Optimal Power Control for Network-Centric and User-Centric Wireless Networks in Interference-Limited Fading Channels

Piplu R. Paul, Ashok K. Karmokar, and Vijay K. Bhargava

Department of Electrical and Computer Engineering, 2356 Main Mall,
University of British Columbia, Vancouver, BC, V6T 1Z4, Canada.
E-mail: {piplup, ashokk, vijayb}@ece.ubc.ca

***Abstract*— We present a new and simple unconstrained optimization based power control algorithm for interference-limited Rayleigh fading wireless networks. We consider optimizing two objective functions, namely, minimizing outage and maximizing utility under bounds on transmission powers. With some transformation techniques, we show that the formulated constrained optimization problem can be converted into an equivalent unconstrained problem. In each case, both the user-centric and network-centric formulation and their iterative solution techniques are given. Our proposed algorithms are very efficient and converge to optimal solution very quickly.**

I. Introduction

As the demand for wireless services increases, optimal allocation of radio resources has become a great challenge. Allocation of power is critical for both longer battery life of the mobile devices and increased utilization of the limited wireless spectrum. Power control provides an effective way of determining the transmitter power to achieve quality of services (QoS) specifications, such as frame error rate (FER), outage probability, etc. Because of these, it has received extensive studies in the literature. Radio resource management (RRM) can be broadly classified as user- centric, where RRM schemes attempt to maximize individual's interests and network-centric, where it optimizes network interests [1]. Distributed power control [2] can be thought of as examples of user-centric RRM, whereas centralized power control [3] falls into the network-centric category. Traditionally, power control schemes update power whenever the channel meanders from one fading state to another. This approach consumes a lot of signal processing energy. Alternative approach is to take the statistical variation of the signal-to-interference and noise ratio (SINR) of each transmitter/receiver (Tx/Rx) pair into account and allocate power. This approach allows power updates to be carried out at a time scale far larger than Rayleigh fading time scale, which is often the log-normal shadowing time-scale [4]. We address the problem of optimal power control for interference-limited wireless networks adopting latter approach. In this paper, we provide formulation and solution technique for optimal power control for both the user-centric and network-centric schemes with the goal of optimizing either outage or utility under constraints on transmission powers of all Tx/Rx pairs.

The remaining part of the paper is organized as follows. In Section II, we describe system and channel model for our work. The formulation of both the outage-based and utility-based network-centric and user-centric problems and their solution techniques are given in Section III. We give simulation results in Section IV and conclude in Section V.

II. System and Channel Modeling

We consider a cellular wireless system that consists of fixed N number of channels (e.g., frequency channels, codes in a code division multiple access (CDMA) systems, or antenna beams in an antenna array). One channel is assigned to a particular transmitter/reciever pair at a particular time. Suppose P_i denote the transmit power level of transmitter i. The receiver associated with transmitter i is denoted as receiver i. Let G_{ij} is a positive number and denotes the path gain from the transmitter j to the receiver i. It can represent distance dependent power attenuation, log-normal shadowing, cross-correlations between codes in a CDMA systems, as well as gain dependency on antenna direction. We assume Rayleigh/Rayleigh fading environment in which both desired signal and interference signals are subject to Rayleigh fading. This assumption of Rayleigh/Rayleigh fading environment is very realistic in urban wireless networking environments since the receiver gets no direct line-of-sight signal components, either from its own transmitter or from the interfering transmitters. The received signal amplitude in a Rayleigh fading environment has a Rayleigh distribution. The channel fading power gain, which is proportional to the square of the signal amplitude, is exponentially distributed and can be written as,

$$p(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}} \text{ for } \gamma \geq 0, \tag{1}$$

where $\bar{\gamma} = \mathbb{E}\{\gamma\}$ is the average fading power gain of the channel and $\mathbb{E}\{.\}$ is the expectation operator. Let γ_{ij} denote the fading gain between transmitter j and receiver i. Whereas, gain γ_{ij} is randomly time-varying, gain G_{ij} does not change much with time. We assume that G_{ij} are constant and hence the analysis holds for a time scale over which the factors that determine G_{ij} are approximately constant. Therefore, the power received at receiver i from transmitter j is an exponentially distributed random variable $\gamma_{ij} G_{ij} P_j$ with expected value $\mathbb{E}[\gamma_{ij} G_{ij} P_j] = G_{ij} P_j$, where the mean value of fading gain $\bar{\gamma}$ is unity. The instantaneous SINR of the receiver i is given by,

$$\text{SINR}_i = \frac{\gamma_{ii} G_{ii} P_i}{\sum_{k \neq i} \gamma_{ik} G_{ik} P_k + \sigma^2}, \tag{2}$$

where, the term in the numerator denotes the desired received power of the receiver i from transmitter i and the denominator is due to the total interference due to received power from all other transmitters and the noise power due to the white channel noise and thermal noise at the receiver.

