Outline

1. Capacity Bounds for Wideband Multipath Fading Channels
   - Basic Assumptions
   - Upper Bound on Capacity
   - Lower Bound on Capacity

2. Mutual Information Achieved by Spread-Spectrum Signaling
   - Motivation and Assumptions
   - Bounds on Mutual Information
   - Practical Implications
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   - Bounds on Mutual Information
   - Practical Implications
Basic Assumptions

- **Wideband multipath fading channel, bandwidth** $W$
  - Wideband: received power spread out over large bandwidth
  - Still narrowband in the sense $W \ll f_c$ ($f_c$: carrier frequency)

- Channel model:
  $$y(t) = \sum_{l=1}^{L} a_l(t) x(t - d_l(t)) + z(t)$$
  - $y(t)$: received waveform, $x(t)$: transmitted waveform, $z(t)$: AWGN
  - $L$: number of physical multipaths
  - $a_l(t)$: path amplitudes, constant during coherence time $T_c$, unknown at receiver
  - $d_l(t)$: path delays, slowly time-varying, perfectly known at receiver

- Capacity derivation:
  - Constraint on average received power $P \Rightarrow$ SNR $= P / N_0$
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Capacity Bounds for Wideband Fading Channels II

Upper Bound on Capacity

- Capacity of infinite bandwidth fading channel with SNR $P/N_0$ and _perfect_ channel state information (CSI) at receiver:

\[ C^* = \frac{P}{N_0} \]

- Wideband multipath fading channel: path amplitudes $a_i(t)$ _unknown_ at receiver $\Rightarrow$

\[ C \leq C^* = \frac{P}{N_0} \]

- $C^*$ corresponds to capacity of infinite bandwidth AWGN channel (non-fading, perfect CSI at receiver):

\[ \lim_{W \to \infty} W \log \left(1 + \frac{P}{N_0 W} \right) \approx \lim_{W \to \infty} W \frac{P}{N_0 W} = \frac{P}{N_0} =: C_{AWGN} \]
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Lower Bound on Capacity I

Design efficient signaling scheme and assess mutual information (MI)

- Choose symbol duration $T_s$ such that $2T_d \leq T_s \leq T_c$ ($T_d$: delay spread, $T_d \ll T_c$ assumed)
- Convey message $m \in \{1, \ldots, M\}$ using signal

$$x_m(t) = \begin{cases} \sqrt{\lambda} \exp(j2\pi f_m t) & 0 \leq t \leq T_s \\ 0 & \text{else} \end{cases}$$

$\Rightarrow$ Single sinusoid at frequency $f_m$ ($\hat{=} \text{FSK scheme}$)

- Receiver correlates received signal against all possible $x_m(t)$, $m \in \{1, \ldots, M\}$ $\Rightarrow$ non-coherent detection
- Choose frequencies as $f_m := n/(T_s - 2T_d)$ ($n$ integer) to obtain orthogonal scheme
- Repeat transmission of $x_m(t)$ on $N$ disjoint time intervals $\Rightarrow$ receiver can average over fading
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5. Repeat transmission of $x_m(t)$ on $N$ disjoint time intervals $\Rightarrow$ receiver can average over fading
Lower Bound on Capacity II

- Using *low duty cycle* above scheme achieves mutual information (MI):

\[
I(x; y|d_i) = \left(1 - 2 \frac{T_d}{T_c}\right) \frac{P}{N_0}
\]

Due to *average* power constraint we have \( \lambda := P/\theta \gg P \) \((\theta \to 0)\)

- Altogether:

\[
\left(1 - 2 \frac{T_d}{T_c}\right) C_{\text{AWGN}} \leq C \leq C_{\text{AWGN}}
\]

\((C_{\text{AWGN}} = P/N_0)\)

- Since \( T_d \ll T_c \), lower and upper bound approximately *coincide*

- Capacity-achieving signaling is “peaky” in time and frequency domain
Lower Bound on Capacity II

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Capacity Bounds for Wideband Fading Channels

\[ 0 \leq t \leq T_s \]

\[ T_s < t < T_I \]

\[ \theta = \frac{T_s}{T_I} \rightarrow 0 \]
Capacity Bounds for Wideband Multipath Fading Channels
- Basic Assumptions
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Mutual Information Achieved by Spread-Spectrum Signaling
- Motivation and Assumptions
- Bounds on Mutual Information
- Practical Implications
Spread-spectrum (SS) schemes (DS-CDMA, code-spread CDMA, ...) commonly used for communication over wideband channels.

