

# Optimal Packet Scheduling over Correlated Nakagami- $m$ Fading Channels with Different Diversity-Combining Techniques

Ashok K. Karmokar, *Student Member, IEEE*, and Vijay K. Bhargava, *Fellow, IEEE*

Department of Electrical and Computer Engineering, 2356 Main Mall,  
University of British Columbia, Vancouver, BC, V6T 1Z4, Canada.  
E-mail: {ashokk,vijayb}@ece.ubc.ca

**Abstract**—We study two cross-layer optimization problems for M-QAM systems that adapt transmission rate with channel state and buffer occupancy. We formulate both problems as constrained Markov decision process problem and give linear programming technique based solutions. In first problem, our objective is to minimize average transmission power under constraint on average delay and packet dropping probability. We minimize average bit error rate (BER) with average delay and packet dropping probability constraints in second problem. The Nakagami- $m$  fading channel with diversity-combining is described as finite state Markov channel. Simulation results show that the system performances can be improved by adapting rate with buffer state and hence delaying packet in the buffer in addition to employing diversity-combining at the receiver.

## I. INTRODUCTION

The demand for wireless data services having different quality of service (QoS) requirements is growing at a rapid pace with the explosive proliferation of web-based services [1]. However, due to notoriously time-varying channel fading gain and scarce wireless resources (e.g., bandwidth and power), delivering such services is a challenging task in wireless networks. Adaptive modulation schemes employing spectrally efficient multilevel modulation, such as MQAM, have been used as a powerful technique to provide high data rate and compensate channel degradation [2]. The key idea of these schemes are the adaptive variation of transmitter power level, symbol transmission rate, constellation size, BER, coding rate/scheme, or any combination of these parameters. Adaptive transmission requires accurate channel estimation at the receiver and a reliable feedback path between the receiver and transmitter. Space diversity is another useful technique to combat fading and can often be combined with adaptive modulation to further improve the system performance [3]. Most adaptive modulation schemes in the literature (e.g., [2],[3], etc.) focus mainly on optimizing throughput considering only physical layer parameters and neglecting higher layer parameters (such as buffer delay, packet dropping probability, etc.). But in a practical wireless system, the arriving packets are non-constant and are stored in a finite size buffer before being transmitted. Therefore, the buffer delay and packet dropping should be considered in practical adaptive transmission schemes. Recently, several authors in the literature have addressed cross-layer optimization issues and have shown that

simultaneous adaptation with physical layer and higher layer parameters can improve the system performance significantly. Early work of [4] gives a dynamic-programming framework for transmission policies over a simple two-state Gilbert-Elliott channel with constraint on average delay and peak power. In [5], buffer and channel adaptive modulation for maximizing throughput is discussed. The minimal power transmission of independent and identically distributed bursty sources over wireless channels with constraint on mean queueing delay is studied in [6]. In our work [7], we have addressed the issue of optimal and suboptimal scheduling over a Rayleigh fading finite state Markov channel (FSMC) and have shown that the cross-layer problem forms a weakly communicating Markov decision process (MDP). The fading gains of wireless links show a significant degree of correlations among consecutive states due to its inherent memory and is important to take into consideration for designing packet transmission protocols [8]. The FSMC model for Rayleigh fading is proposed in [9] and is much more accurate than two-state Gilbert-Elliott model. For packet/block level communications when the block length is large, this first order Markov model accurately represents practical fading channel with memory [10] and is popular due to its good balance between accuracy and complexity. A generalized FSMC model for Nakagami- $m$  fading channel with different space diversity on receive is given in [11].

In this paper, we present two optimal packet scheduling schemes for M-QAM systems over Nakagami- $m$  fading channels with different receive diversity-combining techniques. We formulate the cross-layer optimization problems as constrained Markov decision process (CMDP) and give linear programming (LP) solutions. We consider finite size buffer and bursty traffic, and adapt the modulation constellations with channel and buffer states to minimize average power/BER putting bounds on average delay and packet dropping probability.

The paper is organized as follows. Section II describes system model and notations used in the paper. The modeling of channel as a FSMC is given in Section III. In Section IV, we formulate the problem as CMDP, give its ingredients and describe its LP solution methodology. Section V provides numerical results and discussions. We conclude in Section VI.