III. Formulation and Solution Techniques

A. Outage Based Optimal Power Control

In wireless networks, the QoS requirement of a particular Tx/Rx pair can be specified by certain minimum acceptable SINR. It can be assumed that the QoS requested is provided when the SINR exceeds a given threshold SINR^{th}. The outage probability of i^{th} transmitter/receiver (Tx/Rx$_i$) pair is given by,

$$\begin{aligned} O_i &= \text{Prob}\left(\mathrm{SINR}_i \leq \mathrm{SINR}^{th}\right) \quad (3) \\ &= \text{Prob}\left(\gamma_{ii}G_{ii}P_i \leq \mathrm{SINR}^{th}(\sum_{k\neq i}\gamma_{ik}G_{ik}P_k + \sigma^2)\right) \end{aligned}$$

Using the identity derived in [4], for N independent exponentially distributed random variables $z_i,\ i = 1, 2, \cdots, N$ with mean $\mathbb{E}(z_i) = 1/\lambda_i$, $\text{Prob}(z_1 > \sum_{i=2}^{N} z_i + c) = e^{-\lambda_1 c}\prod_{i=2}^{N}\left(\frac{1}{1+\frac{\lambda_1}{\lambda_i}}\right)$, the outage probability of Tx/Rx$_i$ can be written as,

$$O_i = 1 - e^{-\frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}P_i}}\prod_{k\neq i}\frac{1}{(1+\frac{\mathrm{SINR}^{th}G_{ik}P_k}{G_{ii}P_i})}. \quad (4)$$

In this section, our objective is to minimize outage probability subject to transmission power constraints. Putting $s_i = 1/(1-O_i)$, the above equation can be given in the following form,

$$s_i = e^{\frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}P_i}}\prod_{k\neq i}(1+\frac{\mathrm{SINR}^{th}G_{ik}P_k}{G_{ii}P_i}). \quad (5)$$

Note that the outage probability in (4) can be minimized by minimizing s_i in (5). Since logarithmic function is monotonically increasing, therefore, taking $\log$ on both side of (5), the objective function can be written as,

$$t_i = \frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}P_i} + \sum_{k\neq i}\log(1+\frac{\mathrm{SINR}^{th}G_{ik}P_k}{G_{ii}P_i}). \quad (6)$$

where, $t_i = \log(s_i)$. In this paper, we consider the optimization problem from two viewpoints, namely, user and system.

1) User-Centric: In user-centric schemes, the objective is to minimize the outage probability of individual Tx/Rx pair. Therefore, the problem can be formulated as,

$$\begin{aligned} \text{minimize} \quad & O_i = 1 - e^{-\frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}P_i}}\prod_{k\neq i}\frac{1}{(1+\frac{\mathrm{SINR}^{th}G_{ik}P_k}{G_{ii}P_i})} \\ \text{subject to} \quad & P_i \geq 0,\ i = 1, 2, \cdots, N \end{aligned}$$

With the transformation technique discuss above, the optimization problem can be written in its equivalent form as,

$$\begin{aligned} \text{minimize} \quad & t_i = \frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}P_i} + \sum_{k\neq i}\log(1+\frac{\mathrm{SINR}^{th}G_{ik}P_k}{G_{ii}P_i}) \\ \text{subject to} \quad & P_i \geq 0,\ i = 1, 2, \cdots, N \end{aligned}$$

Above constrained optimization problem can be converted into an unconstrained problem if we let $P_i = x_i^2$, where x_i is any real number. Therefore,

$$\text{minimize } t_i = \frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}x_i^2} + \sum_{k\neq i}\log(1+\frac{\mathrm{SINR}^{th}G_{ik}x_k^2}{G_{ii}x_i^2}) \quad (7)$$

