On the Spreading Sequences Allocation for DS/CDMA

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Outline of the Presentation

1. Introduction, problems, CDMA model
2. Classical Techniques for Sequence Allocation
3. Information-Theoretic Sequence Allocation
   3.1. Sum Capacity Maximization
   3.2. Symmetric Capacity Maximization
1. Introduction

- Spreading Codes provide differentiation among users in a DS/CDMA system

- CDMA systems are interference limited

- The criteria for the choice of spreading sequences:
  1) Autocorrelation of the sequences
  2) Crosscorrelation between sequences
  3) Peak-to-average power ratio
  4) Maximizing throughput with and without single user QoS constraints
Uncoded Synchronous CDMA Multiple-Access Channel

Channel\textsubscript{1}

Channel\textsubscript{2}

Channel\textsubscript{K}

Matched Filter\textsubscript{1}

Matched Filter\textsubscript{2}

Matched Filter\textsubscript{K}

\textbf{s}_1(t)

\textbf{s}_2(t)

\textbf{s}_K(t)

\textbf{b}_1

\textbf{b}_2

\textbf{b}_K

\hat{\textbf{b}}_1

\hat{\textbf{b}}_2

\hat{\textbf{b}}_K

On the Spreading Sequence Allocation for DS/CDMA
Synchronous CDMA Model (cont’d)

Vector channel model: \( \mathbf{r} = \mathbf{S}\mathbf{a} + \mathbf{n} \)

- \( N \) - processing gain
- \( K \) - number of users in the system

\( \mathbf{S} \) is \( N \times K \) matrix of sequences:

\[
\mathbf{S} = [s_1, s_2, \cdots, s_K]
\]
2. Classical Techniques for Sequence Allocation
2. Classical Algebraic Techniques for Sequence Allocation

**m-sequences**

- The main goal here is to minimize the autocorrelation

- Linear Feedback Shift Register (LFSR)
- Function defined by the characteristic function of the LFSR
- May have large cross-correlations

\[ R = S^T S \]
2. Algebraic Techniques (cont’d)

Gold sequences

- These sequences are \( m \)-sequences and its delayed version
- For smaller number of users and processing gains these sequences can
- Have smaller crosscorrelations than purely random sequences

Kasami sequences

- Large and small set of Kasami sequences
- Small set satisfies the Welch lower bound on cross-correlation
2. Lower Bounds on Cross-correlation

- Welch Lower Bound on the Total Periodic Crosscorrelation
  [L. Welch, Trans. On Inf Theory, 1974]

\[
\sum_{i=1}^{K} \sum_{j=1}^{K} (s_i^T s_j)^2 \geq \frac{K^2}{N}
\]

Applicable when sequences are of unit energy.

\[
s_i^T s_i = 1, \forall i
\]
2. Classical Techniques - Orthogonal Sequences

Orthogonal Sequences

2. Classical Techniques - Orthogonal Sequences

Orthogonal Sequences + variable length codes

- Different rate (QoS) requirements for different users

Example:

User 1, \( R_1 = R \)
User 2, \( R_2 = 2R \)
User 3, \( R_3 = 4R \)
3. Information-Theoretic Sequence Allocation
CDMA Multiple-Access Channel

Vector channel model: \( r = S A d + n \)

- \( N \) - processing gain
- \( K \) - number of users in the system
CDMA Multiple-Access Channel

Capacity region (vector channel) : Verdu (1986)

\[
C_{Gauss} = \bigcap_{J \subset \{1, \ldots, K\}} \left\{ \begin{bmatrix} R_1, \ldots, R_K \end{bmatrix}; \sum_{i \in J} R_i \leq \frac{1}{2} \log \left| I + \frac{\sum_{i \in J} P_i s_i s_i^T}{\sigma^2} \right| \right\}
\]

The Analyzed case: Synchronous channel + Flat Fading

Let:

\[
W = diag [P_1, \ldots, P_K]
\]
CDMA Multiple-Access Channel: 
Sum and Symmetric Capacity

Sum Capacity is defined as:

\[ C_{\text{sum}} = \max_{R \in C_{\text{Gauss}}} \sum_{i=1}^{K} R_i = \frac{1}{2} \log \left| I + \frac{SWS^T}{\sigma^2} \right| \]

Symmetric Capacity is defined as:

\[ C_{\text{sym}} = \max_{[R,R,\ldots,R] \in C_{\text{Gauss}}} R \]

