Time–Domain Transmit Beamforming for
MIMO–OFDM Systems with Finite Rate Feedback

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Abstract

Transmit beamforming (BF) and receive combining are simple and popular methods for performance enhancement in multiple–input multiple–output orthogonal frequency division multiplexing (MIMO–OFDM) systems. In this paper, we propose a novel time–domain BF (TD–BF) scheme for MIMO–OFDM systems which uses cyclic BF filters (C–BFFs). Assuming perfect channel state information (CSI) at the transmitter, the C–BFFs are optimized for maximization of the average mutual information (AMI) and for minimization of the average bit error rate (BER), respectively. If the C–BFF length \(L_g\) is equal to the number of sub–carriers \(N_c\), closed–form solutions to both optimization problems exist. For the practically relevant case of \(L_g < N_c\) we present numerical methods for calculation of the optimum C–BFFs. Using a global vector quantization (GVQ) approach we design C–BFF codebooks for practical finite–rate feedback channels. Simulation and numerical results for typical IEEE 802.11n channels confirm the excellent performance of the proposed scheme and show that TD–BF has a more favorable performance/feedback rate trade–off than previously proposed frequency–domain BF (FD–BF) schemes.

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1 Introduction

Transmit beamforming (BF) and receiver combining are simple yet efficient techniques for exploiting the benefits of multiple-input multiple-output (MIMO) systems to mitigate the effects of fading in wireless communications [1]. BF generally requires channel state information (CSI) at the transmitter. In practical systems, ideal BF is not possible since the amount of information that can be fed back from the receiver to the transmitter is limited. Therefore, BF for quantized CSI and finite-rate feedback channels has recently received considerable attention [2]–[6].

To avoid complex equalization at the receiver, MIMO is often combined with orthogonal frequency division multiplexing (OFDM) which converts broadband frequency–selective channels into a number of parallel narrowband frequency–flat channels [7]. MIMO–OFDM has been adopted for example in various recent standards such as IEEE 802.11 (WLAN) and IEEE 802.16 (WiMAX). Transmit BF techniques proposed for narrowband channels can be easily extended to broadband MIMO–OFDM systems by applying independent BF to each sub–carrier [8, 9]. However, the obvious drawback of this approach is that the amount of CSI data that has to be fed back from the receiver to the transmitter will be prohibitively large for practical OFDM systems with moderate–to–large number of sub–carriers $N_c$ (e.g. $N_c \geq 64$).

Since the fading gains as well as the corresponding BF vectors are correlated across OFDM sub–carriers, in [10] it was proposed to reduce the amount of feedback by only feeding back the BF vectors for a small number of sub–carriers. The remaining BF vectors are obtained by modified spherical interpolation. This approach significantly reduces the required amount of feedback at the expense of some loss in performance. The required number of feedback bits of this frequency–domain BF (FD–BF) scheme can be further reduced by post–processing of the feedback bits [11] and/or by adopting improved interpolator designs such as Grassmannian interpolators [12] or geodesic interpolators [13]. However, fundamentally for all of these FD–BF schemes the required amount of feedback to achieve a certain performance is proportional to the number of OFDM sub–carriers. This may be problematic in OFDM systems with a large number of sub–carriers and stringent limits on the affordable amount of feedback.

In this paper, we propose a novel time–domain (TD) approach to BF in MIMO–OFDM systems. The motivation for considering a TD approach is that the fading correlations in the FD, which are exploited for interpolation in [10, 12, 13], have their origin in the TD. Namely, these correlations are due to the fact that the number of sub–carriers is typically much larger than the number of non–
zero channel impulse response (CIR) coefficients. Therefore, tackling the problem directly in the TD is a natural choice. The proposed TD–BF scheme employs cyclic BF filters (C–BFFs) of length \( L_g \leq N_c \). The C–BFFs are optimized for maximization of the average mutual information (AMI) and for minimization of the average uncoded bit error rate (BER), respectively. While other C–BFF optimization criteria are certainly possible (e.g. maximum cut–off rate, minimum coded BER), the adopted criteria can be considered as extreme cases in the sense that they cater to systems using very powerful (ideally capacity–achieving) forward error correction (FEC) coding (AMI criterion) and systems with weak or no FEC coding (uncoded BER criterion), respectively. For perfect CSI both criteria lead to (different) nonlinear eigenvalue problems for the C–BFF coefficient vectors, and we show that closed–form solutions to both problems exist for \( L_g = N_c \). However, for the practically more interesting case of \( L_g < N_c \), a closed–form solution does not exist for either problem, and we provide efficient numerical methods for calculation of the C–BFFs. Furthermore, for the case of a finite–rate feedback channel we draw from the findings in [14, 15] and propose a global vector quantization (GVQ) algorithm for maximum AMI and minimum BER codebook design, respectively.

We note that TD pre–processing for MIMO–OFDM has been considered in different contexts before. For example, TD–BF schemes with one scalar BF weight per antenna (as opposed to C–BFFs) have been proposed to reduce the number of inverse discrete Fourier transforms (IDFTs) required at the transmitter of MIMO–OFDM systems, cf. e.g. [16] and references therein. Similarly, cyclic delay diversity, which is a simple form of space–time coding, cf. e.g. [17, 18], may be viewed as a TD MIMO–OFDM pre–processing technique. However, the concept of employing C–BFFs for (limited feedback) BF is novel and has not been considered before.

**Organization:** In Section 2, the considered system model is presented. The optimization of the C–BFFs for maximization of the AMI and minimization of the average BER is discussed in Sections 3 and 4, respectively. In Section 5, a GVQ algorithm for finite–rate feedback TD–BF and a detailed comparison between TD–BF and FD–BF are presented. Simulation results are provided in Section 6, and some conclusions are drawn in Section 7.

**Notation:** In this paper, \((\cdot)^T, (\cdot)^H, (\cdot)^*, 0_X, I_X, \text{ and } \mathcal{E}\{\cdot\}\) denote transpose, Hermitian transpose, complex conjugate, the all–zero column vector of length \( X \), the \( X \times X \) identity matrix, and statistical expectation, respectively. In addition, \( \det(\cdot) \) and \( \text{diag}\{x_1, x_2, \ldots, x_N\}\) denote the determinant of a matrix and a diagonal matrix with \( x_1, x_2, \ldots, x_N \) on the main diagonal, respectively.
2 System Model

We consider a MIMO–OFDM system with $N_T$ transmit antennas, $N_R$ receive antennas, and $N_c$ OFDM sub–carriers. The block diagram of the discrete–time overall transmission system in equivalent complex baseband representation is shown in Fig. 1. In the next four subsections, we introduce the models for the transmitter, the channel, the receiver, and the feedback channel.