The unconstrained optimization problem (7) can easily and very efficiently solved using rank-two *Davidon-Fletcher-Powell* (DFP) method or *Broyden-Fletcher-Goldfarb-Shanno* (BFGS) method. The foundation of DFP and BFGS methods are classical *Newton method* and are referred to as *quasi-Newton method* (sometimes also called *variable metric method*). Unlike Newton method, quasi-Newton methods do not require explicit expression for the second order derivatives. Therefore, evaluation of Hessian Matrix and its inversion, which is computationally very expensive, are not required. Another advantage of DFP and BFGS methods are they do not need to check the positive definiteness in each iteration. However, they are almost as efficient as Newton method and quite tolerant to line search imprecision. The algorithm for finding optimal variable $\mathbf{x} = [x_1, x_2, \cdots, x_N]$ employing DFP method is described below:

Algorithm 1:

Step 1: Initialize optimization variable $\vec{x}^{(0)}$ and input tolerance ϵ. Set iteration index $k = 0$ and matrix $\mathbf{S}^{(0)} = \mathbf{I}_N$, where $\mathbf{I}_N$ is a $N \times N$ identity matrix. Compute gradient vector $\vec{g}^{(0)}$ at initial point.

Step 2: Compute DFP direction vector using $\vec{d}^{(k)} = -\mathbf{S}^{(k)}\vec{g}^{(k)}$. Find $\alpha^{(k)}$, the value of α (a small constant) that minimizes $t_i(\vec{x}^{(k)} + \alpha\vec{d}^{(k)})$, using a line search method (e.g., Fletcher line search). Set optimal direction vector $\vec{\delta}^{(k)} = \alpha^{(k)}\vec{d}^{(k)}$ and find updated value of variable $\vec{x}^{(k+1)} = \vec{x}^{(k)} + \vec{\delta}^{(k)}$.

Step 3: If $|\vec{\delta}^{(k)}| < \epsilon$, then output $\vec{x}^{*} = \vec{x}^{(k+1)}$, $t_i^{*} = t_i(\vec{x}^{(k+1)})$ and $O_i^{*} = 1 - e^{-t_i^{*}}$, and stop.

Step 4: Compute gradient vector at updated point $\vec{g}^{(k+1)}$ and set $\vec{\beta}^{(k)} = \vec{g}^{(k+1)} - \vec{g}^{(k)}$. Compute $\vec{S}^{(k+1)}$ using following updating formula: $\vec{S}^{(k+1)} = \vec{S}^{(k)} + \frac{\vec{\delta}^{(k)}(\vec{\delta}^{(k)})^T}{(\vec{\delta}^{(k)})^T\vec{\beta}^{(k)}} - \frac{\vec{S}^{(k)}\vec{\beta}^{(k)}(\vec{\beta}^{(k)})^T\vec{S}^{(k)}}{(\vec{\beta}^{(k)})^T\vec{S}^{(k)}\vec{\beta}^{(k)}}$. Set $k = k+1$ and repeat from Step 2.

2) Network-Centric: In network-centric schemes, the system outage probability, defined as the worst outage probability over all Tx/Rx pairs, is minimized with constraint on the transmission powers. Therefore, the problem of minimizing outage probability of the system can be expressed as,

$$\begin{aligned} \text{minimize} \quad & O = \max_i O_i \\ \text{subject to} \quad & O_i = 1 - e^{-\frac{\sigma^2\mathrm{SINR}^{th}}{G_{ii}P_i}}\prod_{k\neq i}\frac{1}{(1+\frac{\mathrm{SINR}^{th}G_{ik}P_k}{G_{ii}P_i})} \\ & P_i \geq 0,\ i = 1, 2, \cdots, N \end{aligned}$$

With $t_i = \log(\frac{1}{1-O_i})$, $P_i = x_i^2;\ i = 1, 2, \cdots, N$ and applying similar transformation techniques as in previous sections, the optimization problem can be written as,

$$\begin{aligned}\text{minimize } t &= \max_i \ t_i, \ i = 1, 2, \cdots, N \\ &= \max_i \frac{\sigma^2 \text{SINR}^{th}}{G_{ii} x_i^2} + \sum_{k \neq i} \log(1 + \frac{\text{SINR}^{th} G_{ik} x_k^2}{G_{ii} x_i^2})\end{aligned} \quad (8)$$

