⇒ Relay assistance attractive to improve coverage and performance of CR systems
Here:
- **Frequency band** chosen by CR system not completely unoccupied, but contains one or more **licensed** narrowband users
- **Distributed** transmit power allocation schemes for CR systems assisted by **cooperating** relays

**Goal**: Optimize performance of CR system while limiting interference level seen by primary user(s)
Overview

- System Model and Problem Formulation
- Distributed Transmit Power Allocation Schemes
- Numerical Performance Results
- Conclusions
Overview

- **System Model and Problem Formulation**
  - Basic Assumptions and Transmission Protocol
  - Illustration of the Centralized Optimization Problem

- Distributed Transmit Power Allocation Schemes

- Numerical Performance Results

- Conclusions
System Model

Basic Assumptions

- Wideband or UWB CR system based on **CDMA**
- Source $S$ and destination $D$ assisted by $N_r$ **perfectly synchronized relays** $R_1, \ldots, R_{N_r}$ using orthogonal spreading codes
- $N_p$ primary users $U_1, \ldots, U_{N_p}$ within frequency band of CR system
  - Bandwidth ratio $\rho_j := B_{U_j} / B_{CR} < 1 \quad (j = 1, \ldots, N_p)$
  - $\xi_j$: Maximum sum interference power tolerated by $U_j$
- **Quasi-static** scenario; **block fading** with channel impulse responses (CIRs)

\[ h_{X,Y} := \left[ h_{X,Y}^{(0)}, \ldots, h_{X,Y}^{(L_{X,Y})} \right]^T, \]

$X, Y \in \{S, D, R_1, \ldots, R_{N_r}, U_1, \ldots, U_{N_p} \}$

$\Rightarrow$ Corresponding CIR energies:

\[ \alpha_{X,Y} := \sum_{l=0}^{L_{X,Y}} |h_{X,Y}^{(l)}|^2 \]
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System Model

Further Assumptions (require some network acquisition phase)

- **D** perfect knowledge of $h_{S,D}$ and $h_{R_i,D}$ ($i = 1, \ldots, N_r$)
- **$R_i$** ($i = 1, \ldots, N_r$) perfect knowledge of $h_{S,R_i}$
- **$S$ and $R_i$** ($i = 1, \ldots, N_r$) aware of channel power gains $\alpha_{S,U_j}$ or $\alpha_{R_i,U_j}$ in direction of $U_1, \ldots, U_{N_p}$
Transmission Protocol

Transmission protocol consists of **two orthogonal time slots**: 

**Time Slot I**
- **S** broadcasts **coded message** to **R_1, ..., R_Nr** and **D**
- **Transmit power** \( P_S \) adjusted such that all interference constraints are met:

\[
\rho_j P_S \alpha_s, u_j \leq \xi_j \quad (j = 1, ..., N_p)
\]

Moreover, maximum power constraint

\[
P_S \leq P_{S,\text{max}}
\]

- **R_i** \( (i = 1, ..., N_r) \) performs **optimal MRC** of signal received from **S**
- Relays receiving coded message with MRC output SNR \( \geq \) threshold value \( \gamma_{th} \) assumed to **decode correctly**
Transmission Protocol

Transmission protocol consists of two orthogonal time slots:

**Time Slot II**

- These $N'_r \leq N_r$ relays broadcast **short beacon signal** to inform other relays and destination.
- Participating relays *re-encode* message and simultaneously *retransmit* it.
- D performs **optimal MRC** of signals from S and $R_i$ ($i = 1, ..., N'_r$).
- **Transmit powers** of participating relays shall be chosen such that all interference and power constraints are met.
- MRC output SNR $\gamma_D$ at destination shall be **maximized** to establish quick connection between S and D.
Optimization Problem