2.1 Transmitter Processing for TD–BF

The modulated symbols $D[n], 0 \leq n < N_c$, are taken from a scalar symbol alphabet $\mathcal{A}$ and have variance $\sigma_D^2 = \mathbb{E}\{|D[n]|^2\} = 1$. The transmit symbol vector $\mathbf{x} \triangleq [x[0] \ x[1] \ldots \ x[N_c - 1]]^T$ after the IDFT operation can be represented as

$$\mathbf{x} \triangleq \mathbf{W} \mathbf{D},$$

where $\mathbf{D} \triangleq [D[0] \ D[1] \ldots D[N_c - 1]]^T$ and $\mathbf{W}$ is the unitary IDFT matrix [19], i.e., $x[k] = \frac{1}{\sqrt{N_c}} \sum_{n=0}^{N_c-1} D[n] e^{j2\pi nk/N_c}$.

At transmit antenna $n_t$ sequence $x[k]$ is filtered with a C–BFF with impulse response $g_{nt}[k]$, $0 \leq k < L_g$, $1 \leq n_t \leq N_T$, of length $L_g \leq N_c$. The resulting OFDM symbol after cyclic filtering is given by

$$\mathbf{s}_{n_t} = \mathbf{G}_{n_t} \mathbf{x},$$

where $\mathbf{G}_{n_t}$ is an $N_c \times N_c$ column–circulant matrix with first column $[g_{nt}[0] \ 0^T_{N_c-L_g}]^T$, $g_{nt}[\cdot] \triangleq [g_{nt}[0] \ g_{nt}[1] \ldots g_{nt}[L_g - 1]]^T$. We note that in practice the cyclic filtering in (2) can be implemented using the following three simple steps:

1. Add a cyclic prefix (CP) of length $L_g - 1$ to $\mathbf{x}$ to generate $\bar{\mathbf{x}} \triangleq [x[N_c - L_g + 1] \ldots x[N_c - 1]]^T$.

2. Pass the elements of $\bar{\mathbf{x}}$ through a linear filter with coefficients $g_{nt}[k], 0 \leq k < L_g$, to generate $\bar{\mathbf{s}}_{n_t} \triangleq [\bar{s}_{n_t}[0] \ \bar{s}_{n_t}[1] \ldots \bar{s}_{n_t}[N_c + L_g - 2]]^T$, where $\bar{s}_{n_t}[k] = \sum_{\kappa=0}^{L_g-1} g_{nt}[\kappa] \bar{x}[k - \kappa]$ and $\bar{x}[k], 0 \leq k < N_c + L_g - 1$, are the elements of $\bar{\mathbf{x}}$ and $\bar{x}[k] = 0$ for $k < 0$.

3. Remove the CP from $\bar{\mathbf{s}}_{n_t}$ to obtain $\mathbf{s}_{n_t} = [\bar{s}_{n_t}[L_g - 1] \ldots \bar{s}_{n_t}[N_c + L_g - 2]]^T$.

After cyclic filtering a CP is added to $\mathbf{s}_{n_t}$. We assume that the CP length is not smaller than $L - 1$, where $L$ is the length of the CIR. We note that due to the cyclic structure of $\mathbf{G}_{n_t}$, TD–BF does
not affect the length requirements of the CP, i.e., the required CP length for TD–BF is identical to that for single–antenna transmission.

2.2 MIMO Channel

We model the wireless channel as a frequency–selective, spatially and temporally correlated MIMO channel. The spatial correlations may be introduced by insufficient antenna spacing and the temporal correlations are due to transmit and receive filtering. The channel between transmit antenna $n_t$ and receive antenna $n_r$ is characterized by its impulse response $h_{n_t,n_r}[l], 0 \leq l < L$. As is typically done in the BF literature, e.g. [2]–[6], [10, 12, 13], we assume that the transmitted data is organized in frames. The channel remains constant during each frame but changes randomly between frames (block fading model).

2.3 Receiver Processing

TD–BF does not affect the processing at the receiver, i.e., standard OFDM receiver processing is applied. After CP removal the discrete–time received signal at receive antenna $n_r, 1 \leq n_r \leq N_R$, can be modeled as

$$r_{n_r} = \sum_{n_t=1}^{N_T} \bar{H}_{n_t,n_r} G_{n_t} x + n_{n_r},$$

(3)

where $\bar{H}_{n_t,n_r}$ is an $N_c \times N_c$ column–circulant matrix with first column $[h_{n_t,n_r}[0] \ldots h_{n_t,n_r}[L-1] 0_{N_c-L}]^T$ and $n_{n_r}$ is an additive white Gaussian noise (AWGN) vector whose entries $n_{n_r}[k], 0 \leq k < N_c$, are independent and identically distributed (i.i.d.) with zero mean and variance $\sigma_n^2$.

After DFT we obtain at antenna $n_r$

$$R_{n_r} = W^H r_{n_r} = \sum_{n_t=1}^{N_T} H_{n_t,n_r} G_{n_t} D + N_{n_r},$$

(4)

where $H_{n_t,n_r} \triangleq W^H \bar{H}_{n_t,n_r} W = \text{diag}\{H_{n_t,n_r}[0] \ldots H_{n_t,n_r}[N_c-1]\}$, $G_{n_t} \triangleq W^H G_{n_t} W = \text{diag}\{G_{n_t}[0] \ldots G_{n_t}[N_c-1]\}$, and $N_{n_r} \triangleq W^H n_{n_r} = [N_{n_r}[0] \ldots N_{n_r}[N_c-1]]^T$. The $N_{n_r}[n], 0 \leq n < N_c$, are i.i.d. AWGN samples with variance $\sigma_n^2$. The FD channel gains $H_{n_t,n_r}[n]$ and the
C–BFF gains $G_n[n]$ are given by

$$
H_{n,t,n}[n] \triangleq \sum_{l=0}^{L-1} h_{n_t,n_r}[l] e^{-j2\pi nl/N_c},
$$

(5)

$$
G_n[n] \triangleq \sum_{l=0}^{L_g-1} g_n[l] e^{-j2\pi nl/N_c}.
$$

(6)

Considering now the $n$th sub–carrier and assuming an $N_R$–dimensional receive combining vector

$$
C[n] \triangleq [C_1[n] \ldots C_{N_R}[n]]^T,
$$

with (4) the combined received signal can be expressed as

$$
Y[n] = C^H[n]H[n]G[n]D[n] + C^H[n]N[n], \quad 0 \leq n < N_c,
$$

(7)

where $N_R \times N_T$ matrix $H[n]$ contains $H_{n_t,n_r}[n]$ in row $n_r$ and column $n_t$, $G[n] \triangleq [G_1[n] \ldots G_{N_T}[n]]^T$, and $N[n] \triangleq [N_1[n] \ldots N_{N_R}[n]]^T$. In this paper, we assume that the receiver has perfect knowledge of $H[n], 0 \leq n < N_c$. In this case, the combining vector $C[n]$ that maximizes the signal–to–noise ratio (SNR) of $Y[n]$ is given by $C[n] = H[n]G[n]$ (maximal–ratio combining).