The infinity-norm of a vector $\vec{t} = [t_1, t_2, \cdots, t_N]^T$ is defined as $\|\vec{t}\|_\infty = \max_i(|t_i|, \text{ for } 1 \leq i \leq N)$. Therefore, the optimization problem can be think of minimization of infinity-norm of vector $\vec{t}$. If p is even and the vector components are real numbers, then p-norm of vector $\vec{t}$ is defined as $\|\vec{t}\|_p = \left(\sum_{i=1}^N t_i^p\right)^{\frac{1}{p}}$. The p-norm and infinity-norm are related by $\lim_{p \mapsto \infty} \|\vec{t}\|_p = \|\vec{t}\|_\infty$. Therefore, the optimization problem (8) can be expressed in terms of p-norm as follows:

$$\text{minimize } t = \lim_{p \to \infty} \left(\sum_{i=1}^N t_i^p \right)^{\frac{1}{p}} \quad (9)$$

For even p, the p-norm of a vector is a differentiable function of its components, but the infinite norm is not. So, when the infinity norm is used in a problem, one can replace it by a p-norm. The difference between the approximate and exact solutions becomes negligible if the power p is sufficiently large. The algorithm for solving problem (9) is given below:

Algorithm 2:

Step 1: Initialize optimization variable $\vec{x}^{(0)}$ and input tolerance ϵ. Set iteration $k = 0$ and power $p = 2$.

Step 2: Find $\vec{x}^{(k+1)}$ using *Algorithm 1* with objective function replaced by (9).

Step 3: If $\|\vec{x}^{(k+1)} - \vec{x}^{(k)}\| < \epsilon$, then output $\vec{x}^* = \vec{x}^{(k+1)}$, $t^* = \left(\sum_{i=1}^N t_i^p\right)^{\frac{1}{p}}$ and $O^* = 1 - e^{-t^*}$, and stop.

Step 4: Increase k by 1, double value of p and repeat from Step 2.

B. Utility Based Optimal Power Control

The QoS objective for a voice terminal is to achieve a minimum acceptable SINR and therefore outage based optimal power control is appropriate for wireless voice systems. However, this approach is not suitable for the efficient operation of wireless data systems. This is because the QoS objective for data signals differs from the QoS objective for telephones. In telephone systems, low delay is essential, and transmission errors are tolerable upto a point. By contrast, data signals can accept some delay but have very low tolerance to errors. The QoS requirements of data communications are given with utility function, which is defined as the number of information bits received successfully per Joule of energy expanded. Let us assume that data bits are packed into frames of F bits containing L information bits and $F - L$ parity bits for error detection. The transmission rate for all Tx/Rx pair is fixed to R bits/second. The utility function of Tx/Rx$_i$ can be given by,

$$u_i = \frac{LRf(\text{SINR}_i)}{FP_i} \quad (10)$$

where $f(\text{SINR}_i)$ is efficiency function that closely approximates the frame success rate (FSR). The efficiency function can be expressed as $f(\text{SINR}_i) = (1 - O_i)$, where O_i is the frame outage probability and given by (4) [6]. Therefore, we can express the utility function for the Tx/Rx$_i$ as,

$$u_i = \frac{LR}{FP_i} e^{-\frac{\sigma^2 \text{SINR}^{th}}{G_{ii} P_i}} \prod_{k \neq i} \frac{1}{(1 + \frac{\text{SINR}^{th} G_{ik} P_k}{G_{ii} P_i})} \quad (11)$$

Letting $t_i = \log(1/u_i)$, equation (11) can be rewritten as,

$$t_i = \log(\frac{FP_i}{LR}) + \frac{\sigma^2 \text{SINR}^{th}}{G_{ii} P_i} + \sum_{k \neq i} \log(1 + \frac{\text{SINR}^{th} G_{ik} P_k}{G_{ii} P_i}) \quad (12)$$