- **Linear** optimization problem:

  maximize \[ \gamma_D = \frac{1}{\sigma_n^2} \left( \sum_{i=1}^{N'_r} P_{R_i} \alpha_{R_i,D} + P_S \alpha_{S,D} \right) \]

  subject to
  \[ \rho_j \sum_{i=1}^{N'_r} P_{R_i} \alpha_{R_i,U_j} \leq \xi_j, \quad j = 1, \ldots, N_p \]
  \[ P_{R_i} \leq P_{R_i,\text{max}}, \quad i = 1, \ldots, N'_r \]

- Parameters \( \rho_j, \xi_j, P_{R_i,\text{max}} \) assumed to be **known** throughout CR network
- **Optimal solution** can be found using linear programming techniques (e.g. Simplex algorithm)
Optimal solution requires **central** node C with knowledge of **all** channel power gains $\alpha_{R_i,D}$ and $\alpha_{R_i,U_j}$ ($i = 1, \ldots, N'_r$, $j = 1, \ldots, N_p$).

Values $\alpha_{R_i,D}$, $\alpha_{R_i,U_j}$ **communicated** to C; then C computes optimal transmit powers and feeds solution **back** to relays.

⇒ Develop **distributed** power allocation schemes to **reduce** signaling overhead.
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Values $\alpha_{R_i,D}$, $\alpha_{R_i,U_j}$ **communicated** to $C$; then $C$ computes optimal transmit powers and **feeds** solution **back** to relays.

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- System Model and Problem Formulation
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- System Model and Problem Formulation

- Distributed Transmit Power Allocation Schemes
  - Fully Decentralized (FD) Transmit Power Allocation
  - Distributed Scheme with Limited Feedback (LF)
  - Distributed Quasi-Optimal (QO) Power Allocation

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Fully Decentralized (FD) Transmit Power Allocation

- Fully decentralized, i.e., performed **solely** by relays
- Based on beacon signals $N_r'$ **known** throughout CR network
- $R_i \ (i = 1, ..., N_r')$ adjusts transmit power as

$$P_{R_i} := \min \left\{ P_{R_i,\text{max}}, \ \min_{j \in \{1, ..., N_p\}} \left\{ \frac{\xi_j}{\rho_j N_r' \alpha_{R_i, U_j}} \right\} \right\}$$

$\Rightarrow$ Sum interference power at $U_j$ at most $\xi_j \ (j = 1, ..., N_p)$, **without** any further **interaction** between relays

Moreover, $P_{R_i} \leq P_{R_i,\text{max}} \ (i = 1, ..., N_r')$ guaranteed
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Distributed Scheme with Little Feedback (LF)

- Very little **feedback** from destination to improve FD solution
- Starts with FD scheme, D **measures** MRC output SNR $\gamma_D$
  - D tests if switching to **single** transmitting relay $R_k$ improves $\gamma_D$ ($\gamma'_D > \gamma_D$)
  - ‘Best’ relay chosen according to largest channel power gain $\alpha_{R_i,D}$ (i.e., largest component of g), disregarding interference constraints
- **Enhanced** little feedback (ELF) scheme: ‘Best’ relay chosen taking interference constraints into account
  - $\gamma'_D > \gamma_D \Rightarrow$ D sends **beacons** to participating relays
- ** ⇒** Relays **re-adjust** transmit powers as
  \[
P_{R_i} := \begin{cases} 
  \min \left\{ P_{R_k,\text{max}}, \min_{j \in \{1, \ldots, N_p\}} \left\{ \frac{\xi_j}{\rho_j \alpha_{R_k,j}} \right\} \right\}, & i = k \\
  0, & \text{else}
\end{cases}
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- $\gamma'_D \leq \gamma_D \Rightarrow$ FD solution **retained**
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Distributed Quasi-Optimal (QO) Power Allocation