### 2.4 Feedback Channel

We assume that a feedback channel from the receiver to the transmitter is available, cf. Fig. 1. In the idealized case, where the feedback channel has infinite capacity, the receiver sends the unquantized C–BFF vector $g$, $g \triangleq [g_1^T \ldots g_{N_T}^T]^T$, to the transmitter (perfect CSI case). In the more realistic case, where the feedback channel can only support the transmission of $B$ bits per channel update, the receiver and the transmitter have to agree on a pre–designed C–BFF vector codebook $\mathcal{G} \triangleq \{\hat{g}_1, \hat{g}_2, \ldots, \hat{g}_N\}$ of size $N = 2^B$, where $\hat{g}_n$ is an $N_T L_g$–dimensional vector. For a given channel vector $h \triangleq [h_{11}[0] h_{11}[1] \ldots h_{11}[L-1] h_{21}[0] \ldots h_{N_T,N_R}[L-1]]^T$ the receiver determines the address $n$ of the codeword (C–BFF vector) $\hat{g}_n \in \mathcal{G}, 1 \leq n \leq N$, which maximizes the prescribed optimality criterion (maximum AMI or minimum BER). Subsequently, index $n$ is sent to the transmitter which then utilizes $g = \hat{g}_n$ for BF. Similar to [10, 12, 13] we assume that the feedback channel is error–free and has zero delay.

### 3 Maximum AMI Criterion

In this section, we optimize the C–BFFs for maximization of the AMI per sub–carrier. After rigorously formulating the optimization problem, we present a closed–form solution for $L_g = N_c$.
3.1 Formulation of the Optimization Problem

Assuming i.i.d. Gaussian input symbols $D[\cdot]$, the mutual information (in bit/s/Hz) of the $n$th sub-carrier is given by [19]

$$C[n] = \log_2 (1 + \text{SNR}[n]).$$

For maximal-ratio combining the SNR of the $n$th sub-carrier can be obtained from (7) as

$$\text{SNR}[n] = \frac{1}{\sigma_n^2} G^H[n] H^H[n] H[n] G[n].$$

We note that $G[n]$ can be expressed as

$$G[n] = F[n] g,$$

where the $n_t$th row of $N_T \times N_T L_g$ matrix $F[n]$ is given by $[0^{T_{(n_t-1)L_g}} f^T[n] 0^{T_{(N_T-n_t)L_g}}]$, $1 \leq n_t \leq N_T$, with $f[n] \triangleq [1 \ e^{-j2\pi n/N_c} \ldots e^{-j2\pi (L_g-1)n/N_c}]^T$. Therefore, the AMI per sub-carrier depends on $g$ and is given by $C = \frac{1}{N_c} \sum_{n=0}^{N_c-1} C[n]$. The optimization problem can now be formulated as

$$\max_g \sum_{n=0}^{N_c-1} C[n]$$

s.t. $g^H g = 1,$

where (12) is a transmit power constraint.

3.2 Solution of the Optimization Problem for $L_g = N_c$

Although in practice $L_g \ll N_c$ is desirable to minimize the amount of feedback, it is insightful to first consider $L_g = N_c$ since in this case a closed-form solution to the optimization problem in (11), (12) exists. In addition, the solution for $L_g = N_c$ serves as a performance upper bound for the practically relevant case $L_g < N_c$. For $L_g = N_c$ matrix $F \triangleq [F^T[0] \ldots F^T[N_c-1]]^T$ is invertible, and for a given $G \triangleq [G^T[0] \ldots G^T[N_c-1]]^T$ the C-BFF vector $g$ can be obtained from

$$g = F^{-1} G,$$
cf. (10). This means (11) and (12) are equivalent to

$$\max_G \sum_{n=0}^{N_c-1} \log_2 \left( 1 + \frac{1}{\sigma_n^2} G^H[n]H^H[n]H[n]G[n] \right)$$

(14)

s.t. \( G^H G = \mathbf{N}_c \).

(15)

The solution to this equivalent problem can be obtained as

$$G[n] = \alpha[n] E_{\text{max}}[n], \quad 0 \leq n < N_c,$$

(16)

where \( E_{\text{max}}[n] \) is that eigenvector of matrix \( H^H[n]H[n] \) which corresponds to the maximum eigenvalue \( \lambda_{\text{max}}[n] \), and \( \alpha[n] \) is obtained from

$$\alpha[n] = \sqrt{N_c \sigma_n^2 \left( \frac{1}{\lambda} - \frac{1}{N_c \lambda_{\text{max}}[n]} \right)^+},$$

(17)

where \( x^+ \triangleq \max(0, x) \) and \( \lambda \) is the solution to the waterfilling equation

$$\sigma_n^2 \sum_{n=0}^{N_c-1} \left( \frac{1}{\lambda} - \frac{1}{N_c \lambda_{\text{max}}[n]} \right)^+ = 1.$$

(18)

Once \( G[n], 0 \leq n < N_c \), has been calculated, the optimum \( g \) can be obtained from (13). Therefore, in this case, TD–BF is equivalent to ideal FD–BF with waterfilling which is not surprising since for \( L_g = N_c \) there are as many degrees of freedom in the TD as there are in the FD.

### 3.3 Solution of the Optimization Problem for \( L_g < N_c \)

For \( L_g < N_c \) the \( N_T N_c \times N_T L_g \) matrix \( F \) is not invertible, i.e., (11) and (14) are not equivalent anymore. For convenience we rewrite (11), (12) as

$$\max_g \sum_{n=0}^{N_c-1} \log_2 \left( 1 + \frac{1}{\sigma_n^2} g^H M[n] g \right)$$

(19)

s.t. \( g^H g = 1 \).

(20)

with \( N_T L_g \times N_T L_g \) matrix \( M[n] \triangleq F^H[n]H^H[n]H[n]F[n] \). Unfortunately, (19) is not a concave function, i.e., (19), (20) is not a convex optimization problem. In fact, (19) and (20) are equivalent to the maximization of a product of Rayleigh coefficients

$$\bar{L}(g) \triangleq \prod_{n=0}^{N_c-1} g^H \left( \sigma_n^2 I_{N_T L_g} + M[n] \right) \frac{g}{g^H g},$$

(21)
which is a well–known difficult mathematical problem that is not well understood for \( N_c > 1 \), cf. e.g. [20, 21].

In the remainder of this subsection, we will first consider a relaxation of (19), (20) to find a suboptimum solution and then provide a numerical algorithm for calculation of the optimum C–BFF vector.