## II. SYSTEM MODELING AND NOTATIONS

The schematic of the adaptive transmission system with diversity is illustrated in Fig. 1. We assume a discrete-time

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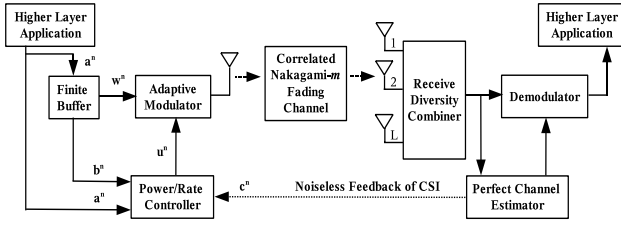


Fig. 1. Schematic of the System

system, where a time-slot (also called stage or block) is corresponding to  $F$  channel uses of a correlated block fading channel. Unless otherwise specified, in this paper we use superscript  $n$  to denote the value of certain variable at time-slot  $n$ . Let  $\bar{P}$  denotes the average transmit signal power,  $\bar{h}$  denotes the average channel power gain, and  $\sigma^2$  denotes the variance of channel noise, which is assumed as additive white Gaussian. The receiver estimates the channel at each time-slot and sends the information to the transmitter over a feedback path. We assume this path to be instantaneous and error-free. Therefore, the channel gain estimate  $\hat{h}^n$  equals the channel gain  $h^n$ . For a constant transmit power  $\bar{P}$ , the instantaneous received SNR is  $\gamma^n = h^n \bar{P} / \sigma^2$ . In the sequel, we will omit the time reference  $n$  relative to  $\gamma$  since  $h^n$  is stationary and we denote the probability distribution function (pdf) of  $\gamma$  by  $f_\Gamma(\gamma)$ . Therefore, at time-slot  $n$  when the transmit power is  $P_T$ , the instantaneous received SNR is given by  $\gamma P_T / \bar{P}$ . We also assume ideal coherent phase detection.

Incoming packets from a higher layer application are placed in a buffer of size  $B$  packets. Suppose  $a^n$  denotes the number of packets arriving at the input of the buffer between time-slot  $n-1$  and  $n$  with average packet arrival rate  $\lambda = \mathbb{E}\{a^n\}$ . It is assumed that  $a^n$  is independent of the channel fading and noise processes. Let  $p^a(a^n)$  denotes the discrete probability distribution of  $a^n$  and  $\mathcal{A} = \{a_0, a_1, \dots, a_A\}$  denotes the state space of the input arrival packet. For Poisson distributed arrival traffic, the probability of  $a_j$  packet arrival in time-slot  $n$  can be expressed as  $p^a(a^n = a_j) = \exp(-\lambda T_B) \frac{(\lambda T_B)^{a_j}}{a_j!}$ ,  $j = 0, 1, \dots, A$ , where  $T_B$  is the block period and  $p^a(a^n = a_A) \mapsto 0$  (a very small number).

Let  $b_k$  denotes  $k$  packets in the buffer, therefore, the state space of the buffer's packet occupancy can be expressed as  $\mathcal{B} = \{b_0, b_1, \dots, b_B\}$ . At the starting of a particular time-slot  $n$ , the scheduler decides action  $u^n$  to be taken depending on the buffer state  $b^n$  as well as channel state  $c^n$ . Assume that there are total  $U$  actions available and  $\mathcal{U} = \{u_1, u_2, \dots, u_U\}$  be the set of all permissible actions. Corresponding to action  $u^n$ , the transmitter takes  $w^n$  packets of size  $G$  bits from the buffer and transforms these packet into  $Gw^n/F$  bits/symbol codeword, which will be sent over the next  $F$  channel uses. All the packets transmitted in a block experiences same channel gain. Note that,  $w^n = \Psi(u^n)$ , i.e.,  $\Psi$  maps action set into a set of number of packet  $\mathcal{W} = \{w_1, w_2, \dots, w_U\}$  to be transmitted. Since there is no transmission when the buffer

is empty and the transmitter cannot take more packet from the buffer than its current occupancy, therefore, a natural constraint on  $u^n$  can be written as  $0 \leq w^n \leq b^n$ . We assume that all packets arrive in time-slot  $n$  cannot be transmitted immediately and can only be transmitted in time-slot  $n+1$  or later. The buffer's packet occupancy in each time-slot can be updated using the following relations,

