The Effect upon Channel Capacity in Wireless Communications of Perfect and Imperfect Knowledge of the Channel

Muriel Medard, Trans. on IT 2000

Reading Group Discussion

March 20, 2008
Time Variation, ISI and multiple access dealt with an error in channel est. perspective.

The general idea is to establish bounds for the asymptotic performance with imperfections in CSI.

Look at Mutual Info. Use of Gaussian codebooks.. hence looking at capacity?

Use perturbation to model error in CSI - Error with known variance.

Will focus on the uplink case . . . and start by looking at the mutual info for a known channel.

General Trick: Upper bound the mutual info. with that of an equal variance Gaussian process
Perfectly Known Channel

Channel Model

- Imp. Resp. at $t' - t$:
  $$g(t', t) = \sum_m g_m(t', t) = \sum_m a^m(t') \delta(\tau^m(t') - t)$$
  where $\tau^m(t) = \tau^m + (B^m/f_0)t$.

- Discrete Time Model: $y[k] = \sum_{\text{all paths}} \sum_l s[k - l] g^m[k, l] + n[k]$

Max. Mutual Info. – Complex Transmission (Single User)

- Mutual Info. given by $l(Y_k; S_k) = h(Y_k) - h(N_k)$
- Eqn.(12) - approx. assumption of time and BW limiting - ???
- Using sampling rate assumptions construct $f^{2k}_{2k'}$.
- Using W.filling Max MI
  $$\frac{1}{2T} \sum_{i=1}^{2k'} \ln \left( 1 + \frac{u_i \lambda_i}{WN_0/2} \right)$$
Multiple Users- Non-cooperating case

Mutual Info

- The case where the antennas cooperate \( \rightarrow \) antenna array (development similar to single user).
- Maximise \( I(\{S_{i_{2k'}}\}_{i=1}^{K}; Y_{2k}) = h(Y_{2k}) - h(N_{2k}) \) (each source has individual power constraints - sum of all M.I / time)
- \( h(Y_{2k}) \) is maximized by using Gaussian inputs.

2-user case

- Maximize (Typo in eqn. 23)
  \[
  \max \ln \left( |f_{2k}^{2k} \Lambda_{S_1} f_{1}^{2kT} + f_{2k}^{2k} \Lambda_{S_2} f_{2}^{2kT} + \Lambda_{N_{2k}}| \right)
  \]
- The above region is proved to be convex.. hence maximization \( \rightarrow \) global max.
Assumptions on channel

- Unknown at Tx.
- Partially known at Rx.
- Idea: Look at asymptotic variations of the channel.

Mutual Info.

- \( Y = FS + N \).

\[
I(Y; S) = h(S) - h(S|Y)
\] (1)

- Known Channel:

\[
I(Y; S|F) = h(S|F) - h(S|Y, F)
\] (2)

- Eqn.2 - Eqn.1 gives (typo in Eqn.34)

\[
I(Y; S|F) - I(Y; S) = I(S; F|Y)
\] (3)
Single User - cont’d (single symbol)

- Channel known within some MSE.
- Therefore $F = \bar{F} + \tilde{F} \ldots$ where $\tilde{F}$ accounts for the meas. error.
- Recall $I(Y; S) = h(Y) - h(Y|S)$ and from $Y = S\bar{F} + S\tilde{F} + N$ we have

$$h(Y|S = s) = h(s\tilde{F} + N) \quad (4)$$

- Integrating over dist. of $\tilde{F}$

Lower Bound on MI using the worst case dist. under a cov. constraint

- Max. $I(Y; S) = h(S) - h(S|Y) \ldots$ Use Gaussian dist. $S$ - might not be max.
- Max $h(S|Y)\ldots$ thus yielding lower bound on $I$.

$$h(S|Y) \leq \frac{1}{2} \ln(2\pi e \text{Var}(S - \alpha Y)) \quad (5)$$
.. and this is the part I don't understand very well

**lower bound**

- \[ \alpha = \frac{E[SY]}{E[Y^2]} = \frac{\bar{F}\sigma_S^2}{\bar{F}^2\sigma_S^2 + \sigma_F^2\sigma_S^2 + \sigma_N^2} \]

- Use above to get \( \text{Var}(S - \alpha Y) \) and hence lower bound on \( I(S; Y) \)

**Upper bound**

- For upper bound use the fact \( I(S; Y) \leq I(S; Y|F) \) and then Jensen’s ineq. to get Eqn.(49).

- Loss in mutual info. due to uncertainty about the channel

\[
I(S; Y|F) - I(S; Y) \leq \frac{1}{2} \ln \left( 1 + \frac{\sigma_F^2\sigma_S^2}{\sigma_N^2} \right)
\]

(6)

**Interpretation**

- Upper bound : Meas. noise = Extra Tx power
- Lower bound : Not useful at all
- Not discussing the extension to multiple symbols but (Interpretations are the same) ..

Ques. why does a matrix have to be invertible if its white (pg. 940).
Multiple Access Case

Channel Assumptions and idea

- Channels of different users are mutually independent.
- If user $k$ can be decoded $\rightarrow$ interference from this user be cancelled with error owing to channel est. errors.

Mutual Info. (2 user case)

- Assumption: All Rx components part of signal ... Therefore you get upper bound on mutual info.
- $I(Y; S_1|S_2) \leq I(Y; S_1|S_2, F_1, F_2) = h(Y|S_2, F_1, F_2) - h(N)$
- Again upper bounded using Gaussian dist.

\[
I(Y; S_1|S_2) \leq \frac{1}{2} E_{F_1}[\ln(\sigma_N^2 + \sigma_{S_1}^2 F_1^2)] - \frac{1}{2} \ln(\sigma_N^2) \\
\leq \frac{1}{2} \ln \left( 1 + \frac{\sigma_{S_1}^2 F_1^2}{\sigma_N^2} \right)
\]
Lower bound

\[ I(Y; S_1 | S_2) = h(S_1 | S_2) - h(S_1 | Y, S_2) \]
\[ = h(S_1) - h(S_1 | Y, S_2) \]
\[ = h(S_1) - h(S_1 | (Y - \bar{F}_2 S_2), S_2) \geq h(S_1) - h(S_1 | (Y - \bar{F}_2 S_2)) \]

... and then use the LMMSE estimate

Interpretation

► Main Result: Uncertainty of channel of other users while doing Interference cancellation - lower bounded by more AWGN.

► Fig. 6 reinstates the widely known result that interference cancellation is better than treating the MAC as an interference channel.