1) Relaxation of the Optimization Problem: A popular approach for solving non–convex optimization problems is to transform the original non–convex problem into a convex one by relaxing the constraints [22]. This leads in general to a suboptimum (but often closed–to–optimum) solution for the original problem. For the problem at hand we may define a matrix \( S \triangleq gg^H \) and rewrite (19), (20) as

\[
\begin{align*}
\max_S & \quad \sum_{n=0}^{N_c-1} \log_2 \det \left( I_{N_R} + \frac{1}{\sigma_n^2} H[n]F[n]SF^H[n]H^H[n] \right) \\
\text{s.t.} & \quad \text{trace}\{S\} \leq 1, \\
& \quad S \succeq 0, \\
& \quad \text{rank}\{S\} = 1,
\end{align*}
\]

where \( S \succeq 0 \) means that \( S \) is a positive–semidefinite matrix. The equivalent optimization problem in (22)–(25) is still non–convex due to the rank condition in (25) but can be relaxed to a convex problem by dropping this rank condition. The resulting relaxed problem is a convex semidefinite programming (SDP) problem which can be solved with standard algorithms, cf. [22]. If the \( S \) found by this procedure has rank one, the corresponding \( g \) is also the solution to the original, non–convex problem. On the other hand, if the optimum \( S \) does not have rank one, the eigenvector of \( S \) corresponding to its maximum eigenvalue can be used as (suboptimum) approximate solution to the original non–convex problem.

Unfortunately, the complexity of the relaxed optimization problem strongly depends on \( N_c \), and for medium numbers of sub–carriers (e.g. \( N_c \geq 64 \)) standard optimization software takes a very long time to find the optimum \( S \). Therefore, this relaxation approach is most useful for the practically less relevant case when the number of sub–carriers is small (e.g. \( N_c < 64 \)).

2) Modified Power Method (MPM): The Lagrangian of (19), (20) can be formulated as

\[
L(g) = \sum_{n=0}^{N_c-1} \log_2 \left( 1 + \frac{1}{\sigma_n^2} g^H M[n]g \right) - \mu g^H g,
\]

where \( \mu \) is a Lagrange multiplier.
where $\mu$ denotes the Lagrange multiplier. The optimum C–BFF vector has to fulfill $\partial L(g)/\partial g^* = 0_{NT \times L_g}$, which leads to the non–linear eigenvalue problem

$$
\left[ \sum_{n=0}^{N_c-1} \frac{M[n]}{\sigma_n^2 + g^H M[n] g} \right] g = \mu g.
$$

(27)

For very low SNRs (i.e., $\sigma_n^2 \to \infty$) the optimum C–BFF vector can be obtained from (27) as that unit–norm eigenvector of $\sum_{n=0}^{N_c-1} M[n]$ which corresponds to the maximum eigenvalue of that matrix, i.e., a closed–form solution exists for this special case. Numerically, the relevant eigenvector can be efficiently calculated with the so–called Power Method [23]. Unfortunately, the low SNR solution for $g$ does not yield a good performance for finite, practically relevant SNRs.

However, the applicability of the Power Method to the low SNR problem motivates us to consider a Modified Power Method (MPM) for solving the original problem in (27) recursively. This MPM is summarized in Table 1. Because of its involved nature, we are not able to prove global convergence of the MPM to the maximum AMI. However, our simulations have shown that the choice of the initial vector $g_0$ is not critical and the algorithm always achieved very similar AMI values for different random $g_0$. Furthermore, for those cases where the relaxation method discussed in 1) found the solution to the original problem (19), (20), i.e., $S$ had rank one, the solution found with the MPM achieved the same AMI. The convergence time of the MPM depends on $L_g$ and $N_T$. For example, for a termination constant of $\epsilon = 10^{-4}$ and $N_T = 2$ the MPM typically terminated after less than 20 and 150 iterations for $L_g = 2$ and $L_g = 4$, respectively.

It is interesting to note that formally the proposed MPM is somewhat similar to the MPM introduced in [15] for BFF calculation for single–carrier transmission and decision–feedback equalization (DFE) at the receiver. The reason for this similarity is that the SNR at the input of the DFE, which was the optimality criterion in [15], is mathematically similar to the AMI for MIMO OFDM, which is the optimality criterion in this paper.

### 4 Minimum BER Criterion

The main criterion considered for C–BFF optimization in this section is the BER averaged over all sub–carriers. However, we will also consider the minimization of the maximum sub–carrier BER for optimization of the C–BFFs. Besides the additional insight that this second BER criterion offers, it also provides a useful starting point for numerical computation of the minimum average BER.
C–BFF filters, cf. Section 4.3.

4.1 Formulation of the Optimization Problems

While closed–form expressions for the BER or/and symbol error rate exist for most regular signal constellations such as $M$–ary quadrature amplitude modulation ($M$–QAM) and $M$–ary phase–shift keying ($M$–PSK), these expressions are quite involved which is not desirable for C–BFF optimization. Therefore, we adopt here the simple yet accurate BER approximations from [24], which allow us to express the approximate BER of the $n$th sub–carrier as

$$\text{BER}[n] \approx c_1 \exp (-c_2 \text{SNR}[n]),$$

(28)

where the $n$th sub–carrier SNR is defined in (9) and $c_1$ and $c_2$ are modulation dependent constants. For example, for square $M$–QAM we have $c_1 = 0.2$ and $c_2 \triangleq 3/[2(M - 1)]$ [24].

1) Average BER Criterion: The (approximate) average BER is given by $\text{BER} = \frac{1}{N_c} \sum_{n=0}^{N_c-1} \text{BER}[n]$. Consequently, the minimum average BER optimization problem can be formulated as

$$\min_{g} \sum_{n=0}^{N_c-1} \text{BER}[n]$$

s.t. $g^H g = 1$.

(29)

(30)

2) Max–Min Criterion: Since the exponential function is monotonic, we observe from (28) that minimizing the maximum sub–carrier BER is equivalent to maximizing the minimum sub–carrier SNR. The resulting max–min problem becomes

$$\max_{g} \min_{\forall n} \text{SNR}[n]$$

s.t. $g^H g = 1$.

(31)

(32)

Since for high SNR, the maximum sub–carrier BER dominates the average BER, we expect that in this case both optimization criteria lead to similar performances.