$$b^{n+1} = \min\{b^n + a^{n+1} - w^n, b_B\}. \quad (1)$$

### III. CORRELATED CHANNEL MODELING

In a land-mobile and indoor-mobile multipath propagation environments, the received signal envelope has Nakagami- $m$  distribution. A slowly varying Nakagami- $m$  fading channel can be modeled as a FSMC by partitioning instantaneous received SNR  $\gamma_{dc}$  at the output of the diversity combiner, which is proportional to the square of the received signal amplitude  $r_{dc}$  at the output of the diversity receiver, into finite  $C$  number of non-overlapping states. Let  $f_\Gamma^{dc}(\gamma_{dc})$  and  $F_\Gamma^{dc}(\gamma_{dc}) = \int_0^{\gamma_{dc}} f_\Gamma^{dc}(y) dy$  denote the probability density function (pdf) and cumulative density function (cdf) respectively of  $\gamma_{dc}$ . Suppose  $\mathcal{C} = \{c_1, c_2, \dots, c_C\}$  denotes the state space of the combined diversity channel and  $\Gamma_{dc} = \{\gamma_{dc_0}, \gamma_{dc_1}, \dots, \gamma_{dc_C}\}$  is the corresponding set of received SNR thresholds of the diversity combiner in increasing order with  $\gamma_{dc_0} = 0$  and  $\gamma_{dc_C} = \infty$ . Then the channel is said to be in state  $c_i$  if  $\gamma_{dc_{i-1}} \leq \gamma_{dc} < \gamma_{dc_i}$ . There are several methods in the literature for partitioning received SNR threshold. Although our analysis can be applied to any partitioning scheme, in the sequel we consider equal probability method (EPM). In EPM, the received SNR are divided so that the stationary probabilities of staying in all states are same i.e.,  $p_{dc_1} = p_{dc_2} = \dots = p_{dc_C} = \frac{1}{C}$  [9]. The stationary probability of channel state  $c_i$   $i = 1, \dots, C$  can be expressed as,

$$p_{dc_i} = \int_{\gamma_{dc_{i-1}}}^{\gamma_{dc_i}} f_\Gamma^{dc}(\gamma_{dc}) d\gamma_{dc} = F_\Gamma^{dc}(\gamma_{dc_i}) - F_\Gamma^{dc}(\gamma_{dc_{i-1}}). \quad (2)$$

Suppose, the channel transition probability from state  $c_i$  to state  $c_j$  be denoted as  $p_{dc}^c(c_j|c_i)$ . When the fading of the channel is slow, the first order finite state Markov channel model can be approximated as a Birth-and-Death process. Therefore, the quantized channel state  $c_i$  can jump only to the adjacent state  $c_j$ , i.e.  $p_{dc}^c(c_j|c_i) = 0$  if  $|j - i| > 1$ . The transition probability  $p_{dc}^c(c_j|c_i)$  for  $|j - i| = 1$  can be approximated by the ratio of the expected number of level crossings at the interfacial received SNR threshold and the average transmission rate in state  $c_i$ . If  $T_B$  denotes the duration of a block in second, the average duration of a block during which the channel is in state  $c_i$  is  $T_{B_i} = T_B / p_{dc_i}$ . Therefore, the crossover probabilities of the channel states can be written as [9],

$$p_{dc}^c(c_{i+1}|c_i) \approx N_{dc_i} T_{B_i}, \quad i = 1, 2, \dots, C-1 \text{ and } (3)$$

$$p_{dc}^c(c_{i-1}|c_i) \approx N_{dc_{i-1}} T_{B_i}, \quad i = 2, 3, \dots, C, \quad (4)$$

where,  $N_{dc_i}$  is the level crossing rate (LCR) at the corresponding SNR threshold  $\gamma_{dc_i}$  either in positive direction or in

negative direction only. The self-transition probabilities of the channel states can be found from

$$p_{dc}^c(c_i|c_i) = 1 - \sum_{j=i-1, j \neq i}^{i+1} p_{dc}^c(c_j|c_i); i = 1, 2, \dots, C, \quad (5)$$

where, redundant probabilities  $p_{dc}^c(c_0|c_1) = p_{dc}^c(c_{C+1}|c_C) = 0$ . We consider two diversity-combining techniques. We assume that the diversity channels are independent and the fading parameters of all channels are identical. Suppose  $L$  denotes the number of antenna employed at the receiver and  $\bar{\gamma} = \mathbb{E}\{\gamma\}$  denote the average received SNR of each branch.

#### A. Selection Combining Diversity

Selection combining (SC) diversity only processes one of the diversity branches, specifically one with the largest instantaneous received SNR. The pdf and cdf of the received output SNR of a  $L$ -branch diversity combiner are [11],

$$f_{\Gamma}^{sc}(\gamma_{sc}) = L \left[ \frac{\gamma(m, \frac{m\gamma_{sc}}{\bar{\gamma}_{sc}})}{\Gamma(m)} \right]^{L-1} \frac{m^m \gamma_{sc}^{m-1}}{\bar{\gamma}_{sc}^m \Gamma(m)} \exp\left(-\frac{m\gamma_{sc}}{\bar{\gamma}_{sc}}\right) \quad (6)$$

$$\text{and } F_{\Gamma}^{sc}(\gamma_{sc}) = \left[ \frac{\gamma(m, \frac{m\gamma_{sc}}{\bar{\gamma}_{sc}})}{\Gamma(m)} \right]^L, \quad (7)$$