More on the Optimal Sequence Allocation in Next seminar
CDMA Multiple-Access Channel Rate Region

\[ C_1 = \frac{1}{2} \log \left( 1 + \frac{P_1}{\sigma^2} \right) \]

\[ C_2 = \frac{1}{2} \log \left( 1 + \frac{P_2}{\sigma^2} \right) \]

\[ C_1 + C_2 = \frac{1}{2} \log \left| I + \frac{SSW^T}{\sigma^2} \right| \]

\[ R_1 = R_2 \]

Question: \[ C_{sym} = C_{sum} \]?
Optimal Sequences that Maximize $C_{\text{sum}}$

$K \leq N$

- orthogonal sequences are assigned to different users
- more interesting is the over-saturated case: $K > N$
Optimal Sequences that Maximize $C_{sum}$ ($K>N$)

1) [Rupf, Massey, Trans. On Inf. Theory, 1994]

$W = \text{diag}[P, \ldots, P]$

The following condition has to be satisfied: maximization of $\frac{1}{2} \log \left| I + \frac{S S^T}{\sigma^2} \right|$

$S S^T = I$

Sequences have to be unit norm: $s_i^T s_i = 1, \forall i$

These sequences satisfy Welch Lower Bound with equality:

$$\sum_{i=1}^{K} \sum_{j=1}^{K} (s_i^T s_j)^2 = \frac{K^2}{N}$$
Optimal Sequences that Maximize $C_{sum}(K>N)$

1) [Rupf, Massey, Trans. On Inf. Theory, 1994] (cont’d)

$$C_{sum} = \frac{1}{2} \log \left| I + \frac{SWS^T}{\sigma^2} \right| \leq \frac{1}{2} \log \left( 1 + \frac{P_{tot}}{\sigma^2} \right)$$
Optimal Sequences that Maximize $C_{\text{sum}} (K>N)$

2) [Viswanath, Anantharam, Trans. On Inf. Theory, 1999]

General case is considered: 

$$W = \text{diag} \ [P_1, \ldots, P_K]$$

- Oversized users: orthogonal sequences
- Non-oversized users: Generalized Welch Bound sequences

$$S_1 W S_1^T = I$$
Optimal Sequences that Maximize $C_{\text{sum}} (K>N)$

2) [Viswanath, Anantharam, Trans. On Inf. Theory, 1999] (cont'd)

If there are oversized users:

$$C_{\text{sum}} = \frac{1}{2} \log \left| I + \frac{SWS^T}{\sigma^2} \right| < \frac{1}{2} \log \left( 1 + \frac{P_{\text{tot}}}{\sigma^2} \right)$$

These sequences also maximize signal-to-noise ratio with linear MMSE receivers.

Construction of these sequences:
- centralized iterative
- decentralized iterative: Total Squared Correlation Minimization
  (Ulukus, Yates, Trans. On Inf. Theory [2001])

$$\sum_{i=1}^{K} \sum_{j=1}^{K} (s_i^T s_j)^2$$
Comparison of Optimal Sequences and Random Sequences in Flat Fading Channels

- Optimal Sequences + No Fading
- Random Sequences + No Fading
- Random Sequences + Rayleigh Fading
- Optimal Sequences + Power Control + Rayleigh Fading
- Optimal Sequences + Rayleigh Fading

![Graph showing comparison of signal to noise ratio (SNR) vs. fading parameter α for different sequence types.

- Power control
- No power control

On the Spreading Sequence Allocation for DS/CDMA
Percentage of Orthogonal Sequences

SNR = 3dB
Power Control
No Power Control
Power Controlled Optimal Sequences that Maximize $C_{\text{sum}} (K>N)$

Unit energy assumption does not have to be satisfied: $s_i^T s_i = 1, \forall i$
Optimal Sequences that Maximize $C_{sym}$

Fairness requirement: single user rates of all users are equal

$$C_{sym} = C_{sum}$$ Correct for equal received power of all users.

$K \leq N$, different received powers

- The best is to allocate orthogonal sequences to all users

- If power control is allowed the best is to equalize received powers
Optimal Sequences that Maximize $C_{sym}$

$K > N$, different received powers

- Cholesky characterization of the vertices
- Optimization of the vertices

$R_1 = R_2$
Extensions

- Multipath fading channels

- Asynchronous systems

- Different QoS requirements

- How does autocorrelation fit in these information-theoretic measures?
Thank you!
Illustration of the convergence of the order statistics to the quantile

\[ \xi_{0.2} \]

\[ K = 1000 \]
\[ K = 100 \]
\[ K = 10 \]