It is clear that maximizing utility function u_i is equivalent to minimizing function t_i. The SINRth depends on the modulation used and specified minimum FSR. The FSR of Tx/Rx$_i$ is the probability of all bits in a frame being received correctly and can be expressed as,

$$\text{FSR}_i = (1 - P_{e_i})^F, \quad (13)$$

where P_{e_i} is the bit error rate (BER) of modulation used for transmission. For BPSK transmission, the BER can be given as,

$$P_{e_i} = \frac{1}{2} \text{erfc}(\sqrt{\text{SINR}_i}) \quad (14)$$

Combining (13) and (14), the SINR$_i$ can be expressed as,

$$\text{SINR}_i = (\text{erfc}^{-1}(2P_{e_i}))^2 = (\text{erfc}^{-1}(2(1 - (\text{FSR}_i)^{1/F})))^2 \quad (15)$$

where $\text{erfc}^{-1}(.)$ is the inverse complementary error function. On the other hand, for M-QAM transmission the BER can approximately given by [5],

$$P_{e_i} = \frac{1}{5} \exp\left(-\frac{1.6 \text{SINR}_i}{M - 1}\right). \quad (16)$$

Hence, the SINR for Tx/Rx$_i$ can be expressed as,

$$\text{SINR}_i = \frac{1 - M}{1.6} \log(5P_{e_i}) = \frac{1 - M}{1.6} \log(5(1 - (\text{FSR}_i)^{1/F})). \quad (17)$$

Therefore, for specified FSR, the SINRth can be obtained from (15) or (17).

1) User-Centric: The objective in this case is to maximize utility of individual Tx/Rx$_i$ pair with constraints on all transmission power as,

$$\begin{aligned}\text{maximize} \quad & u_i = \frac{LR}{FP_i} e^{-\frac{\sigma^2 \text{SINR}^{th}}{G_{ii} P_i}} \prod_{k \neq i} \frac{1}{(1 + \frac{\text{SINR}^{th} G_{ik} P_k}{G_{ii} P_i})} \\ \text{subject to} \quad & P_i \geq 0, \ i = 1, 2, \cdots, N\end{aligned}$$

Eliminating the nonnegativity bound by replacing $P_i = x_i^2$ and applying $t_i = \log(1/u_i)$, we get the following unconstrained minimization problem,

$$\text{minimize } t_i = \log(\frac{Fx_i^2}{LR}) + \frac{\sigma^2 \text{SINR}^{th}}{G_{ii} x_i^2} + \sum_{k \neq i} \log(1 + \frac{\text{SINR}^{th} G_{ik} x_k^2}{G_{ii} x_i^2}) \quad (18)$$

Above optimization problem can be solved using *Algorithm 1* described in Section III-A.1 with objective function (18).

2) Network-Centric: Our goal in this problem is to maximize the minimum utility over all Tx/Rx pairs with transmission power constraint as,

$$\begin{aligned} \text{maximize} \quad & U = \min_i u_i \\ \text{subject to} \quad & u_i = \frac{LR}{FP_i} e^{-\frac{\sigma^2 \text{SINR}^{th}}{G_{ii}P_i}} \prod_{k \neq i} \frac{1}{(1 + \frac{\text{SINR}^{th} G_{ik} P_k}{G_{ii} P_i})} \\ & P_i \geq 0, \ i = 1, 2, \cdots, N \end{aligned}$$

After similar transformation and formulation as discussed in Section III-A.2, the equivalent problem is given by,

$$\text{minimize } t = \lim_{p \to \infty} \left(\sum_{i=1}^{N} t_i^p \right)^{\frac{1}{p}} \tag{19}$$

where, t_i is given by (12). With objective function (19), we can solve the above problem using *Algorithm 2*.