where  $\bar{\gamma}_{sc} = \mathbb{E}\{\gamma_{sc}\}$  is the average received SNR at the output,  $\Gamma(m) = \int_0^{\infty} t^{m-1} e^{-t} dt$  and  $\gamma(m, x) = \int_0^x t^{m-1} e^{-t} dt$  are the complete and the lower incomplete Gamma function respectively. The Nakagami- $m$  distribution spans over a wide range of fading conditions and fading severity parameter  $m$  ranges from  $\frac{1}{2}$  to  $\infty$ . In a special case,  $m = 1$  corresponds to Rayleigh distribution and as  $m \rightarrow +\infty$ , the Nakagami- $m$  fading channel converges to a non-fading AWGN channel. After combining (2) and (7), the received SNR thresholds can be obtained numerically from the following equation,

$$\left[ \gamma(m, \frac{m\gamma_{sc_i}}{\bar{\gamma}_{sc}}) \right]^L - \left[ \gamma(m, \frac{m\gamma_{sc_{i-1}}}{\bar{\gamma}_{sc}}) \right]^L = \frac{[\Gamma(m)]^L}{C}. \quad (8)$$

The average received SNR of state  $c_i$  can be given by,

$$\bar{\gamma}_{sc_i} = \frac{1}{p_{sc_i}} \int_{\gamma_{sc_{i-1}}}^{\gamma_{sc_i}} \gamma_{sc} f_{\Gamma}^{sc}(\gamma_{sc}) d\gamma_{sc}, \quad (9)$$

and the LCR of state  $c_i$  can be given by,

$$N_{sc_i} = \frac{\sqrt{2\pi} f_m L}{[\Gamma(m)]^L} \left( \frac{m\gamma_{sc_i}}{\bar{\gamma}_{sc}} \right)^{m-\frac{1}{2}} e^{-\frac{m\gamma_{sc_i}}{\bar{\gamma}_{sc}}} \left[ \gamma(m, \frac{m\gamma_{sc_i}}{\bar{\gamma}_{sc}}) \right]^{L-1}. \quad (10)$$

In (10),  $f_m = v_{mt}/\lambda_{rw}$  is the maximum Doppler frequency, where  $v_{mt}$  and  $\lambda_{rw}$  denote the speed of the mobile terminal and the wavelength of the radio wave respectively.

#### B. Maximal Ratio Combining Diversity

The SC diversity is the simplest form of ‘‘space diversity on receive’’ techniques and yields suboptimal performance. MRC with perfect combining (perfect knowledge of the branch amplitudes and phases) is the optimal diversity scheme among linear diversity-combining techniques, but it requires knowledge of all branch parameters and independent processing of

each branch. The pdf and cdf of the received output SNR of MRC schemes are given by [11],

$$f_{\Gamma}^{mrc}(\gamma_{mrc}) = \left( \frac{m_{mrc}}{\bar{\gamma}_{mrc}} \right)^{m_{mrc}} \frac{\gamma_{mrc}^{m_{mrc}-1}}{\Gamma(m_{mrc})} e^{-\frac{m_{mrc}\gamma_{mrc}}{\bar{\gamma}_{mrc}}} \quad (11)$$

$$\text{and } F_{\Gamma}^{mrc}(\gamma_{mrc}) = \frac{\gamma(m_{mrc}, \frac{m_{mrc}\gamma_{mrc}}{\bar{\gamma}_{mrc}})}{\Gamma(m_{mrc})} \quad (12)$$

where,  $m_{mrc} = mL$  and  $\bar{\gamma}_{mrc} = \bar{\gamma}L$ . The average received SNR for state  $c_i$  can be given by,

$$\bar{\gamma}_{mrc_i} = \frac{\bar{\gamma}_{mrc}}{m_{mrc} p_{mrc_i} \Gamma(m_{mrc})} \left[ \gamma(m_{mrc} + 1, \frac{m_{mrc}\gamma_{mrc_i}}{\bar{\gamma}_{mrc}}) - \gamma(m_{mrc} + 1, \frac{m_{mrc}\gamma_{mrc_{i-1}}}{\bar{\gamma}_{mrc}}) \right] \quad (13)$$

and the LCR corresponding to state  $c_i$  can be given by,

$$N_{mrc_i} = \frac{\sqrt{2\pi} f_m}{\Gamma(m_{mrc})} \left( \frac{m_{mrc}\gamma_{mrc_i}}{\bar{\gamma}_{mrc}} \right)^{m_{mrc}-\frac{1}{2}} e^{-\frac{m_{mrc}\gamma_{mrc_i}}{\bar{\gamma}_{mrc}}}. \quad (14)$$

