Cognitive Radio: An Information Theoretic Perspective

Jovičić and Vishwanath, Trans. on IT
(Submitted 2006.. cited N times)

Reading Group Discussion

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Overview

Intro.

- Basic Idea: Allocated spectrum is underutilized..in some cases it being as low as 5%..so use it either
  - Opportunistically or
  - Simultaneously → (overlay/underlay).

- In both cases the idea is to not decrease the performance/level of satisfaction of a primary user.

- What is this paper about : What does IT tell us about what enhancements in system utilization is possible

- Assumptions in the paper
  - Generate NO interference for primary user in vicinity.
  - No changes to the decoder for primary user - there could be multiple reasons for this.

(Note that in general interference does not have to be zero for the primary user.)
Transmission Model

Who knows what

- Secondary user knows primary user’s message (..how is this relevant in practice)
- INR quantizes the interference caused the primary’s base station by secondary.
- All noise + interference is iid (usually this will not be the case → implications ??)

Main Result

- Precoding helps - how?? → rest of the paper.
- Largest rate at which \textit{reliable} transmission is possible.
- Closest to → Interference Channel with degraded message sets.
The basics

Received Signal at each base station

- We get

\[ \tilde{Y}_p = p\tilde{X}_p + f\tilde{X}_c + \tilde{Z}_p \]
\[ \tilde{Y}_c = g\tilde{X}_p + c\tilde{X}_c + \tilde{Z}_s \]

- The above equations are transformed into standard form to have unity gains for the desired users. Resulting channel \( \rightarrow (1, a, b, 1) \) channel

Achievable Rate \( R_p \)

- \( R_p \) is achievable if \( \exists \) encoding map \( E_p : \{1, 2, \ldots, 2^{nR_p}\} \rightarrow X_p^n \)
- In contrast a Cognitive encoding code map is

\[ E_c^n : \{1, 2, \ldots, 2^{nR_p}\} \times \{1, 2, \ldots, 2^{nR_p}\} \rightarrow X_c^n \]

Defn. 2.2 and 2.3 define achievability and capacity for the cognitive user (same as the conventional defns.)
Main Result : Mathematically

Capacity

- The capacity of the (1,a,b,1) cognitive radio channel is

\[ R_c^* = \frac{1}{2} \log(1 + (1 - \alpha^*)P_c) \]

for \( a \leq 1 \) and \( \alpha \) from Eqn.(17) in paper.

- Above holds for a low interference gain regime \( \rightarrow \frac{f}{\sqrt{N_p}} \leq \frac{c}{\sqrt{N_s}} \)

Other than the factor \((1 - \alpha^*)\) everything is the same as the conventional cap. formula

A little digression: 2-user conventional GMAC

- If individual rates are \( R_1 \) and \( R_2 \) you get \( R_1 + R_2 \leq 2C(P/\sigma^2_N) \) where \( C(x) \) is Shannon Cap. and \( P \) is average power constraint.

- Achievable rate pair
  - Naive TDMA : \([R_1, R_2] = \{\alpha C(P/\sigma^2), (1 - \alpha) C(P/\sigma^2)\}\)
  - Smart TDMA : \([R_1, R_2] = \{\alpha C(P/\alpha\sigma^2), (1 - \alpha) C(P/(1 - \alpha)\sigma^2)\}\)
Strategies for achieving capacity

Coding options

- Use Gaussian codebook for primary user according $N(0, P_p)$.
- Use superposition coding (refer your notes on multiterminal source coding) with

$$X^n_c = \hat{X}_c^n + \sqrt{\frac{\alpha P_c}{P_p}} X^n_p$$

thereafter use DPC (writing on Dirty paper). This channel is considered similar to Costa’s Channel (non-causal CSIT).

Value of $\alpha$

- $P_e$ for primary user vanishes as $n \to \infty$ for

$$R_p < \frac{1}{2} \log \left( 1 + \frac{(\sqrt{P_p} + a\sqrt{\alpha P_c})^2}{1 + a^2(1 - \alpha)P_c} \right)$$

Eqn(14) from paper

- Now when you equate this with $C(P_p)$ you get $\alpha$ as the root of the resulting quadratic eqn.
Issues

- Joint code design is advocated (Sect 4.2.5):
  - For high interference gain $a \geq 1$
    - P knows $m_c$ and C knows $m_p$ ..everybody cancels out everything and the world is good..
    - PROBLEM: How do you propose to do this !!!
- This brings us to system level considerations.

Optimal Scheme

- Avoids Hidden Terminal problem or Why CR will never be truly opportunistic . . .
- Robust to noise statistics - I am not convinced (argument is vague).
What is practical

Obtaining Side - Information

- Estimates of $p$ and $f$ are required for implementing the scheme.
- Cognitive radio tuned to primary user’s control channel and gets the feedback of $p$ from it. (Highly improbable)
- It is assumed that the CR can decode $m_p$ faster than $P$’s base station due to physical proximity. (Again.. is this practical?)

All of Section 5 sounds like a fairy tale ... (although one can learn a lot from the excellent proofs in this paper.. I am still learning..)

Improvisation on an Interference Channel

- Causality has got a great role to play.
- Knowing all of $m_p$ might be an over-idealization.
- Cooperation is key... However the assumption of non-causal information is ????
Where do we stand now?

Current focus - Finite State channels (and please correct me here if you know more)

- Number of papers where the cognitive transmitter is assumed to have non-causal information about the primary user
- Adv: Costa proved that if you non-causally know the state of the channel at transmitter, $C^{\text{CSIT}} = C^{\text{AWGN}}$
- Recent variation: Cognitive interference channel with finite state sequence $S^n$ (Verdu, ISIT08)
- But rate region again obtained with $S^n$ known non-causally.. relevant? why?

Open Issues

- We know that information needs to be shared - ¿ Efficient ways? (Public/Private messages)
- Above is impossible where secondary keeps moving b/w multiple cells: Too much control information