IV. Numerical Results

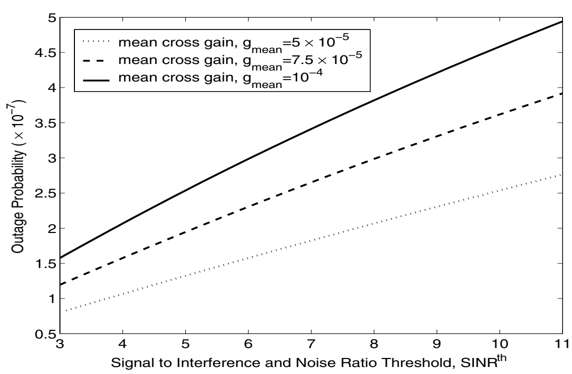

Fig. 1. Effect of different cross-gain on individual outage probability

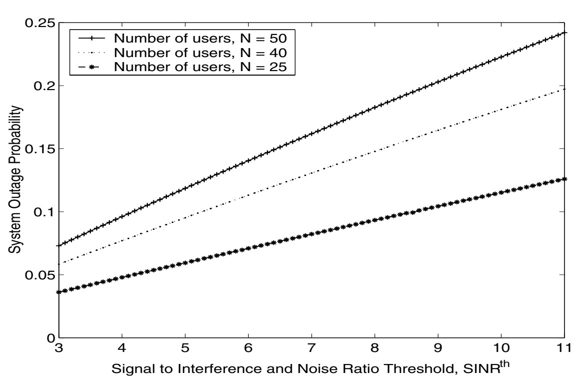

Fig. 2. Effect of number Tx/Rx pairs on system outage probability

We present some simulation results for gains $G_{ii} = 1, \ i = 1, 2, \cdots, N$. The cross gains are uniformly distributed random variable with mean value g_{mean}. In Fig. 1 and 2, we show the results for outage based optimal power control with noise power $\sigma^2 = 0$. We varied SINR^{th} from 3 to 11. It is seen from Fig. 1 that as SINR^{th} increases the outage probability increases. The outage probability is also increased as cross-gain increases. Fig. 2 shows that the system outage increases with the number of Tx/Rx pairs in the system. We show the effect of different transmission and FSR in Fig. 3 with $\sigma^2 = 5 \times 10^{-15}$, transmission rate $R = 10^4$, information bits/packet $L = 64$ and packet size $F = 80$ bits. It is seen that higher order modulation has lower utility and utility decreases as FSR increases. Unless specified otherwise, for all the simulation, the number of user $N = 50$ and $g_{mean} = 5 \times 10^{-4}$.

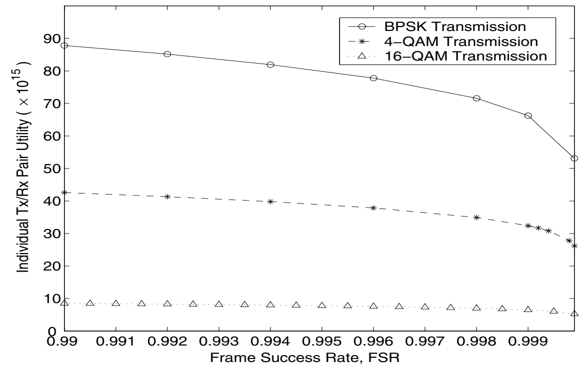

Fig. 3. Effect of different modulation on individual utility function

V. Conclusions

We have addressed the optimal power control for wireless networks in which both the desired and interference signal have Rayleigh distribution. The problems have been formulated both from user and system point of views. We have considered two objective functions that appropriately meet the QoS requirements of both the voice and data networks respectively. Simulation results are given to show some performances. Individual Tx/Rx outage has been seen to increase with both cross-gains and SINR threshold. System outage also increases with number of Tx/Rx pairs. Utility depends on modulation and decreases with FSR.

Acknowledgment

This work is supported by the Natural Sciences and Engineering Research Council (NSERC) of Canada under a strategic project grant.

References

[1] N. Feng, S.-C. Mau, and N. B. Mandayam, “Pricing and power control for joint network-cenric and user-centric radio resource management,” *IEEE Trans. Commun.*, vol. 52, pp. 1547-1557, Sep. 2004.
[2] J. Zander, “Performance of optimum transmitter power control in cellular radio systems,” *IEEE Trans. Veh. Technol.*, vol. 41, pp. 57-62, Feb. 1992.
[3] S. A. Grandhi, R. Vijayan, D. J. Goodman, J. Zander “Centralized power control in cellular radio systems,” *IEEE Trans. Veh. Technol.*, vol. 42, pp. 466-468, Nov. 1993.
[4] S. Kandukuri, and S. Boyd, “Optimal power control in interference-limited fading wireless channels with outage-probability specifications,” *IEEE Trans. Wireles Commun.*, vol. 1, pp. 46-55, Jan. 2002.
[5] S. T. Chung, and A. J. Goldsmith, “Degrees of freedom in adaptive modulation: a unified view,” *IEEE Trans. Commun.*, vol. 49, No. 9, pp. 1561-1571, Sep. 2001.
[6] S. Dey, and J. Evans “Optimal power control in wireless data networks with outage-based utility guarantees,” in *Proc. of IEEE Conf. Decision and Control*, Maui, USA, vol. 1, pp. 570-575, Dec. 2003.