### IV. BUFFER COST CONSTRAINT OPTIMAL RATE, POWER AND BER ADAPTATION

We consider two transmission rate adaptation problems for MQAM systems. We formulate both problems as CMDP problem that has the following ingredients: a set of time-slots  $\mathcal{T} = \{1, 2, \dots, m\}$ , a set of system states  $\mathcal{S} = \mathcal{B} \times \mathcal{C} = \{s_1, s_2, \dots, s_S\}$  with total number of states  $S = (B+1) \times C$ , a set of actions, a set of state and action dependent transition probabilities and a set of state and action dependent immediate costs. The actions are the set of different modulation schemes/MQAM constellations. First action corresponds to no transmission, second action corresponds to BPSK transmission, and next higher actions are corresponding to a particular MQAM constellation. At the start of time-slot  $n$ , the scheduler takes a particular action  $u_i$  and move to system state  $s_j$  from state  $s_i$  according to state transition probability,

$$p_{s_i s_j}(u_i) = p_{dc}^c(\chi(s_j)|\chi(s_i)) \sum_{k=0}^A \delta(\psi(s_j) - \psi(s_i) - a_k + \Psi(u_i)) \quad (15)$$

where, function  $\delta(x)$  returns 1 if  $x = 0$  and 0 otherwise. Function  $\chi(s)$  gives the corresponding channel state of the composite state  $s$  and can be expressed as,

$$\chi(s) = \sum_{i=1}^C c_i \delta\left(\left\lceil \left(\frac{s}{B+1}\right) \right\rceil - i\right) \quad (16)$$

where, the function  $\lceil x \rceil$ , called the ceiling function of  $x$ , gives the smallest integer that is greater than or equal to  $x$ . The buffer state corresponding to the system state  $s$  can be expressed as,

$$\psi(s) = \sum_{j=0}^{C-1} \sum_{i=1}^{B+1} b_{i-1} \delta(s - (B+1)j - i). \quad (17)$$

The problem at hand forms a weakly communicating average cost MDP [7]. A MDP is said to be *weakly communicating*

if there exists a closed set of states, with each state in that set accessible from every other state in that set under some deterministic stationary policy, plus a possibly empty set of states which is transient under every policy [12]. A policy is defined as the decision rule to be used at all time-slot,  $\pi = \{\mu^1, \mu^2, \dots, \mu^m\}$ . Let  $\Pi$  denote the set of all admissible policies  $\pi$ . We assume that the immediate costs and transition probabilities do not vary with respect to time-slot  $n$ . A policy is called stationary if it does not vary with time-slot  $n$  and has the form  $\pi = \{\mu, \mu, \dots, \mu\}$ ; for brevity we denote it by  $\mu$ .

#### A. Optimal Rate and Power Adaptation with Constant BER

In the first problem, we adapt transmission rate and power with the channel condition and buffer occupancy keeping the BER of all channel states to a specified equal value. We are interested in minimizing three objectives, namely, average power, average delay, and average packet dropping probability. Since the objectives are conflicting, in CMDP problem, we minimize average power with bounds on average delay and average packet dropping probability. The objective in this case is to find the optimal stationary policy  $\mu^*$  so that,

$$\begin{aligned} \min_{\mu} J_P(\mu) &= \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{n=1}^m \mathbb{E}[P_T(s^n, \mu^n(s^n))] \quad (18) \\ \text{subject to:} & \quad \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{n=1}^m \mathbb{E}[d^1(s^n, \mu^n(s^n))] \leq B_{\text{delay}} \\ \text{and} & \quad \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{n=1}^m \mathbb{E}[d^2(s^n, \mu^n(s^n))] \leq P_{\text{dropping}}, \end{aligned}$$

where  $P_T(s^n, \mu^n(s^n))$  is the immediate transmission power cost and  $d^i(s^n, \mu^n(s^n))$ ,  $i = 1, 2$  are the buffer related cost described in Section IV-D.1.  $B_{\text{delay}}$  and  $P_{\text{dropping}}$  are the maximum tolerable average delay and packet dropping probability.

#### B. Optimal Rate and BER Adaptation with Constant Power

In the second problem, the transmitter power is assumed to be always same, and we adapt transmission rate and BER with the channel condition and buffer occupancy. Our goal is to minimize average BER, average delay, and average packet dropping probability. We formulate the problem as CMDP where we minimize average BER with constraint on average delay and average packet dropping probability. Therefore, the objective is to find the optimal stationary policy  $\mu^*$  so that,

$$\begin{aligned} \min_{\mu} J_e(\mu) &= \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{n=1}^m \mathbb{E}[\bar{P}_e(s^n, \mu^n(s^n))] \quad (19) \\ \text{subject to:} & \quad \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{n=1}^m \mathbb{E}[d^1(s^n, \mu^n(s^n))] \leq B_{\text{delay}} \\ \text{and} & \quad \lim_{m \rightarrow \infty} \frac{1}{m} \sum_{n=1}^m \mathbb{E}[d^2(s^n, \mu^n(s^n))] \leq P_{\text{dropping}}, \end{aligned}$$

where  $\bar{P}_e(s^n, \mu^n(s^n))$  is the immediate BER cost.