4.2 Solution of the Optimization Problems for $L_g = N_c$

For the solution of the optimization problem we exploit again the fact that for $L_g = N_c$ matrix $F$ is invertible, i.e., for a given $G$ the C–BFF vector $g$ can be obtained from (13).
1) Average BER Criterion: Eq. (13) implies that (29) and (30) are equivalent to
\[
\begin{align*}
\min_{G} & \quad \sum_{n=0}^{N_c-1} \exp \left( -\frac{c_2}{\sigma_n^2} g^H[n] M[n] M[n] g[n] \right) \\
\text{s.t.} & \quad G^H G = N_c.
\end{align*}
\tag{33}
\]
Formulating (33) and (34) as a Lagrangian, it can be shown that the optimum \(G[n]\) is again proportional to \(E_{\text{max}}[n]\), i.e., (16) is still valid. However, now \(\alpha[n]\) in (16) is given by
\[
\alpha[n] = \sqrt{\frac{\sigma_n^2}{c_2 \lambda_{\text{max}}[n]} \left[ \ln \left( \frac{\lambda_{\text{max}}[n]}{\lambda} \right) \right]^+},
\tag{35}
\]
where \(\lambda\) is the solution to the waterfilling problem
\[
\frac{\sigma_n^2}{c_2 N_c} \sum_{n=0}^{N_c-1} \left[ \ln \left( \frac{\lambda_{\text{max}}[n]}{\lambda} \right) \right]^+= 1.
\tag{36}
\]
For high SNR, i.e., \(\sigma_n^2 \ll 1, \lambda_{\text{max}}[n] > \lambda, 0 \leq n < N_c\), holds and the sub-carrier BER can be calculated as \(\text{BER}[n] = c_1 \lambda / \lambda_{\text{max}}[n]\), where \(\lambda = \exp((\sum_{n=0}^{N_c-1} \ln(\lambda_{\text{max}}[n]) / \lambda_{\text{max}}[n]) - c_2 N_c / \sigma_n^2) / \left[ \sum_{n=0}^{N_c-1} 1 / \lambda_{\text{max}}[n] \right]\), cf. (9), (28), (35), and (36). This means for high SNR the sub-carrier BER is inversely proportional to the maximum sub-carrier eigenvalue \(\lambda_{\text{max}}[n]\).

2) Max–Min Criterion: Exploiting (13) also for the max–min criterion, it can be shown that the optimum solution has again the general form given by (16) with
\[
\alpha[n] = \left( \frac{\lambda_{\text{max}}[n]}{N_c} \sum_{n=0}^{N_c-1} \frac{1}{\lambda_{\text{max}}[n]} \right)^{-\frac{1}{2}}.
\tag{37}
\]
This means that for the max–min criterion and \(L_g = N_c\) all sub-carrier SNRs are equal to \(\text{SNR}[n] = N_c / (\sigma_n^2 \sum_{n=0}^{N_c-1} 1 / \lambda_{\text{max}}[n])\). Therefore, in contrast to the minimum average BER solution, for the max–min solution all sub–carriers have identical BERs.

4.3 Solution of the Optimization Problems for \(L_g < N_c\)

Since \(F\) is not invertible for \(L_g < N_c\), we present alternative approaches for solving the BER optimization problems in this subsection.

1) Average BER Criterion: For convenience we rewrite (29), (30) as
\[
\begin{align*}
\min_{g} & \quad \sum_{n=0}^{N_c-1} \exp \left( -\frac{c_2}{\sigma_n^2} g^H M[n] g \right) \\
\text{s.t.} & \quad g^H g = 1.
\end{align*}
\tag{38}
\]
\[
\begin{align*}
\min_{g} & \quad \sum_{n=0}^{N_c-1} \exp \left( -\frac{c_2}{\sigma_n^2} g^H M[n] g \right) \\
\text{s.t.} & \quad g^H g = 1.
\end{align*}
\tag{39}
\]
where $M[n]$ was defined in Section 3. Unfortunately, (38) is not a convex function, i.e., (38), (39) is not a convex optimization problem. Therefore, similar to Section 3.3, we first pursue a relaxation approach to find a suboptimum solution to the problem. In particular, letting again $S = gg^H$ we can rewrite (38), (39) as

$$\min_S \sum_{n=0}^{N_c-1} \exp \left(-\frac{c_2}{\sigma_n^2} \text{trace} \left( H[n]F[n]SF^H[n]H^H[n]\right) \right)$$

s.t. $\text{trace}\{S\} \leq 1$, $S \succeq 0$, $\text{rank}\{S\} = 1$. (40)–(43)

The equivalent optimization problem (40)–(43) is still non-convex due to the rank condition in (43) but can be relaxed to a convex SDP problem by dropping this rank condition. The resulting convex problem has similar properties as the relaxed convex problem in the AMI case. In particular, a (possibly suboptimum) solution to the original minimum BER problem is given by that eigenvector of the optimum $S$ which corresponds to its maximum eigenvalue. Furthermore, the complexity of the relaxed problem again strongly depends on $N_c$, and becomes prohibitive for a moderate number of sub-carriers (e.g. $N_c \geq 64$).

2) Max–Min Criterion: For the max–min criterion, we may rewrite (31), (32) as

$$\max g \min \forall n g^H M[n] g$$

s.t. $g^H g = 1$. (44)–(45)

which constitutes a quadratic objective quadratic constraint (QOQC) NP-hard problem [25]. This problem can be restated in equivalent form as [25]

$$\max t$$

s.t. $\text{trace}\{S\} \leq 1$, $\text{trace}\{M[n]S\} \geq t, \forall n$, $S \succeq 0$, $\text{rank}\{S\} = 1$. (46)–(50)

By dropping the rank condition (50) the optimization problem (46)–(50) can be relaxed to an SDP problem. Unlike the SDP problems for the maximum AMI and the minimum average BER criteria,
the complexity of the SDP problem (46)–(49) is dominated by $L_g$ and not by $N_c$. Since we are mainly interested in the case where $L_g \ll N_c$, the relaxed problem for the max–min criterion can be solved even for large $N_c$ (e.g. $N_c \geq 256$) using standard software.

3) Gradient Algorithm: Unfortunately, for both relaxed optimization problems presented in this section the resulting $S$ has a high rank most of the time, and the dominant eigenvector of $S$ is a sub–optimum solution which may entail a significant performance degradation. However, a gradient algorithm (GA) may be used to recursively improve the initial C–BFF vector found through relaxation. In Table 1, we provide the GA for the average BER criterion since this is our primary BER–related criterion. However, if the average BER SDP problem (40)–(42) cannot be solved since the number of sub–carriers $N_c$ is too large, we use the solution found for the max–min SDP problem (46)–(49) for initialization of the GA.

We note that the speed of convergence of the GA depends on the adaptation step size $\delta_i$, which has to be empirically optimized, cf. e.g. [26] for guidelines for step size optimization of GAs. However, in practice, the speed of convergence of the GA is not critical, since in the realistic finite–rate feedback case, the GA is only used to find the C–BFF codebook, which is done off–line.

5 Finite–Rate Feedback and Comparison

In this section, we briefly discuss codebook design for finite–rate feedback channels based on the GVQ algorithm in [15]. Furthermore, we also compare TD–BF with interpolation–based FD–BF [10, 12, 13].