- **Feedback** from the destination (similar to LF scheme)
- **Congenrous** primary users, i.e., $\xi_1/\rho_1 = \ldots = \xi_{N_p}/\rho_{N_p} =: \theta$
- Starts with FD scheme, $D$ measures MRC output SNR $\gamma_{D,i}$ for each relay
  - From this $D$ determines applied transmit powers $P_{R_i}$
  - Worst-case estimates for $\alpha_{R_i,U_j}$ ($j = 1, \ldots, N_p$):
    $$\tilde{\alpha}_{R_i,U_j} := \frac{\theta}{N_t' P_{R_i}} \geq \alpha_{R_i,U_j}$$
  - $D$ computes (quasi-)optimal transmit powers (using values $\tilde{\alpha}_{R_i,U_j}$) and feeds solution back to relays (similar to optimal centralized scheme)
  - Interference constraints (and power constraints) always met
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Simulation Setup

- All link lengths **normalized** w.r.t. S–D link; S and D have **fixed positions** at \((-0.5, 0)\) and \((+0.5, 0)\), respectively.

- \(N_r = 20\) relays and \(N_p \geq 1\) primary users; **random positions** within square areas of side length 0.8, center points \((0, 0)\) and \((x_p, 0)\), respectively.

- Identical **maximum transmit powers** \(P_{\text{max}}\) for all nodes within CR network.

- Quasi-static **Rayleigh** fading, path-loss exponent \(p = 2\)
  - Links associated with primary users: flat fading \((L_{X,Y} = 0)\)
  - Links within CR network: frequency-selective fading \((L_{X,Y} = 9)\)

- All simulation results **averaged** over 1,000 random locations of relays and primary users; 100 channel realizations per spatial constellation.
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Example:

Maximum transmit power $P_{\text{max}} = 1$

Threshold SNR $\gamma_{\text{th}} = 10 \text{ dB}$

$N_p = 1$, $\xi_1 = 0.1$, $\rho_1 = 0.5$

$x_p = 0$, ------ $x_p = 5$

$\Rightarrow$ Already FD scheme significantly outperforms direct transmission; QO scheme performs very close to optimum centralized solution
Example:

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$\Rightarrow$ For larger distances of primary users FD, LF, and ELF scheme similar; significantly larger gains over direct transmission than for $x_p = 0$
Example:

Threshold SNR $\gamma_{th} = 10$ dB

$N_p = 1$, $\xi_1 = 0.1$, $\rho_1 = 0.5$, $x_p = 1$

SNR $1/\sigma_n^2 = 10$ dB

⇒ For low values of $P_{max}$ all schemes perform very close to optimum; for large values of $P_{max}$ flat behavior due to interference constraint
Example:

Maximum transmit power $P_{\text{max}} = 1$

Threshold SNR $\gamma_{\text{th}} = 10$ dB

$\xi_j = 0.1$, $\rho_j = 0.5$ for all $j$

$x_p = 0$, - - - $x_p = 5$

SNR $1/\sigma_n^2 = 15$ dB

⇒ For $x_p = 0$ notable performance degradation as $N_p$ grows; still significant improvements over direct transmission
Example:

Maximum transmit power $P_{\text{max}} = 1$

Threshold SNR $\gamma_{\text{th}} = 10$ dB

$\xi_j = 0.1$, $\rho_j = 0.5$ for all $j$

--- $x_p = 0$, --- $x_p = 5$

SNR $1/\sigma_n^2 = 15$ dB

$\Rightarrow$ For larger $x_p$ performance degrades gracefully with growing $N_p$;

LF and ELF scheme similar, QO scheme still close to optimum
Conclusions

- **Distributed** transmit power allocation schemes for **relay-assisted CR systems** in the presence of single or multiple **primary users**
  - **FD scheme:** Significant **performance improvements** over direct transmission, without interaction between relays or feedback from destination
  - **LF schemes:** Further **performance gains** utilizing little feedback from destination
  - **QO scheme:** Performance very **close to optimal** centralized solution

- **Future work:**
  - More sophisticated solutions for multiple non-congenerous primary users?
  - Impact of non-perfect channel knowledge and time-varying channel conditions?