#### C. Solution Techniques

Linear programming (LP) methodology can be used to solve the CMDP problem (18) and (19) formulated above [12]. There is a one-to-one correspondence between feasible (and hence optimal) solution of the LP and the feasible (and hence optimal) solution of CMDP. LP is feasible if and only if CMDP is feasible [13]. Let  $\nu(s, u)$  represents the ‘‘steady-state’’ probability that the process is in state  $s$  and action  $u$  is applied. We seek to find the control policy which is represented in terms of probability distribution  $\nu$  over  $\mathcal{S} \times \mathcal{U}$ . The optimal policy  $\nu^*$  can be obtained by solving the linear program,

$$\begin{aligned} \min_{\nu} & \quad \sum_{s \in \mathcal{S}, u \in \mathcal{U}_s} g(s, u) \nu(s, u) \quad (20) \\ \text{s.t.:} & \quad \sum_{s \in \mathcal{S}, u \in \mathcal{U}_s} d^1(s, u) \nu(s, u) \leq B_{\text{delay}} \\ & \quad \sum_{s \in \mathcal{S}, u \in \mathcal{U}_s} d^2(s, u) \nu(s, u) \leq P_{\text{dropping}} \\ & \quad \sum_{u \in \mathcal{U}_s} \nu(s, u) = \sum_{s \in \mathcal{S}, u \in \mathcal{U}_s} \nu(s, u) p_{st}(u), \quad \forall t \in \mathcal{S} \\ & \quad \sum_{s \in \mathcal{S}, u \in \mathcal{U}_s} \nu(s, u) = 1; \nu(s, u) \geq 0, \forall s \in \mathcal{S} \text{ and } u \in \mathcal{U}_s, \end{aligned}$$

where,  $\mathcal{U}_s$  is the set of actions that are allowed in state  $s$  and  $g(s, u)$  represents either immediate power cost  $P_T(s, u)$  or BER cost  $\bar{P}_e(s, u)$  depending on the problem. Suppose there exists an optimal solution  $\nu^*$  to the LP problems (20). Then there exists a stationary policy  $\mu^*$  that is optimal for the respective CMDP problem. The optimal policy  $\mu^*$  for CMDP is randomized and is uniquely characterized with probability  $\theta_{\mu^*(s)}(u)$  of applying policy  $u \in \mathcal{U}_s$  in state  $s \in \mathcal{S}$ , where

$$\theta_{\mu^*(s)}(u) = \frac{\nu^*(s, u)}{\sum_{u' \in \mathcal{U}_s} \nu^*(s, u')} \text{ if } \sum_{u' \in \mathcal{U}_s} \nu^*(s, u') > 0. \quad (21)$$

If  $\sum_{u' \in \mathcal{U}_s} \nu^*(s, u') = 0$  for some  $s \in \mathcal{S}$ , an action that drives the system to  $\mathcal{S}_{\nu^*} = \{s \in \mathcal{S} : \sum_{u' \in \mathcal{U}_s} \nu^*(s, u') > 0\}$  is chosen in each state [12]. In general, LP can handle problems with large number of variables and above linear programs can easily be solved using interior-point methods [14].

#### D. Evaluation of Immediate Costs for the Above Problems

When the scheduler takes an action in a particular state, it incurs a one step immediate cost  $\mathcal{G} : \mathcal{K} \mapsto \mathbb{R}$ , where  $\mathcal{K} = \{(s, u) : s \in \mathcal{S}, u \in \mathcal{U}_s\}$  to be the set of state-action pairs. The costs for the above two problems are given below.

1) *Buffer Related Costs:* We consider two different cost associated with the buffer, namely, buffer delay cost and buffer packet dropping probability cost. The average packet delay in the buffer is related to the average buffer occupancy via Little’s theorem as  $\bar{D} = \frac{1}{\lambda} \mathbb{E}[b^n]$ . Therefore, we can write immediate buffer delay cost as follows,

$$d^1(b^n) = \frac{b^n}{\lambda}. \quad (22)$$

Note that buffer occupancy is updated in the next time-slot according to the action taken in the present time-slot, and

$d^1(s^n, \mu(s^n)) = d^1(b^n), \forall c^n$  and  $u^n$ . The packet dropping probability cost does not depend on channel state directly, and therefore for all channel state  $c^n$ , it can be expressed as,

$$d^2(b^n, u^n) = \sum_{k=B-A+1}^B I_{\{b^n - u^n = k\}} \sum_{j=B-k+1}^A p^a(a^n = a_j), \quad (23)$$

where  $I_{\{\cdot\}}$  is the indicator function, and returns 1 when the equality holds and 0 otherwise.