5.1 Finite–Rate Feedback Case

Vector quantization can be used to design a codebook $\mathcal{G}$ of size $N$ for the finite–rate feedback channel case, cf. Section 2.4. Here, we adopt the GVQ algorithm introduced in [15]. For this purpose a set $\mathcal{H} \triangleq \{h_1, h_2, \ldots, h_T\}$ of $T$ channel vectors $h_n$ is generated. Thereby, the $N_T N_R L$–dimensional vector $h_n$ contains the CIR coefficients of all $N_T N_R$ CIRs of the $n$th MIMO channel realization. For each of these channel realizations the corresponding C–BFF vector $g = \bar{g}_n$ is generated using the MPM (maximum AMI criterion) or the GA (minimum BER criterion), cf. Table 1, yielding the set $\mathcal{G}_T \triangleq \{\bar{g}_1, \bar{g}_2, \ldots, \bar{g}_T\}$. The vector quantizer can then be represented as a function $Q: \mathcal{G}_T \rightarrow \mathcal{G}$. Ideally, this function is optimized for minimization of the mean quantization
error

\[
\text{MQE} \triangleq \frac{1}{T} \sum_{i=1}^{T} d(Q(\bar{g}_i), \bar{g}_i),
\]

(51)

where \(d(\hat{g}_m, \bar{g}_i)\) denotes the distortion caused by quantizing \(\bar{g}_i \in G_T\) to \(\hat{g}_m \in G\). The distortion measure depends on the optimization criterion and is given by

\[
d(\hat{g}_m, \bar{g}_i) \triangleq - \sum_{n=0}^{N_c-1} \log_2 \left( 1 + \text{SNR}(\hat{g}_m, h_i)[n] \right)
\]

(52)

and

\[
d(\hat{g}_m, \bar{g}_i) \triangleq \sum_{n=0}^{N_c-1} \exp \left( -c^2 \text{SNR}(\hat{g}_m, h_i)[n] \right)
\]

(53)

for the maximum AMI and the minimum BER criterion, respectively. Here, \(\text{SNR}(\hat{g}_m, h_i)[n]\) is defined in (9) and the subscripts indicate that \(G[n]\) and \(H[n]\) have to be calculated for \(\hat{g}_m\) and \(h_i\), respectively. With this definition for the distortion measure the GVQ algorithm given in [15, Section IV] can be straightforwardly applied to find \(G\). We omit here further details and refer the interested reader to [14, 15] and references therein.

Once the off-line optimization of the codebook is completed, \(G\) is conveyed to the transmitter and the receiver. For a given channel realization \(h\) the receiver selects that C–BFF \(\hat{g}_m \in G\) which minimizes the distortion measure (52) [AMI criterion] or (53) [BER criterion] and feeds back the corresponding index to the transmitter.

### 5.2 Comparison with FD–BF

We compare TD–BF with FD–BF in terms of feedback requirements and computational complexity.

1) Feedback Requirements: The required number of complex feedback symbols \(S\) for TD–BF, interpolation–based FD–BF [10, 12, 13], and ideal FD–BF are summarized in Table 2, where \(K\) denotes the cluster size in interpolation–based FD–BF [10], i.e., \(N_c/K\) is the number of sub–carriers for which CSI is assumed to be available at the transmitter. Table 2 illustrates the fundamental difference between TD–BF and FD–BF. While for all FD–BF schemes the number of complex feedback symbols is proportional to the number of sub–carriers \(N_c\), it is independent of \(N_c\) for TD–BF.

2) Computational Complexity: The calculation of the C–BFFs and the GVQ–based codebook design for the proposed TD–BF scheme are more involved than the calculation of the BF weights
and the codebook design method adopted in [10, 12, 13] for FD–BF, respectively. However, in practice, codebook design is done very infrequently. In fact, if the statistical properties of the MIMO channel do not change (as is typically the case in downlink scenarios), the codebook has to be designed only once. Therefore, in practice, the computational effort for C–BFF calculation and codebook design can be neglected. The interpolation of BF weights in FD–BF has to be done in every frame. The interpolation complexity is generally proportional to $N_c$ but strongly depends on the interpolator used. For example, modified spherical interpolation requires a grid search whereas Grassmannian and geodesic interpolation do not. Assuming a codebook of size $N$ selecting the beamformer index at the receiver requires evaluation of $N$ and $NN_c/K$ distortion measures for TD–BF and interpolation–based FD–BF, respectively. However, a fair quantitative comparison of the associated complexities is difficult since the required $N$ to achieve a similar performance may be very different in both cases.

Similar to [16] we assume that the inverse IDFTs and the BF itself dominate the complexities of TD–BF and FD–BF. As is customary in the literature, we adopt the required number of complex multiplications as measure for complexity and assume that the (I)DFT is implemented as a (inverse) fast Fourier transform ((I)FFT). Following [10] we assume that one (I)FFT operation requires $N_c \log_2(N_c)/2$ complex multiplications. Therefore, since FD–BF requires $N_T$ IFFT operations and $N_T N_c$ complex multiplications for BF, a total of

$$M_{FD} = \frac{N_T N_c}{2} \log_2(N_c) + N_T N_c$$

(54)

compact multiplications are obtained. In contrast, assuming a straightforward TD implementation of convolution,

$$M_{TD} = \frac{N_c}{2} \log_2(N_c) + L_g N_T N_c$$

(55)

compact multiplications are required for TD–BF. A comparison of $M_{FD}$ and $M_{TD}$ shows that the complexity of TD–BF is lower than that of FD–BF if

$$L_g < \frac{N_T - 1}{2N_T} \log_2(N_c) + 1.$$  

(56)

For example, assuming $N_c = 512$ sub–carriers and $N_T = 2$, $3 \leq N_T < 9$, and $N_T \geq 9$ TD–BF requires a lower complexity than FD–BF for $L_g \leq 3$, $L_g \leq 4$, and $L_g \leq 5$, respectively. Our results in Section 6 show that generally a high performance can be achieved with these small values of $L_g$. 
6 Numerical and Simulation Results

In this section, we present numerical and simulation results for the AMI and the BER of MIMO–OFDM with TD–BF. Besides the uncoded BER, we also consider the BER of a coded system employing the popular bit–interleaved coded modulation (BICM) concept, since the combination of BICM and OFDM has been adopted in various recent standards, cf. e.g. [7]. However, first we briefly discuss the parameters used in our simulations.