**Key result**
- Capacity-achieving signaling for wideband multipath fading channels *maximal different* from SS signaling
  - ⇒ SS signals are “white-like” and non-peaky in time

**Question**
- How good is SS signaling for wideband multipath fading channels?
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**Key result**
- Capacity-achieving signaling for wideband multipath fading channels *maximal different* from SS signaling
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**Question**
- How good is SS signaling for wideband multipath fading channels?
Assumptions

- Discrete-time channel model:

\[ Y_i = \sqrt{\frac{\mathcal{E}}{K_c}} \sum_{l=1}^{\tilde{L}} G_l X_{(i-D_l)} + Z_i \]

- \( Y_i \): received sample, \( X_i \): transmitted symbol, \( Z_i \): AWGN sample
- \( \tilde{L} \): number of resolvable multipaths at system bandwidth \( W \) \((\tilde{L} \leq L)\)
- \( G_l, D_l \): amplitudes/delays of resolvable multipaths
- \( \mathcal{E} := PT_c/N_0 \), \( K_c \) normalization factor

- Two different notions of “white-like” signals
  - info symbols modulated on pseudo-random spreading sequences with near-perfect auto-correlation (\( \hat{=} \) DS-CDMA)
  - info symbols spread onto wide bandwidth using low-rate FEC (\( \hat{=} \) code-spread CDMA)
MI bounds for Spread-Spectrum Signaling II

Assumptions

- **Discrete-time channel model:**

  \[ Y_i = \sqrt{\frac{\mathcal{E}}{K_c}} \tilde{L} \sum_{l=1}^{\tilde{L}} G_l X_{(i-D_l)} + Z_i \]

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  - info symbols modulated on pseudo-random spreading sequences with near-perfect auto-correlation (\( \wedge \) DS-CDMA)
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Upper bound on MI per unit time (holds for large $W$ and large $\tilde{L}$; equal average path energies assumed):

$$I(X; Y|D_l) \leq \frac{\mathcal{E}^2}{T_c^2 \tilde{L}}$$

Lower bound on MI per unit time (holds for large $W$ and any $\tilde{L}$):

$$I(X; Y|D_l) \geq \frac{\mathcal{E}}{T_c} - \frac{\tilde{L}}{T_c} \log \left(1 + \frac{\mathcal{E}}{\tilde{L}}\right)$$

- If $\tilde{L} \ll \mathcal{E}$, lower bound close to $\mathcal{E}/T_c = P/N_0 = C_{AWGN}$, i.e., SS signaling near-optimal
- If $\tilde{L} \gg \mathcal{E}$, upper bound holds and is close to zero, i.e., SS signaling highly suboptimal (!)

$\Rightarrow \mathcal{E} =: \tilde{L}_{\text{crit}}$ critical system parameter indicating overspreading
Bounds on MI

- Upper bound on MI per unit time (holds for large $W$ and large $\tilde{L}$; equal average path energies assumed):

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Critical Parameter $\tilde{L}_{\text{crit}}$

- Critical parameter also plays *key role* for detection error probability of (specific) binary orthogonal modulation schemes ($W \to \infty$)

- Interpretation of case $\tilde{L} \gg \tilde{L}_{\text{crit}}$:
  - $\Rightarrow$ energies of resolvable paths very small
  - $\Rightarrow$ poor estimates of complex gains
  - $\Rightarrow$ effective multipath combining at the receiver *difficult*

Question

- How good are DS-UWB systems?
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