2) *Power and BER Costs:* M-QAM scheme is a useful modulation technique for achieving high data rate transmission without increasing the bandwidth of wireless communications systems. An approximate expression of BER for M-QAM, valid for both low and high SNR, can be expressed as [15],

$$P_{e, \text{MQAM}} = \frac{2}{v} \left(1 - \frac{1}{\sqrt{M}}\right) \sum_{i=1}^{\frac{\sqrt{M}}{2}} \text{erfc} \left( (2i-1) \sqrt{\frac{3v\gamma P_T}{2(M-1)\bar{P}}} \right), \quad (24)$$

where  $v = \log_2(M)$  is the number of bits that modulates a  $2^v$ -QAM symbol. For BPSK transmission, the BER is

$$P_{e, \text{BPSK}} = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{\gamma P_T}{P}} \right). \quad (25)$$

*Power Cost:* For a certain channel state  $c_k$  and action  $u_i$ , and with a fixed specified BER for all channel state, the power cost  $P_T$  can be found numerically either from (24) and (25) using average channel SNR or from the following equation,

$$\bar{P}_e(c_k, u_i) = \frac{1}{p_{dc_k}} \int_{\gamma_{dc_{k-1}}}^{\gamma_{dc_k}} P_{e, \text{BPSK/M-QAM}}(c_k, u_i) f_{\Gamma}^{dc}(\gamma_{dc}) d\gamma_{dc}. \quad (26)$$

*BER Cost:* Similarly with a fixed transmitter power  $P_T$ , for a certain channel state  $c_k$  and action  $u_i$ , the BER cost  $\bar{P}_e$  can be found either from (24) and (25) using average channel SNR value or from (26) numerically. Note in (26)  $dc = sc$  or  $mrc$ .

## V. NUMERICAL RESULTS

In this section, we present simulation results for both problems employing SC and MRC diversity techniques at the receiver. All the results are given for average transmit signal power  $\bar{P} = 1$ , average received SNR of each branch  $\bar{\gamma} = 1$ , packet size to number of channel uses ratio  $G/F = 1$ , block duration  $T_B = 10^{-4}$ , number of channel state  $C = 10$ , buffer size  $B = 100$ , number of actions  $U = 6$ , and maximum allowable packet dropping probability  $P_{\text{dropping}} = 10^{-3}$ . The set of packet transmission rate is  $\mathcal{W} = \{0, 1, 2, 4, 6, 8\}$ . The first action corresponds to no transmission, second action corresponds to BPSK transmission and  $i^{\text{th}}$ ,  $i = 3, 4, \dots, U$  action corresponds to square constellation  $2^{2i-4}$ -QAM transmission. We also assume that the input traffic is Poisson distributed with average value  $\lambda = 2$  and maximum number of packet arrival in a slot  $A = 10$ . For first problem, the power cost is determined using average BER,  $\bar{P}_e = 10^{-4}$ .

In Fig. 2, we present the variation of average power with average delay for SC diversity with Nakagami- $m$  parameter  $m = 2$ . It is seen that the average power decreases as the

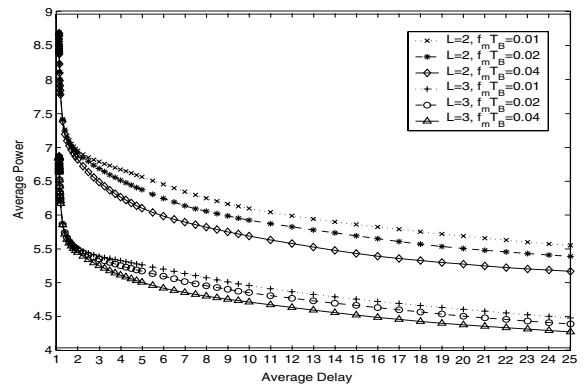


Fig. 2. Influences of  $L$  and fading rates on power vs. delay for SC

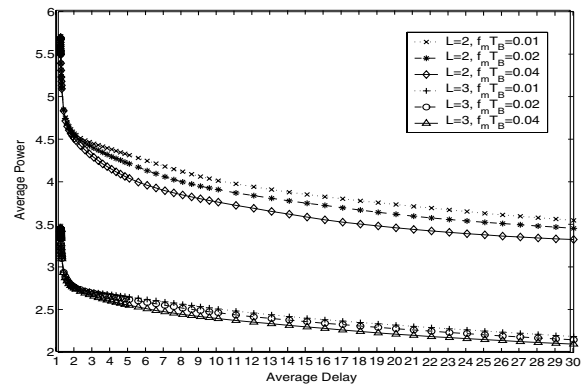


Fig. 3. Influences of  $L$  and fading rates on power vs. delay for MRC

delay increases. Also, instead of transmitting in the immediate next time-slot (time-slot 1), by delaying packet only by 1 more time-slot, the scheduler can save more than 50% of transmitter power. The rate of decrease of power with delay increases as fading rate increases. When the number of selection diversity branch increases, the receiver has more flexibility to choose a branch with best received SNR. Therefore, as expected, the average transmitter power decreases for the same delay as number of branch  $L$  increases from 2 to 3.