6.1 Simulation Parameters

Throughout this section we consider a MIMO–OFDM system with \( N_T = 2 \) or \( N_T = 3 \) transmit antennas, \( N_R = 1 \) receive antenna, and \( N_c = 512 \) OFDM sub–carriers. If BICM is employed, the data bits are encoded with the quasi–standard \((171,133)_8\) convolutional code of rate \( R_c = 1/2 \), possibly punctured, interleaved, and Gray mapped to the data symbols \( D[·] \) [7, 9]. At the receiver standard Viterbi soft decoding is applied. For all BER results 16–QAM was used. For practical relevance we adopted for our simulations the IEEE 802.11n Channel Model B with \( L = 9 \) assuming a carrier frequency of 2.5 GHz and a transmit antenna spacing of \( \lambda_0/2 \), where \( \lambda_0 \) is the wavelength [27]. All simulation results were averaged over 20,000 independent channel realizations. For \( L_g < N_c \) the C–BFF vectors were calculated with the algorithms given in Table 1. The all–ones vector and the solution of the relaxed max–min problem were used for initialization of the MPM and the GA, respectively. For \( L_g = N_c \) (equivalent to ideal FD–BF) the closed–form solutions for the C–BFF provided in Sections 3.2 and 4.2 were used. For the finite–rate feedback case the C–BFF vector codebook was generated with the GVQ algorithm discussed in Section 5.1 based on a training set of \( T = 1000 \) independent channel realizations.

6.2 Maximum AMI Criterion

We first consider TD–BF with AMI–optimized C–BFFs and compare its performance with that of FD–BF with modified spherical (MS–FD–BF) [10] and geodesic (GD–FD–BF) [13] interpolation, respectively. We note that in [10] an AMI criterion is used for interpolator optimization, whereas the interpolator optimization in [13] is not directly tied to the AMI or BER. Throughout this subsection \( N_T = 2 \) is valid.

Fig. 2 shows the AMI per sub–carrier vs. \( E_s/N_0 \) (\( E_s \): energy per received symbol, \( N_0 \): power
spectral density of underlying continuous–time passband noise process) for the proposed TD–BF, MS–FD–BF, and GD–FD–BF for the case of perfect CSI at the transmitter. To facilitate a fair comparison between TD–BF with C–BFFs of length $L_g$ and FD–BF with cluster size $K$, we have included in the legend of Fig. 2 the respective required number of complex feedback symbols $S$, cf. Table 2. As can be observed, TD–BF provides a better performance/feedback trade–off than interpolation–based FD–BF. For example, TD–BF with $S = 2$ ($L_g = 1$) outperforms MS–FD–BF and GD–FD–BF with $S = 6$ ($K = 256$) and $S = 4$ ($K = 256$), respectively. MS–FD–BF with $S = 24$ ($K = 64$) is necessary to outperform TD–BF with $S = 8$ ($L_g = 4$) which performs only less than 0.5 dB worse than ideal FD–BF.

In Fig. 3, we consider the AMI of TD–BF with finite–rate feedback channel as a function of the number of feedback bits $B$ (solid lines) for an SNR of $10 \log_{10}(E_s/N_0) = 10$ dB. For comparison, Fig. 3 also contains the AMI for TD–BF with perfect CSI (dashed lines). For $B = 0$ the codebook has just one entry and no feedback is required. As can be observed from Fig. 3, finite–rate feedback TD–BF approaches the performance of the perfect CSI case as $B$ increases. Furthermore, as expected, the number of feedback bits required to approach the perfect CSI case increases with increasing $L_g$.

Fig. 4 shows the BERs of a coded MIMO–OFDM system ($R_c = 1/2$) employing TD–BF, MS–FD–BF, and GD–FD–BF vs. $E_b/N_0$, where $E_b$ denotes the average energy per information bit. Perfect CSI is assumed at the transmitter. As expected from the AMI in Fig. 2 for similar or identical $S$ TD–BF outperforms the FD–BF schemes. For example, at a BER of $10^{-4}$ TD–BF with $S = 6$ yields performance gains of more than 1.7 dB and 0.7 dB over MS–FD–BF with $S = 6$ and GD–FD–BF with $S = 8$, respectively.

In Fig. 5, we show the BER of a coded MIMO–OFDM system ($R_c = 1/2$) employing TD–BF with finite–rate feedback and $L_g = 2$. A C–BFF vector codebook optimized for $10 \log_{10}(E_b/N_0) = 10$ dB was used for all $E_b/N_0$ values. Fig. 5 shows that TD–BF with $L_g = 2$ and $B = 7$ feedback bits outperforms TD–BF with $L_g = 1$ and perfect CSI and closely approaches the performance of TD–BF with $L_g = 2$ and perfect CSI.

Finally, we note that it is very difficult to fairly compare TD–BF and interpolation–based FD–BF for the finite–rate feedback case as the performance of both schemes strongly depends on the adopted quantization scheme. For example, the numbers of feedback bits considered in [10, 12, 13] are much higher than those in this paper but can be reduced with the method in [11].
Since, in this paper, our main interest is not the comparison of different quantization schemes, but the investigation of the fundamental differences between TD–BF and FD–BF, we do not show simulation results for finite–rate feedback FD–BF.

6.3 Minimum BER Criterion

Now, we shift our attention to TD–BF with BER–optimized C–BFFs. $N_T = 2$ is still valid.

Assuming perfect CSI we show in Fig. 6 the average BERs for the average BER criterion and the max–min criterion, respectively. As expected, for $L_g = N_c$ (ideal FD–BF) the average BER criterion leads to a lower average BER than the max–min criterion. However, the difference between both criteria is less than 1 dB at BER $= 10^{-3}$. For $L_g = 1$ and $L_g = 5$ we show the average BER obtained for the relaxed max–min criterion. As can be observed the performance is quite poor in this case and a comparison with single–antenna transmission ($N_T = 1$) suggests that the diversity offered by the second antenna is not exploited. However, Fig. 6 clearly shows that this diversity can be exploited if the GA is used to improve the relaxed max–min solution. In this case, the BER approaches the BER of the limiting $L_g = N_c$ case as $L_g$ increases. For example, for $L_g = 5$ the performance loss compared to $L_g = N_c = 512$ is less than 1.5 dB at BER $= 10^{-3}$.

In Fig. 7, we investigate the effect of a finite–rate feedback channel on the average BER. In particular, we show the average BER as a function of the number of feedback bits $B$ (solid lines) for an SNR of $E_b/N_0 = 10$ dB. For comparison, Fig. 7 also contains the BERs for perfect CSI (dashed lines). As can be observed, finite–rate feedback BF approaches the performance of the perfect CSI case as $B$ increases. Furthermore, as expected, the number of feedback bits required to approach the perfect CSI case increases with increasing $L_g$. Therefore, smaller $L_g$ are preferable if only few feedback bits can be afforded.

In Fig. 8 we show the average BER for uncoded and coded ($R_c = 1/2$) transmission with finite–rate feedback TD–BF and TD–BF with perfect CSI, respectively. C–BFFs of length $L_g = 2$ were used and the C–BFF vector codebook was optimized for $E_b/N_0 = 10$ dB. Interestingly, for coded transmission significantly fewer feedback bits are required to approach the performance of the perfect CSI case than for uncoded transmission. For example, for BER $= 10^{-4}$ and $B = 3$ feedback bits the performance loss compared to perfect CSI is 0.45 dB and 3.8 dB for coded and uncoded transmission, respectively.
6.4 Comparison of Maximum AMI and Minimum BER Criteria

In Fig. 9, we compare the average BERs of uncoded and coded MIMO–OFDM systems employing minimum average BER (dashed lines) and maximum AMI (solid lines) TD–BF, respectively. We assume perfect CSI, \(N_T = 3\), \(L_g = 2, 4\), and \(N_c\) (ideal FD–BF). As one would expect, for uncoded transmission the minimum average BER criterion yields a significantly better performance than the maximum AMI criterion. However, although the employed convolutional codes are by no means capacity achieving, for the coded case the maximum AMI criterion yields a lower BER than the minimum average BER criterion.