Fig. 3 shows the average power vs. average delay tradeoff for MRC diversity with  $m = 2$ . Like SC case, we show the influences of fading rate and number of diversity branches in this case as well. These parameters also have similar effect on the performance curve. However, power savings for MRC as compared to SC increases as number of branches increases. Since MRC take optimally all diversity branch into account, it is seen that the average power is less for MRC than SC for same average delay. Also, the rate of decrease of average power with delay for MRC is less than SC.

The effect of the Nakagami- $m$  fading severity parameter  $m$  on the power/delay curve is shown in Fig. 4. The curve is plotted for 2-branch SC diversity combiner and different fading rates. Note that the behavior of the curve in terms of power savings improves as the value of  $m$  increases. This is expected because as the value of  $m$  increases, the channel condition improves toward non-fading channel. Therefore, the

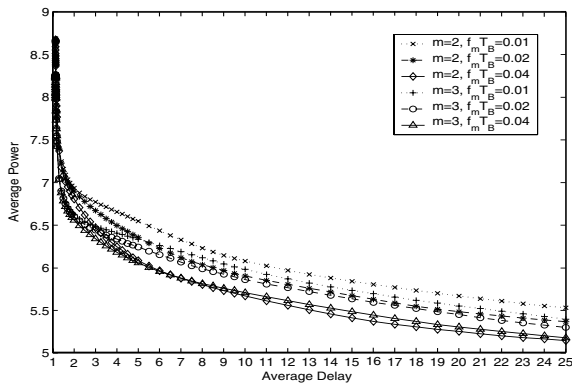


Fig. 4. Power vs. delay for SC with different Nakagami fading parameter

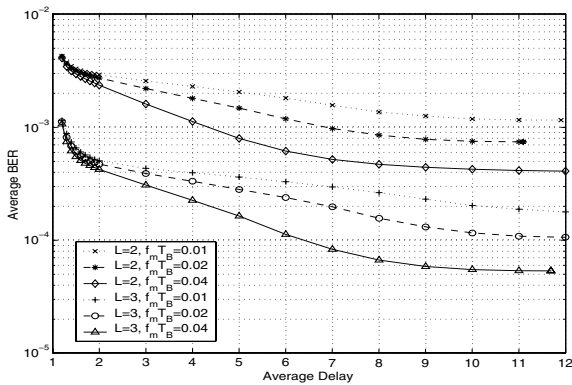


Fig. 5. Effect of  $L$  and fading rates on BER/Delay tradeoff curve for SC

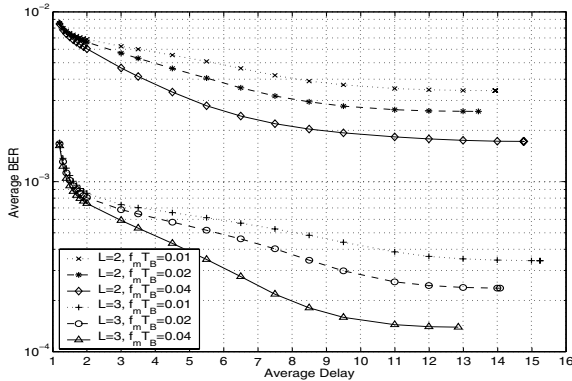


Fig. 6. Effect of  $L$  and fading rates on BER/Delay tradeoff curve for MRC

received SNR has less fluctuations. However, the gain of increasing number of diversity branch is much more than the gain of increasing parameter  $m$  of the channel.

Fig. 5 depicts the average BER performance as a function of average delay for SC diversity receiver with parameter  $m = 1$ , corresponds to Rayleigh fading. The transmitter power is kept at  $P_T = 8$  mW and curves are given for two different number of branches, namely,  $L = 2$  and 3. The influence of fading rate is shown and it is seen that as fading rate increases, the average BER decreases for the same delay. Also for larger number of branches, the rate of decreasing BER increases as

fading rate increases.

The BER/delay tradeoff curve for MRC diversity scheme is shown in Fig. 6. Like SC case, this curve is also plotted for  $m = 1$  and shows similar effect of parameters  $L$  and  $f_m$ . However, transmitter power in this case is kept at  $P_T = 4$  mW. It can be noted that the adaptive MRC diversity MQAM scheme shows almost similar BER/delay performance curve as adaptive SC diversity MQAM scheme, but with 50% transmitter power. However, it is obvious that this gains of MRC comes with increased receiver complexity.

## VI. CONCLUSIONS

We have presented two constrained Markov decision process based transmission adaptation techniques over diversity Nakagami- $m$  fading channel with memory. In each problem, performance results are shown for selection-combining and maximal ratio combining techniques with different fading rates, Nakagami- $m$  parameter, and number of branches. The results show that the scheduler can save power or reduce BER by adapting transmission rate with buffer occupancy in addition to adapting only with channel conditions.

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