7 Conclusions

In this paper, we have proposed a novel TD approach to BF in MIMO–OFDM systems. The C–BFFs have been optimized for maximization of the AMI and for minimization of the average BER. For both criteria we have provided efficient numerical algorithms for calculation of the C–BFFs for the practically relevant case where the length of the C–BFFs \(L_g\) is much smaller than the number of OFDM sub–carriers \(N_c\). For the case of a finite–rate feedback channel a GVQ algorithm has been presented for codebook design. Numerical and simulation results for the IEEE 802.11n Channel Model B have shown that TD–BF achieves a more favorable performance/feedback rate trade–off than interpolation–based FD–BF schemes. In particular, in contrast to FD–BF, for TD–BF the number of complex feedback symbols to be conveyed to the transmitter is independent of \(N_c\). Furthermore, while the minimum average BER criterion is preferable for uncoded MIMO–OFDM systems, the maximum AMI criterion yields a better performance if BICM with convolutional codes is employed.

References


Tables and Figures:

Table 1: Calculation of the optimum C–BFFs $g$ for the maximum AMI (MPM) and the minimum average BER (GA) criterion, respectively. Termination constant $\epsilon$ has a small value (e.g. $\epsilon = 10^{-4}$). $i$ denotes the iteration and $\delta_i$ is the adaptation step size necessary for the GA.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Let $i = 0$ and initialize the C–BFF vector with some $g_0$ fulfilling $g_0^H g_0 = 1$.</td>
</tr>
<tr>
<td>2</td>
<td>Update the C–BFF vector:</td>
</tr>
<tr>
<td>AMI:</td>
<td>$\tilde{g}<em>{i+1} = \left[ \sum</em>{n=0}^{N_c-1} \frac{M[n]}{\sigma_n^2 + g_i^H M[n] g_i} \right] g_i$</td>
</tr>
<tr>
<td>BER:</td>
<td>$\tilde{g}<em>{i+1} = g_i + \delta_i \left[ \sum</em>{n=0}^{N_c-1} \exp \left( -\frac{c_2}{\sigma_n^2} g_i^H M[n] g_i \right) M[n] \right] g_i$</td>
</tr>
<tr>
<td>3</td>
<td>Normalize the C–BFF:</td>
</tr>
<tr>
<td></td>
<td>$g_{i+1} = \frac{\tilde{g}<em>{i+1}}{\sqrt{\tilde{g}</em>{i+1}^H \tilde{g}_{i+1}}}$</td>
</tr>
<tr>
<td>4</td>
<td>If $1 -</td>
</tr>
<tr>
<td>5</td>
<td>$g_{i+1}$ is the desired C–BFF vector.</td>
</tr>
</tbody>
</table>

Table 2: Feedback Requirements for TD–BF, ideal FD–BF, and FD–BF with modified spherical (MS), Grassmannian (GS), and geodesic (GD) interpolation.

<table>
<thead>
<tr>
<th>BF Scheme</th>
<th>Number of Complex Feedback Symbols per Frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal FD–BF</td>
<td>$S = N_c N_T$</td>
</tr>
<tr>
<td>MS–FD–BF [10]</td>
<td>$S = \frac{N_c}{K} (N_T + 1)$</td>
</tr>
<tr>
<td>Proposed TD–BF</td>
<td>$S = N_T L_g$</td>
</tr>
</tbody>
</table>
Figure 1: MIMO–OFDM system with TD–BF. P/S: Parallel–to–serial conversion. S/P: Serial–to–parallel conversion. CE: Channel estimation.
Figure 2: AMI of TD–BF (AMI criterion), MS–FD–BF [10], and GD–FD–BF [13] with perfect CSI. $N_T = 2$, $N_R = 1$, $N_c = 512$, and IEEE 802.11n Channel Model B. For comparison the AMIs for ideal FD–BF and single–input single–output (SISO) transmission ($N_T = 1$, $N_R = 1$) are also shown.

Figure 3: AMI of TD–BF (AMI criterion) vs. number of feedback bits $B$ per channel update. $N_T = 2$, $N_R = 1$, $N_c = 512$, and IEEE 802.11n Channel Model B.
Figure 4: BER of coded MIMO–OFDM system with TD–BF (AMI criterion), MS–FD–BF [10], and GD–FD–BF [13]. Perfect CSI, $N_T = 2$, $N_R = 1$, $N_c = 512$, $R_c = 1/2$, and IEEE 802.11n Channel Model B. For comparison the BERs for ideal FD–BF and SISO transmission ($N_T = 1$, $N_R = 1$) are also shown.

Figure 5: BER of coded MIMO–OFDM system with TD–BF (AMI criterion). Perfect CSI (dashed lines) and finite–rate feedback channel (solid lines), $N_T = 2$, $N_R = 1$, $N_c = 512$, $R_c = 1/2$, and IEEE 802.11n Channel Model B. For comparison the BER for SISO transmission ($N_T = 1$, $N_R = 1$) is also shown.
Figure 6: Average BER of uncoded MIMO–OFDM system with TD–BF. Minimum average BER criterion (solid lines) and max–min criterion (dashed lines), perfect CSI, $N_T = 2$, $N_R = 1$, $N_c = 512$, and IEEE 802.11n Channel Model B. For comparison the BERs for ideal FD–BF and SISO transmission ($N_T = 1$, $N_R = 1$) are also shown.

Figure 7: Average BER of uncoded MIMO–OFDM system with TD–BF (average BER criterion) vs. number of feedback bits $B$ per channel update. GA was used for C–BFF optimization. $N_T = 2$, $N_R = 1$, $N_c = 512$, and IEEE 802.11n Channel Model B.
Figure 8: Average BER of uncoded and coded MIMO–OFDM system with TD–BF (average BER criterion). GA was used for C–BFF optimization. Perfect CSI (bold lines) and finite–rate feedback channel, $N_T = 2$, $N_R = 1$, $N_c = 512$, and IEEE 802.11n Channel Model B.

Figure 9: Average BER of uncoded and coded MIMO–OFDM system employing TD–BF with perfect CSI. Average BER criterion (dashed lines) and AMI criterion (solid lines), $N_T = 3$, $N_R = 1$, $N_c = 512$, and IEEE 802.11n Channel Model B.